Homework #7

Problem 1: Numerical Solution of 1st order ODE

Consider the following ODE:

$$\frac{dy}{dt} = -y \text{ with initial condition } y(0) = 5.$$

a) Use a 4th order Runge-Kutta method to solve the ODE. Use h = 0.1 and compute the first two steps. Show all your work (including computation of the k_i).
b) Now code the 4th order Runge-Kutta method in MATLAB for this problem. Use h = 0.1 from t = 0 until t = 2. Present all intermediate steps (y_0, y_1, y_2, etc.).

Problem 2: Numerical Solution of 2nd order ODE

Consider the following ODE:

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0 \text{ with initial conditions } y(0) = 5,\ y'(0) = -1.$$

Numerically solve this problem using a 4th order Runge-Kutta method.

a) Transform the system into canonical form.
b) Use a 4th order Runge-Kutta method to solve the ODE. Use h = 0.1 and compute the first two steps. Show all your work (including computation of the k_i).
c) Use ODE45 in MATLAB to solve the ODE from t = 0 until t = 2. Plot the solution.

Problem 3: Numerical Solution of 2nd order ODE

Consider the following ODE:

$$\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y = 0 \text{ with initial conditions } y(0) = 10,\ y'(0) = -1.$$

Numerically solve this problem using the Euler predictor-corrector method from t = 0 until t = 2 using h = 0.1. Plot the solution.

Problem 4: State Transition Matrix

You are given the following system of linear ODEs:

$$\frac{dx}{dt} = Ax \quad \text{with initial conditions} \quad x(0) = \begin{pmatrix} x_1(0) & x_2(0) & x_3(0) \end{pmatrix}^T = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^T$$

and $A = \begin{pmatrix} -1 & 1 & 2 \\ 0 & -2 & 3 \\ 0.2 & 0 & -3 \end{pmatrix}$

a) Compute the eigenvalues and normalized eigenvectors of the A matrix.
b) State the individual components of the state transition matrix for this specific case.
c) Evaluate x(1) using the expression from part b).