PH320: Condensed Matter Physics II

Centre for Condensed Matter Theory, Physics Department, IISc Bangalore
Semester I, 2015–2016

PROBLEM SET 5, DEADLINE: NOV. 17, 2015

5/1. **(C/T) More second quantization:** Consider particles moving in 3D free space with two-body interactions. Find a second quantized expression for the particle current operator.

5/2. **(C/T) Density correlation function:** Consider spinless fermions hopping on a simple 1D tight binding chain with filling f. Using techniques of second quantization, find an expression $\langle n_i n_j \rangle$ where i and j are site indices at zero temperature.

5/3. **(C/T) Density response:** For the free Fermi gas in d-dimensions with density ρ_0 at zero temperature, find the density response function $\chi_0(\mathbf{q}, \omega)$. Make contour plots of the imaginary part of χ_0 in the $\omega - |\mathbf{q}|$ plane for $d = 1, 2$ and 3.

5/4. **(C) Screening (Thomas-Fermi theory):** Consider a 3D jellium of spinless fermions (uniform positive background and a gas of negatively charged fermions) of density ρ_0. A test charge Z is introduced at the origin. Following the discussion in the class, show the effective potential of this test charge has a Yukawa form $e^{-r/\lambda}/r$. Obtain an expression for λ in terms of ρ_0.

5/5. **(C) RPA:** Recall our in-class discussion of RPA of the 3D jellium. Carefully, reproduce the arguments that leads to

$$\chi(\mathbf{q}, \omega) = \frac{\chi_0(\mathbf{q}, \omega)}{1 - U(\mathbf{q})\chi_0(\mathbf{q}, \omega)}$$

where $U(\mathbf{q})$ is the Fourier transform of the Coulomb interaction. Focus on zero temperature.

(a) Find an expression for the dielectric function $\varepsilon(\mathbf{q}, \omega)$. Show that this result is consistent with the Thomas Fermi theory.

(b) Find the poles of $\chi(\mathbf{q}, \omega)$ and obtain the dispersion of the plasmon modes. Note that for large enough $\mathbf{q}$, the plasmon modes become damped.

(c) Use classical physics to obtain the frequency of the plasmon modes at small $\mathbf{q}$ (compared to what?).

———— ⋆ ————