Mathematics borrows from everyday language

Jim Thorpe explores everyday and mathematical language and hints at classroom consequences.

Mathematicians, claimed Goethe, are like Frenchmen:
whatever you say to them, they translate it into their own language, and forthwith it means something entirely different.

Among my endeavours to support students studying Open University modules ME627 *Developing geometric thinking* and MU123 *Discovering mathematics*, I found myself reflecting on how everyday words also used in mathematics may obstruct, or, contribute to learning and I contemplated potential confusion emanating from dual or overlapping meanings. Not only words, the mixture of informal and technical talk may also be a stop or step to learning.

My interaction with students often starts with their email query. Lacking the quick-fire repartee of the classroom, my replies must at least implicitly invite dialogue. Students may one-sidedly terminate an email exchange; no captive audience, they may ignore me when they choose.

Miss-takes (miss-gives?) I think, vary in their capacity to confuse, and accordingly the ease with which the opaque may be clarified. Aware that this diagram (figure 1):

Figure 1: Viewed by learner - named by teacher.

shows a right angle, a learner who describes this (figure 2):

Figure 2: What might learners call this angle?

as a "left angle" displays descriptive power but mathematical innocence of how it is that mathematical terms are not so much descriptive as tied to mathematical properties. Unless unwilling to let go of their customary descriptive power, learners may readily accept our attribution of "right angle" to both diagrams as "left angle" is neither everyday nor mathematical usage. We might remark that "right" is short for "upright" in our attempt to drag learners away from their idiomatic description, saying what they see. By acceding to our dismissal of left angle, our learners have implicitly, or explicitly if told, begun to buy into the mathematical idea that properties of shapes are unaffected by position, invariant under isometries as the saying goes.

What presently concerns me is the clash in the minds of learners between the everyday and the mathematical, a conflict which may confuse, or lead to angry dismissal of mathematics as nonsensical. Further, I would like to know how to enable the new to be added to learners' natural descriptive powers, but not in any way disparage natural competence, perhaps pompously with, "We use words more precisely in mathematics." Should we allow learners to stay with left angle until encountering transformations and congruence?

We present to learners idioms commonplace among mathematics aficionados, but is our dialect any more transparent than colloquial turns of phrase are to foreigners learning our native language? About to return home at the end of WWII, Polish airmen lined up for a valedictory address by their leader to their RAF counterparts, their hosts for the duration. The Polish leader's address ended with "... and may God pickle you all!". A similar error (misconception?), innocent of context, is attested to by a translation into Russian of a journal article which contained the maxim, "Out of sight, out of mind". At a later date the article was needed in English; the maxim was translated back as, "Blind and mad".

We teachers seem to get away with the mathematical use of "property", perhaps because it is semantically so distant from a home, or from one's residential cash-cow; or is it because we just *use* rather than define "property", referring to the angle-sum property of triangles, or the defining property of a tangent as a line touching a curve and its consequential property of being normal to a radius of a circle at the point of touch? … normal in the sense of perpendicular, not normal as in probability distribution, nor in topology.

When learners misinterpret mathematical terms, relying on everyday interpretation of what they perceive, their answer to “How many diagonals has this rectangle?” (figure 3).

Figure 3: Rectangle displayed to learner.

may well be “None.”, but when asked about this rectangle (figure 4):

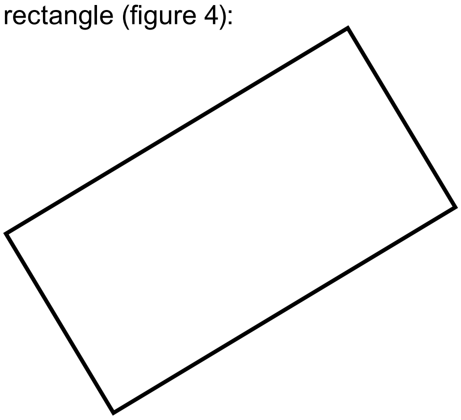

Figure 4: Rectangle tilted.

a frequent answer is “Four.” David Pimm points out that, in everyday use, “diagonal” describes something sloping, in contrast to vertical or horizontal: But in mathematics “diagonal” is a property of shapes (Pimm, 1978). Rectangles have diagonals, even when none are shown, as in figure 3.

Mathematics adopts and adapts everyday words, so “symmetrical” becomes “symmetric” in statistics, though both words are used variously in algebra and in geometry. Learners may be nonplussed on encountering familiar words used as mathematical objects or concepts. In everyday speech we say, “A third of a bar of chocolate” to describe how much chocolate. In mathematics we have the concept of one-third, now no longer a description of quantity. Undergraduates, as Alcock and Simpson (2009) point out, may also experience uncertainty about mathematical use of everyday words such as “limit”, as well as confusion arising from their dual use in mathematics as process and as value.

> Is 2^3 a number, or is it an instruction to calculate one?

A word or phrase may point towards the mathematics or it may distract us. I asked a student to place a band to show the triangle in figure 5, describing it during a phone tutorial, and asked her:

> “How may you find its area?”

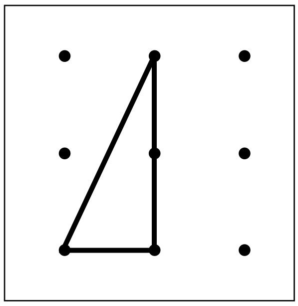

Figure 5: Band on a Geoboard.

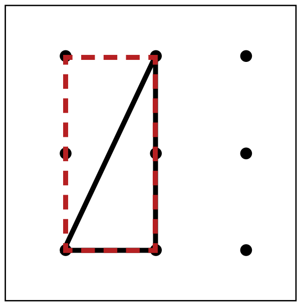

Figure 6: Imagined bounding rectangle.

“Halfbasetimesheight”, she swiftly replied, to which I responded, “Could we relate the triangle to its bounding rectangle?”, which we may have both thought of as the diagram in figure 6, but I chose not to check. It was enough that the conversation continued to work. At least I think I know to what she was attending, but not in what way. Perhaps she saw the rectangle as two of the triangles put together, or the diagonal as the line creating the two triangles, or focused on an area relationship, and so on. “Yes”, replied the student, “the triangle is half the rectangle.”

The original “half”, which with sophistication we may describe as the multiplicative operator “one half of” was for her part of the ritual incantation of the formula. With the second “half” my student related two shapes, as geoboard work facilitates. In geometry, diagrams may be worked with, not merely provide springboards for algorithmic activity.

The semantic distance between everyday and mathematical meaning affects, I suggest, how easily learners cope with dual use. As illustration I will contrast the question, “Does a table have legs, or does it have columns?” with the notion of invariance.

A table to eat off has legs, but a table in mathematics has columns. Confusing the two is unlikely because their application, albeit not their etymology, is so different. “Invariance”, on the other hand, is close enough in its everyday use and in mathematics to provoke confusion.

In everyday English, “invariant” simply means unchanging. In mathematics it is a bit different. It is about what is unchanging under certain conditions. An invariant, ubiquitous in mathematics, is a property that stays the same when some change takes place, for example, a geometrical transformation.

I was alerted to confusion with its non-mathematical use when a student erroneously declared that everything about the geometrical shape and the geoboard being used for the student’s task was invariant. Also, the student employed “variance” as the opposite to “invariance”. Sensible enough in everyday parlance but in mathematics “variance” is a term from statistics not the opposite of invariance.

Mathematical terms relate to their everyday cousins in diverse ways. In textbooks we see these definitions (figure 7):

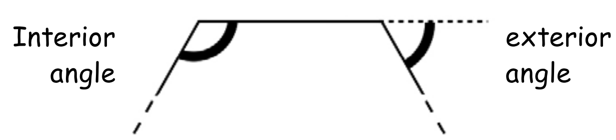

Figure 7: Interior and exterior angles of polygons.

although the following interpretation of exterior angle is plausible in the light of the provided definition of interior angle (figure 8):

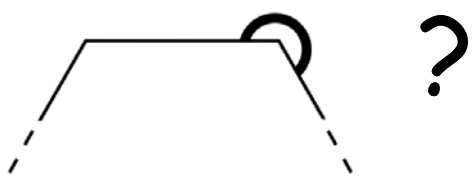

Figure 8: What might learners call this angle?

I see one text proclaims, “This is *not* an exterior angle”. (Heirs to Magritte, do texts in France announce, “Ceci n’est pas un angle extérieur?”)

Other terms may place even greater demands on learners’ capacity to comprehend the bizarre: “expanding brackets?” Suspension of disbelief?

In response to a task asking for the distance a ball is thrown and which is found from a quadratic equation with solutions –300 and 67, students who are aware that the negative value has to be ignored, sometimes write, “The distance is 67m. The –300 is negligible.”, unaware of, or ignoring the mathematical meaning of negligible.

Along with words with multiple meanings, the status of rules such as, “two minuses make a plus”, is problematic. Are they mathematical rules, or simply aides memoires?

A student addicted to rules asked me why -20 + -10 made -30 not the +30 she expected, a conflict with her rule about two negatives, remembered from earlier times. I suspect many students rely on prior learning, ignoring the course texts. I emailed my student this diagram (figure 9):

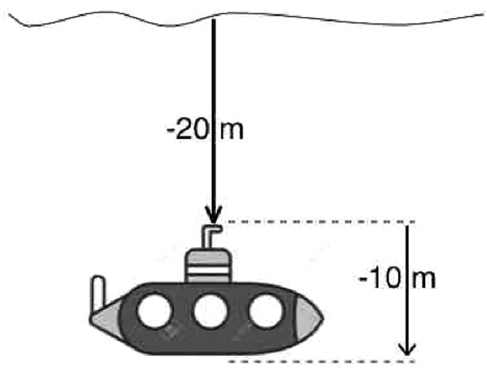

Figure 9: Sketch sent to student.

“I understand from your diagram”, the student replied, “but that means my rule doesn’t always work.” Perhaps I did the student no favour, undermining her regular certainty.

Learning may at times be enabled by a mathematical term or a mnemonic; in contrast, exemplifying their potential to disable, learners may know “tangent” in trigonometry, but “SOHCAHTOA” obscures its link to tangent to a unit circle. I wonder if SOHCAHTOA is best avoided?

As well as learners’ discomfort with unusual use of their familiar words, we teachers may find that ambiguity obscures our on-the-spot assessment of understanding. Lacking our own well-worn identification of representation and data-spread my student’s designation of his boxplot as “symmetrical” may refer merely to its visible symmetry, not to its statistical meaning. A different student, however, left me in no doubt as to what she meant, although describing her own boxplot as “right-skewered”.

Both everyday and mathematical utterance figure in the classroom, so learners must cope with both. More than merely cope, we actually esteem non-specialist talk when we encourage our learners to argue points of view, agree, disagree, without pressure to meet canons of formal reasoning. We may not, according to situation, leave it there in the shared agreement of friends, but take different or narrower paths from conjecture to proof, echoed in the mantra, “convince yourself, convince a friend, convince a sceptic.”

Adrian Pinel (see MT217) alludes to his way of shifting learners’ everyday descriptives to mathematical terms:

> … whilst adding to their vocabulary subtly. They would say “turn” and I would say, “yes, turn, … rotate”. The next time they would say, “turn, rotate” and later just “rotate”. This approach led them to adopting my language, but with understanding … .

I wonder how Adrian reacted to his learners when at times they partially reverted to their former colloquial utterance? Not at all I suspect.

We are told that high attainers in mathematics cope better with the treachery of multivalent words. Is this because they are better able to discount their everyday discourse, their socially held knowledge?

If terms were used in only one way, learners’ lives might be easier but as Poincaré points out:

> Mathematics is the art of giving the same name to different things.

In his review of *Infinity and the mind*, Dick Tahta reminds us that:

> People say things multiply when there is increase. Mathematicians also say they multiply when there is a decrease (times half) or when neither increase nor decrease is in question (times a matrix).

I do not see that learners simply need to cope with mathematical terms, dismissing their everyday use. As it happens, mathematical texts employ both the technical and the informal. Learners may be told that a solution to a quadratic equation should not be written

$x = 3$ and $x = 2$,

on logical grounds, but as

$x = 3$ or $x = 2$,

as seen in the texts, although on the next line the text announces, “So the solutions are $x = 3$ and $x = 2$”, returning from logical to everyday.

My examples, amongst other experience, lead me to believe that relationships between language and mathematics are not straightforward, a belief echoed by the observation:

> Mathematical concepts and topics are not simply ‘ideas’, nor are they simply ‘definitions’ or techniques. Rather, the words are labels for a complex tapestry of interwoven thoughts and images, connections and links, behavioural practices and habits, emotions and excitements.’ (Johnston-Wilder S and Mason J, 2005)

I do not believe that direct instruction is in general an optimal way to address everyday and mathematical discourse in the classroom. Also, no approach should denigrate learners’ socio-linguistic heritage. The subtlety of prompting seems more respectful of the natural powers of learners.

Although I am relaxed about mixing ordinary with specialised language others appear less liberal. Thornton (1970) says, “any must go” in an article of that name, the dictionary definition of “any” allowing it to do duty for the mathematical “for all” as well as for “for at least one”. Thornton, however, is more subtle than this example suggests. He writes:

> As with so many important aspects of the body of mathematics, the needs of the learner are in conflict with the conscience and convenience of the teacher. Which of the subtleties and refinements can be deferred until later, without actually misleading, in order to make the broad ideas accessible to beginners? (p. 221)

Perhaps Thornton has it about right, there is no single answer, decisions about everyday and mathematical classroom talk being, to turn a phrase, “horses for cases”.

References

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