Data Structures and Algorithms

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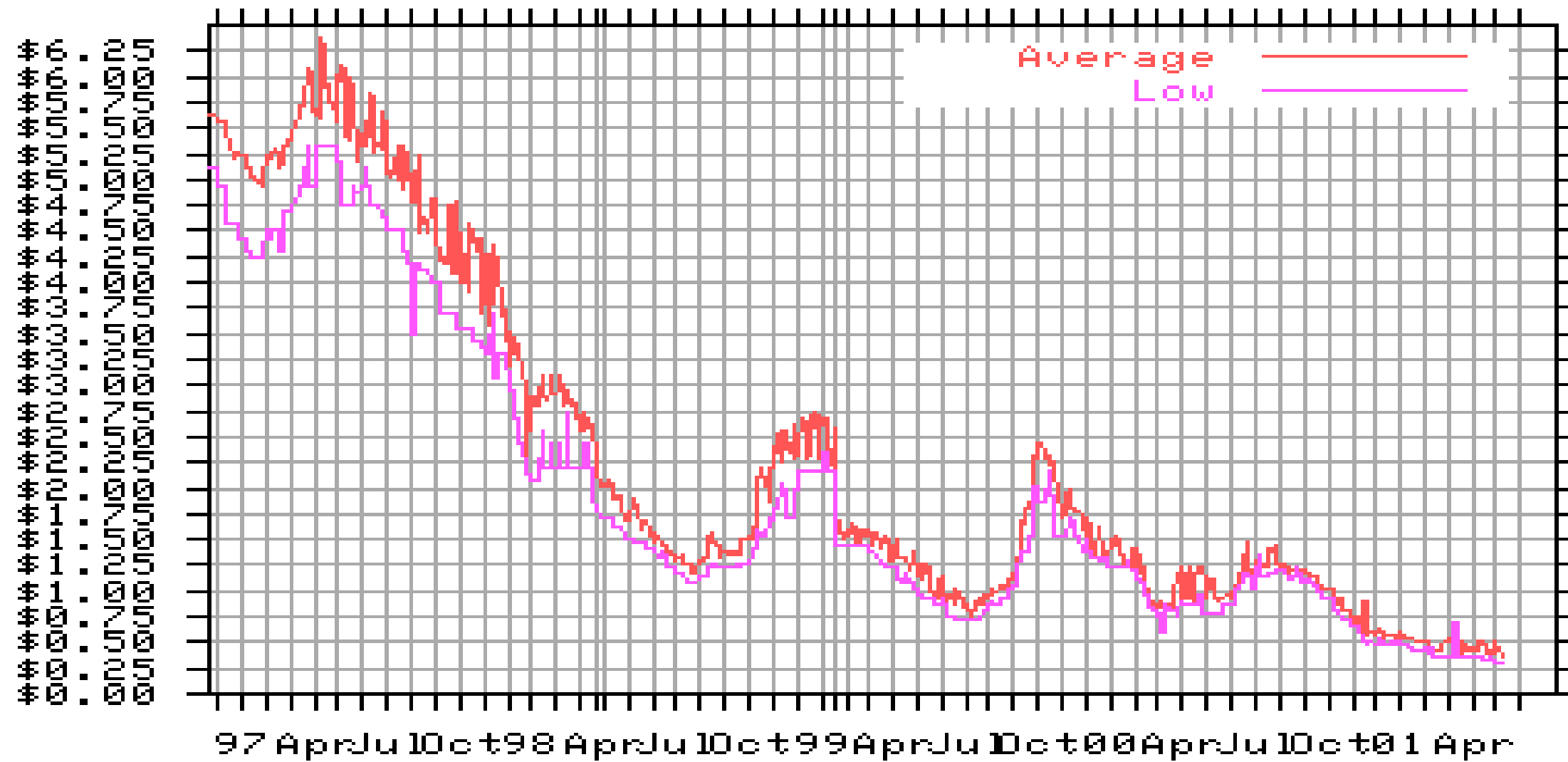**Web: raoumer.com**

outline

Graphs

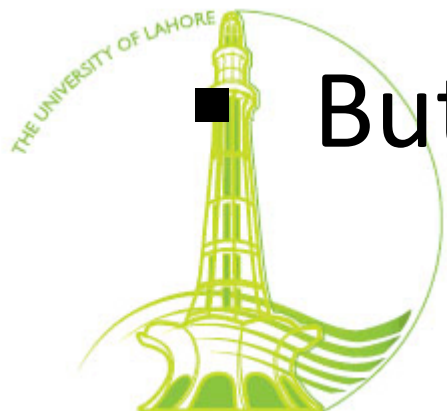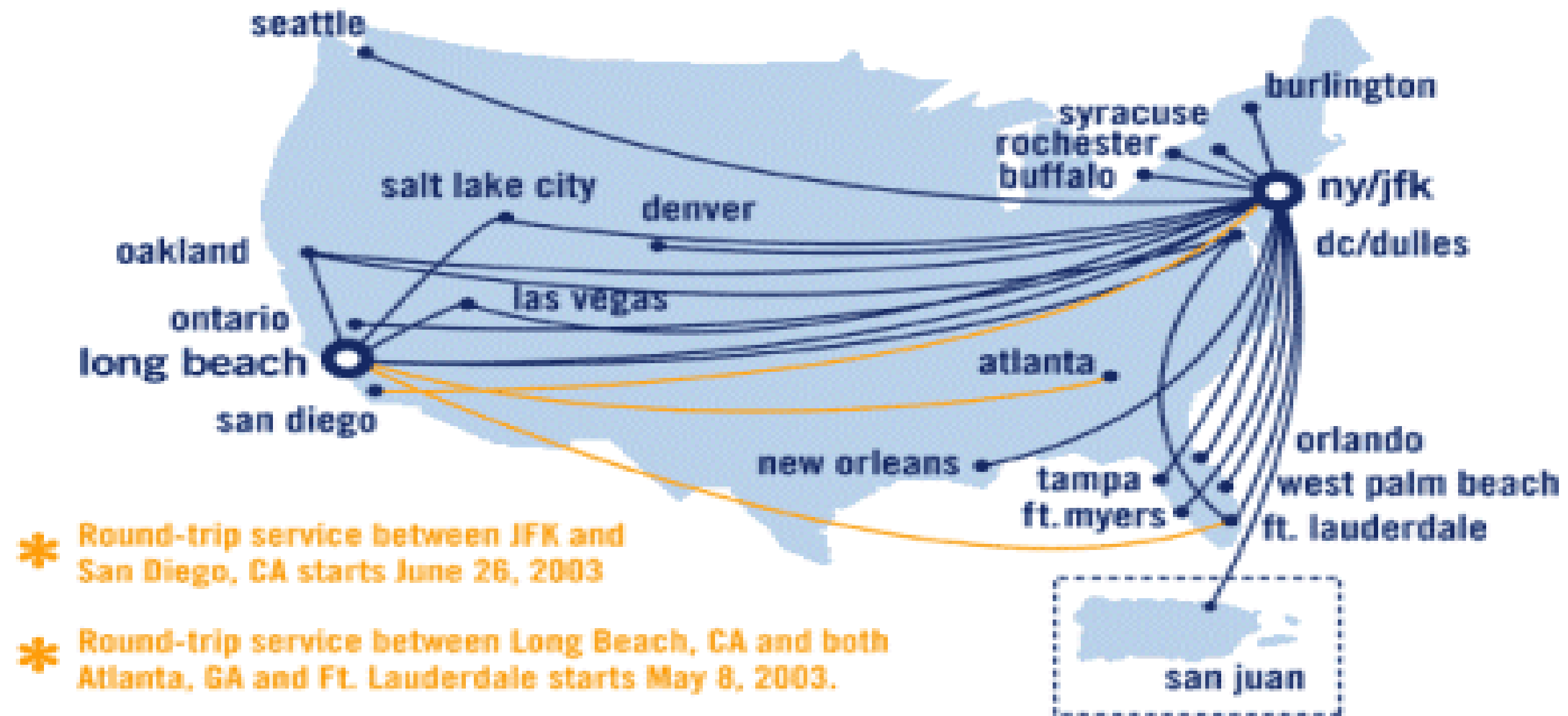- **What are Graphs?**
- **Graphs' Terminologies**

What are graphs?

- Yes, this is a graph….

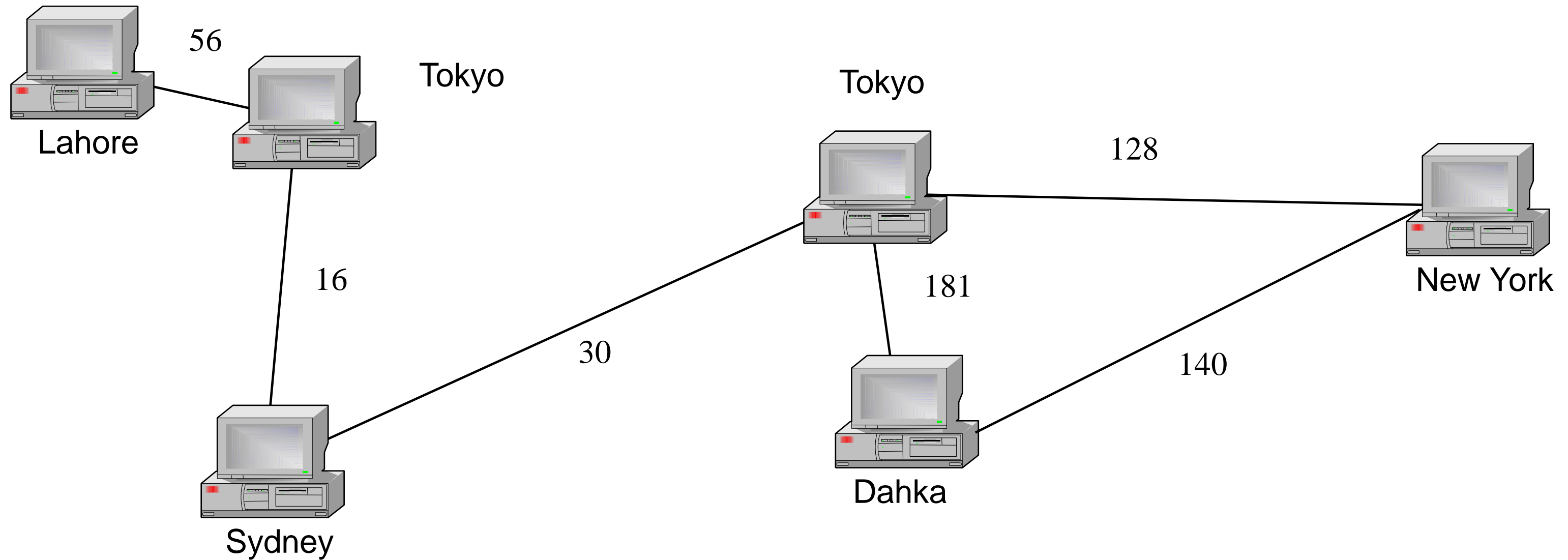- But we are interested in a different kind of "graph"

Airline Routes

Nodes = cities

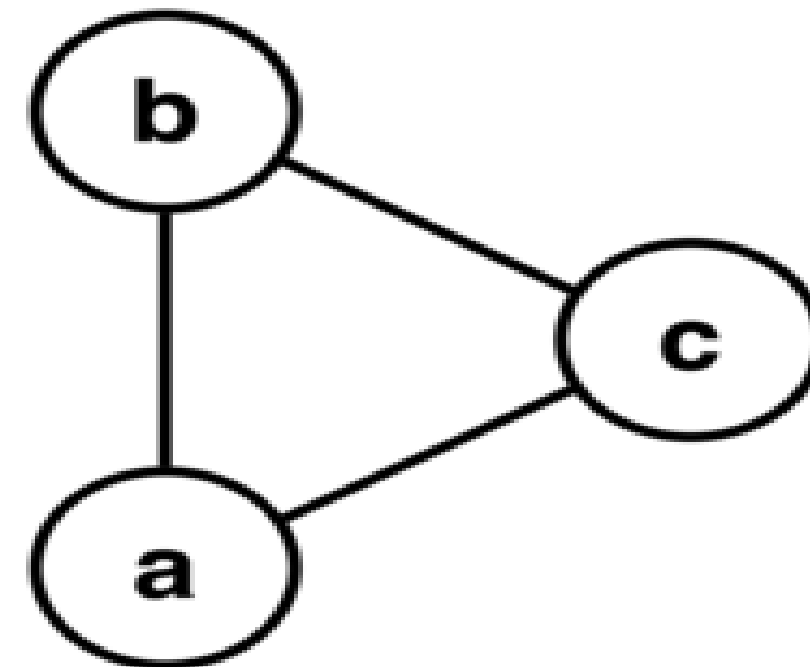**Edges = direct flights**

4

Computer Networks

Nodes = computers

Edges = transmission rates

Graphs

- **Graph**: a data structure containing
  - a set of vertices V
  - a set of edges E, where an edge represents a connection between 2 vertices
    - edge is a pair (v, w) where v, w in V

- Denote graph as $G = (V, E)$
- Example:

$G = (V, E)$ where

$V = \{a, b, c\}$ and $E = \{(a, b), (b, c), (c, a)\}$

Paths

- **path**: a path from vertex *A* to *B* is a sequence of edges that can be followed starting from *A* to reach *B*

  - Can be represented as vertices visited or edges taken

  - Example: path from V to Z: {b, h} or {(v,x), (x,z)} or {V, X, Z}

- **reachability**: v_2 is *reachable* from v_1 if a path exists from v_1 to v_2

- **connected graph**: one in which it is possible to reach any node from any other
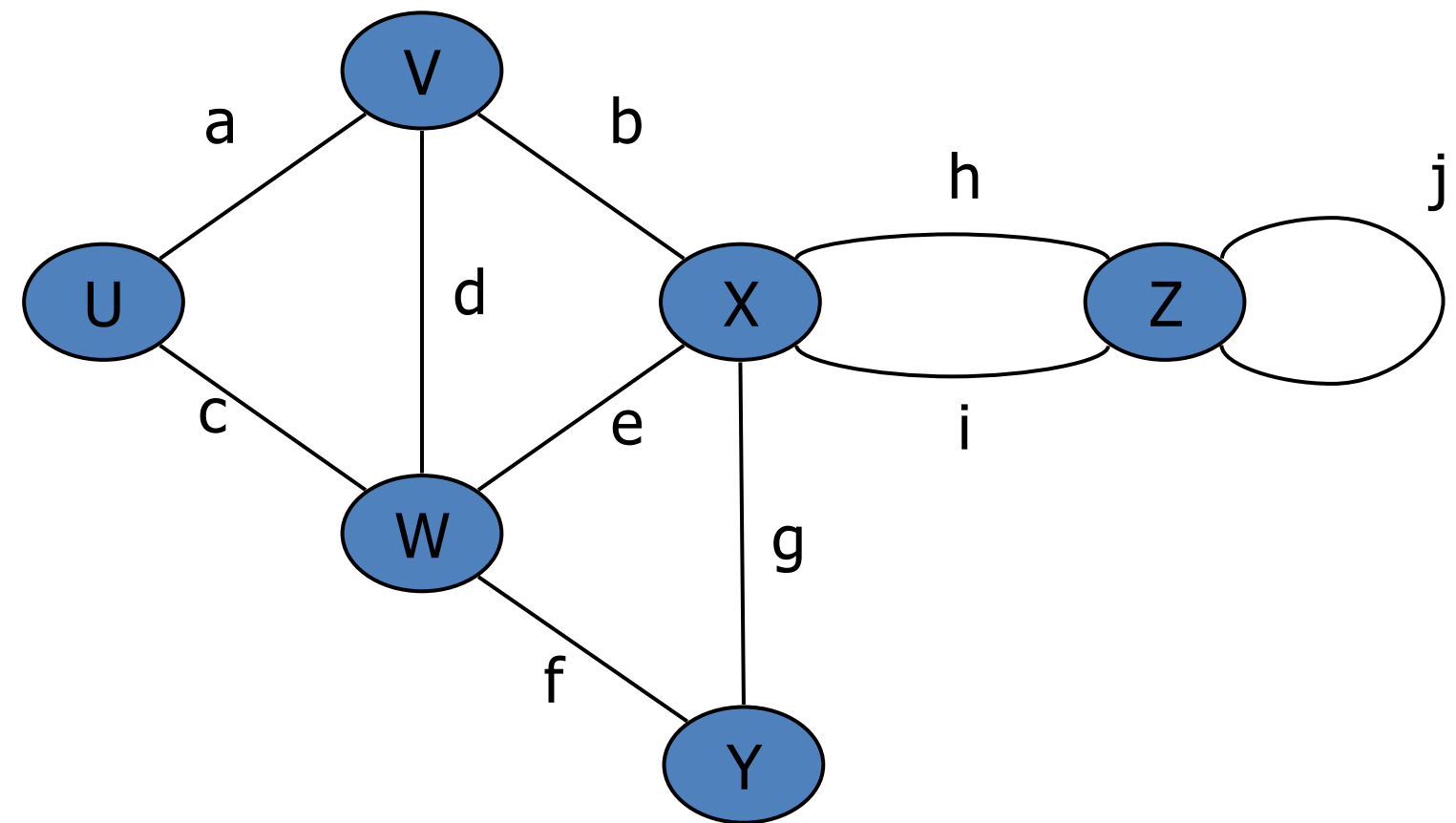
  - Is this graph connected?
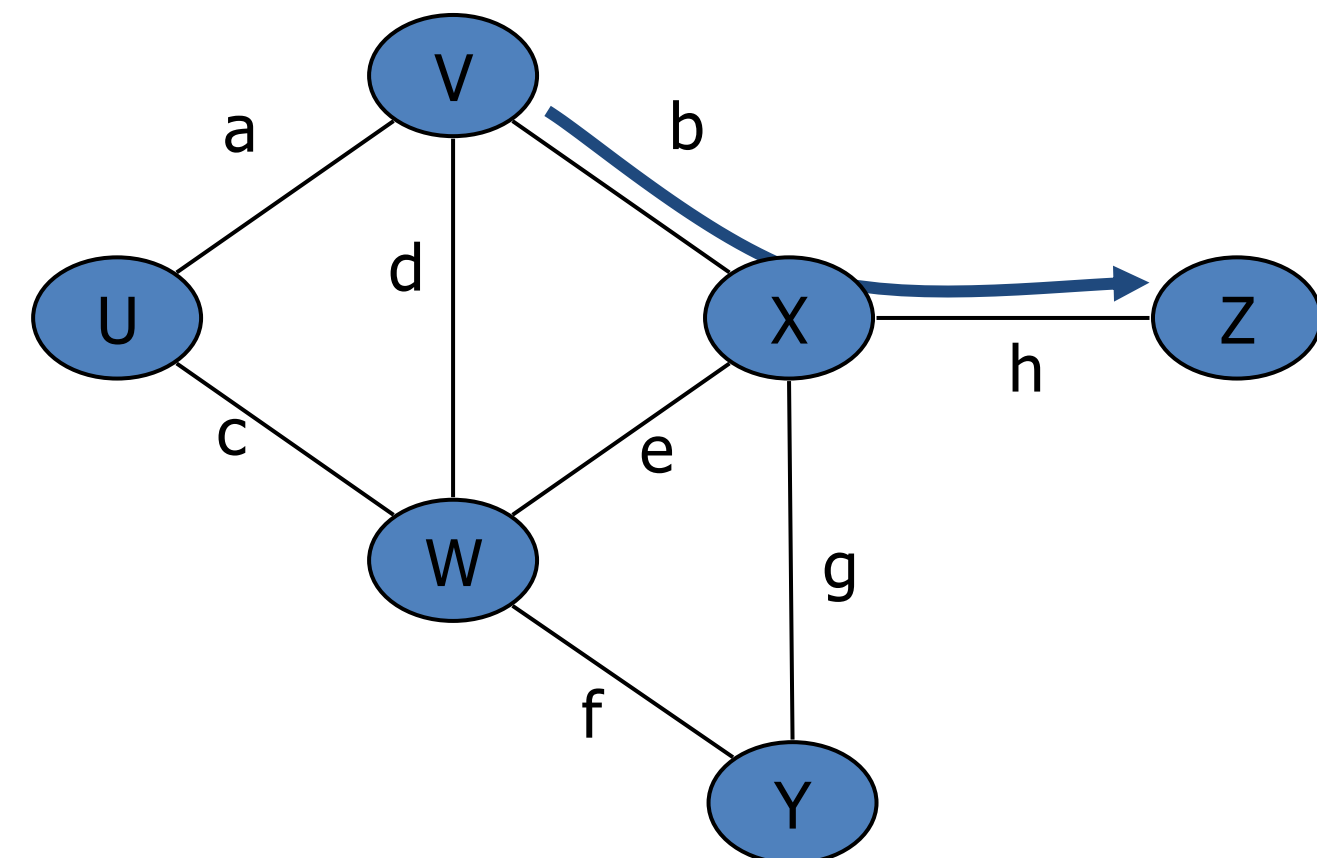
Cycles

- **cycle**: path from one node back to itself
  - Example: {V, X, Y, W, U, V}

- **loop**: edge directly from node to itself
 - Many graphs don't allow loops
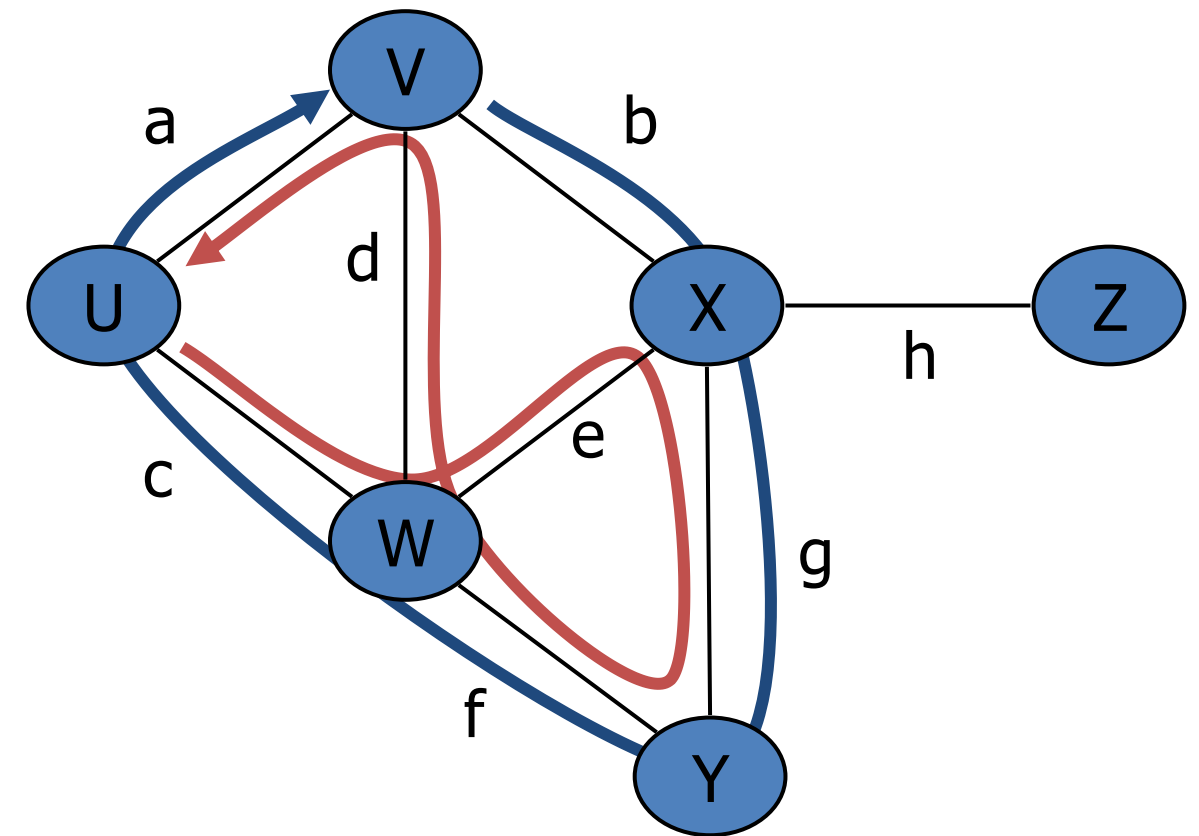
More terminology

- **degree**: number of edges touching a vertex
  - Example: W has degree 4
  - What is the degree of X? of Z?

- **adjacent vertices**: vertices connected directly by an edge
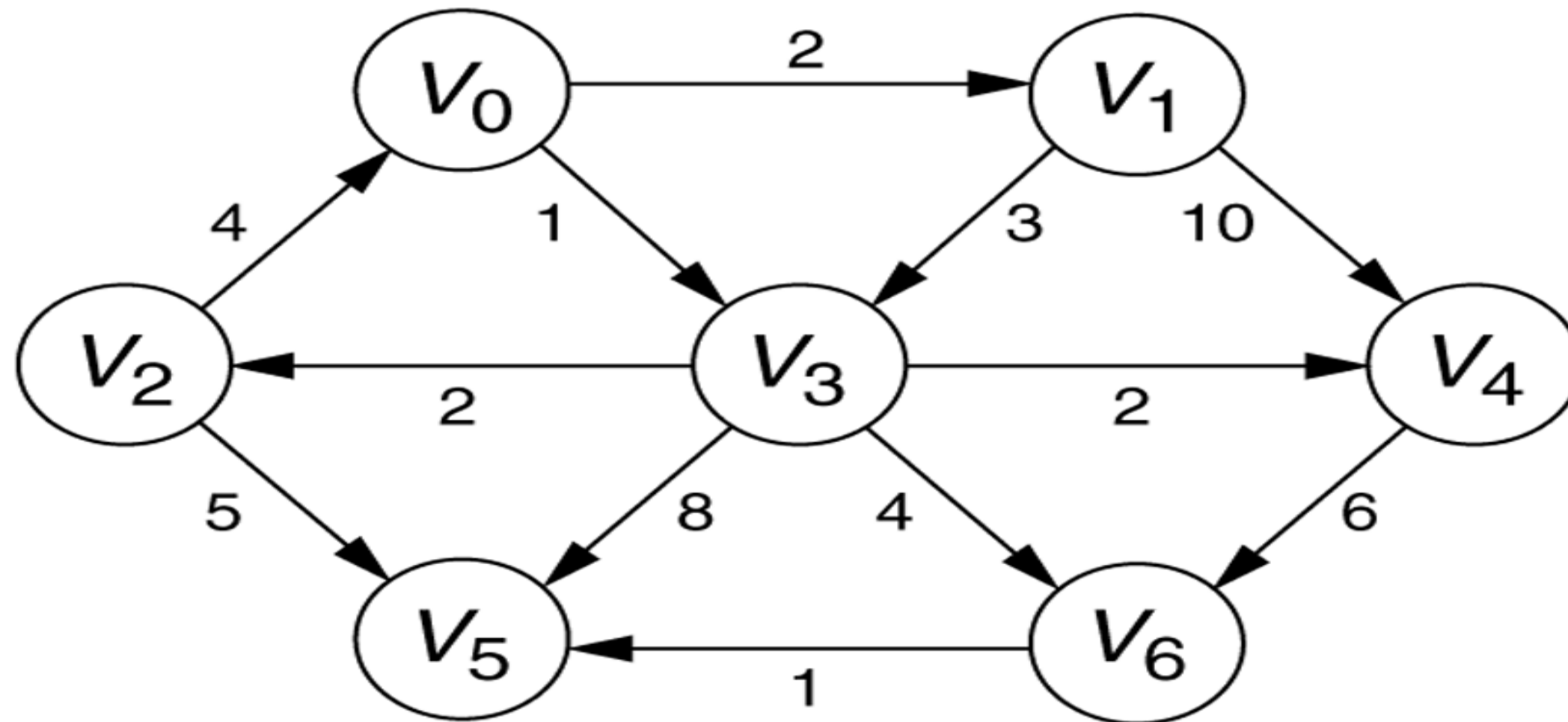
Weighted graphs

- **weight**: cost associated with a given edge

- Example: graph of airline flights
  - If we were programming this graph, what information would we have to store for each vertex / edge?
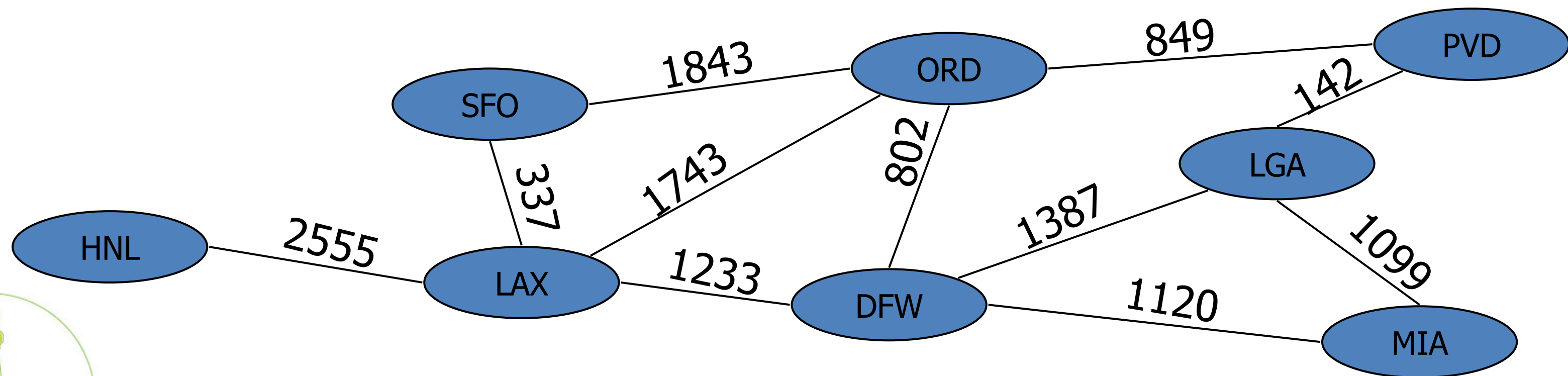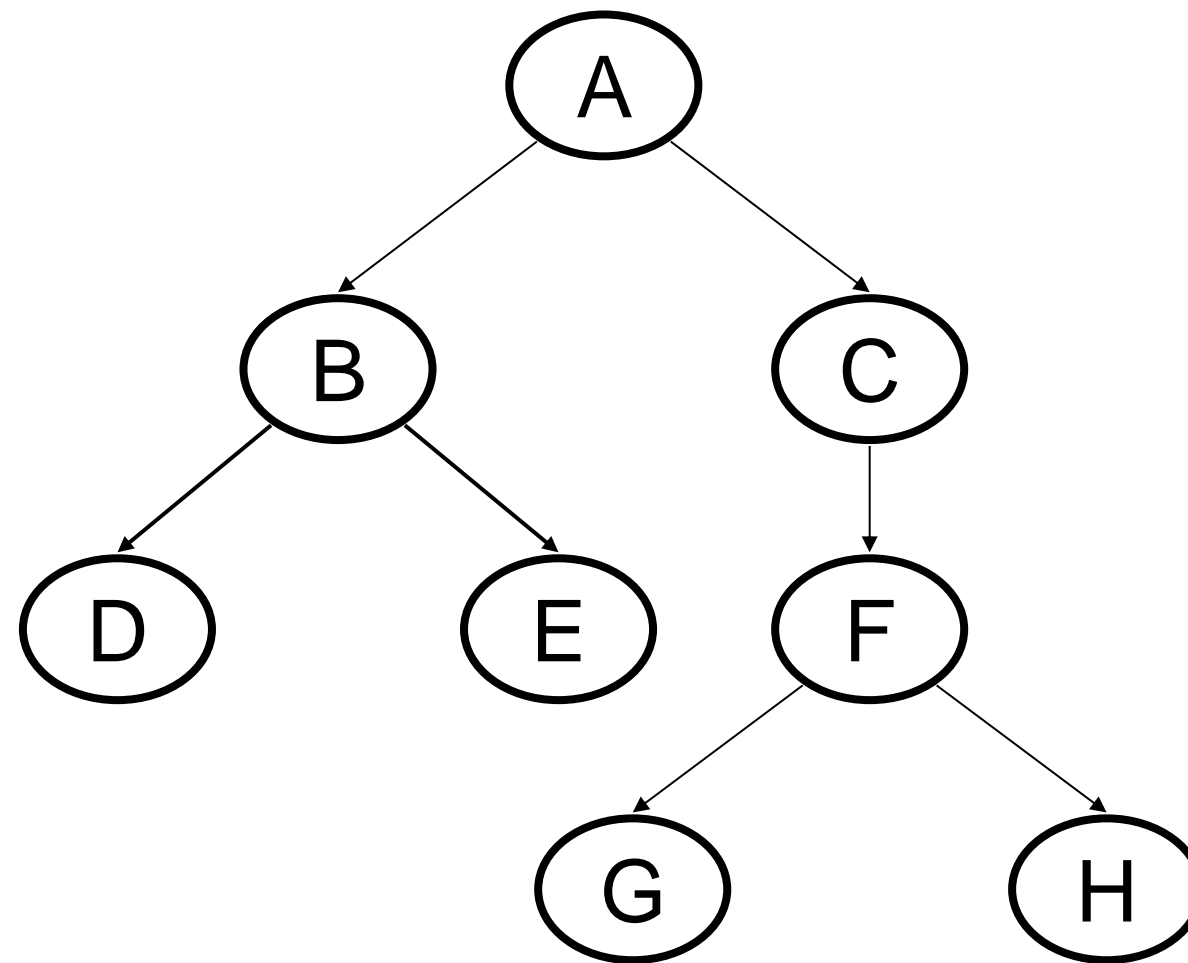
Directed graphs

- **directed graph (digraph)**: edges are one-way connections between vertices
  - If graph is directed, a vertex has a separate *in/out* degree

Trees as Graphs

- Every **tree** is a **graph** with some restrictions:
  - The tree is **directed**
  - There is exactly one **directed path** from the root to every node

Graph questions

- Are the following graphs directed or undirected?

    - **Buddy graphs** of instant messaging programs?
      (vertices = users, edges = user being on another's buddy list)

    - **bus line graph** depicting all of Lahore's bus stations and routes

    - **graph of movies** in which actors have appeared together

- Are these graphs potentially **cyclic**? Why or why not?

Graph exercise

- Consider a **graph of instant messenger buddies**.
    - What do the vertices represent? What does an edge represent?

    - Is this graph directed or undirected? Weighted or unweighted?

    - What does a vertex's degree mean? In degree? Out degree?

    - Can the graph contain loops? Cycles?

Graph exercise

- Consider this graph data:
  - Jessica's buddy list: Meghan, Alan, Martin.
  - Meghan's buddy list: Alan, Lori.
  - Toni's buddy list: Lori, Meghan.
  - Martin's buddy list: Lori, Meghan.
  - Alan's buddy list: Martin, Jessica.
  - Lori's buddy list: Meghan.

  - Compute the in/out degree of each vertex. Is the graph connected?
  - Who is the most popular? Least? Who is the most antisocial?
  - If we're having a party and want to distribute the message the most quickly, who should we tell first?

Graph exercise

- Consider a graph of **Facebook friends**.
  - What do the vertices represent? What does an edge represent?

  - Is this graph directed or undirected? Weighted or unweighted?

  - What does a vertex's degree mean? In degree? Out degree?

  - Can the graph contain loops? Cycles?