



### Instructions:

1. Please read the problems carefully and answer **ALL** questions.
2. Verify that your exam contains 4 pages + Smith chart.
3. Present your solutions *neatly*. Show the relevant steps, so that partial credit can be awarded.  
**BOX** your final answers where applicable. Draw figures wherever necessary.

### QUESTION 1 [20 Marks]

**1A** The S-parameters of a 2-port network are given by

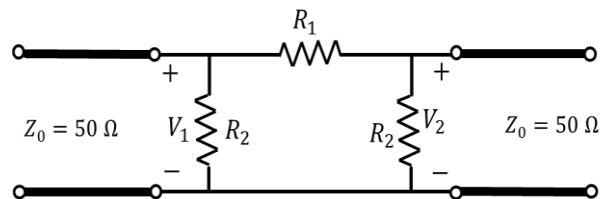
$$[S] = \begin{bmatrix} 0.5 + j0.5 & 0.15 - j0.05 \\ 0.95 + j0.25 & 0.5 - j0.5 \end{bmatrix}$$

The system impedance is  $Z_0 = 50 \Omega$ .

- (a) Is this network loss-free? Justify your answer. [2]
- (b) Is this network reciprocal? Justify your answer. [2]
- (c) If port 2 is matched, find the reflection coefficient  $\Gamma_1$  at port 1 as well as the input impedance  $Z_{in1}$  at port 1. [2]
- (d) Assuming that the network is fed at port 1 and the load at port 2 is  $Z_0$ , what is the internal impedance of the generator at port 1 such that maximum power is delivered to the network? [2]
- (e) If port 2 is short-circuited, find the reflection coefficient  $\Gamma_1^{sc}$  at port 1 as well as the input impedance  $Z_{in1}^{sc}$  at port 1. [4]

**1B** For the “ $\Pi$ ” network shown below,  $R_1 = 18 \Omega$  and  $R_2 = 291 \Omega$ . Find

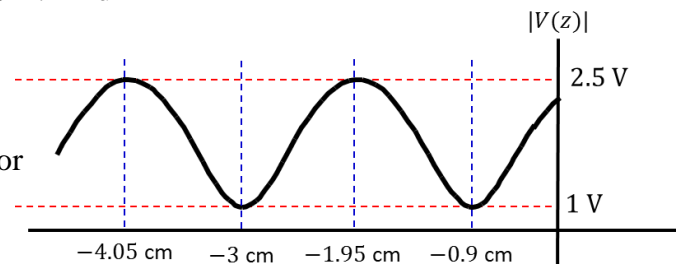
- (a) The scattering matrix. [4]
- (b) The reflected-to-incident power ratio. [1]
- (c) The transmitted-to-incident power ratio. [1]
- (d) The loss-to-incident power ratio. [1]
- (e) What is the function of this device? [1]



### QUESTION 2 [20 Marks]

**2A** The results of a slotted-line experiment are plotted in the following figure. The length of the line is  $\ell = 8.4$  cm; its characteristic impedance is  $Z_0 = 50 \Omega$ . Find

- (a) The reflection coefficient at the load. [3]
- (b) The load impedance. [1]
- (c) The input impedance. [3]
- (d) The reflection coefficient at the generator terminals. [1]
- (e) The percentage time-average incident power that is absorbed by the load. [2]



**2B** An engineer started bounce diagram shown below, but being interrupted by a Facebook posting, never finished his work. As ECE 323 student, find (show your calculations):

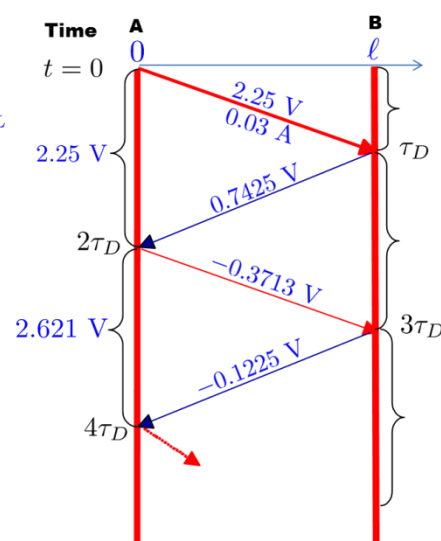
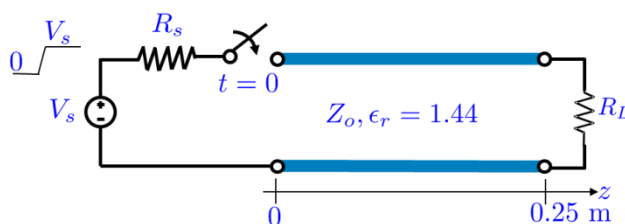
(a)  $\Gamma_L = \underline{\hspace{2cm}}$  [2]

(b)  $Z_0 = \underline{\hspace{2cm}}$  [2]

(c)  $R_L = \underline{\hspace{2cm}}$  [2]

(d)  $\tau_D = \underline{\hspace{2cm}}$  [2]

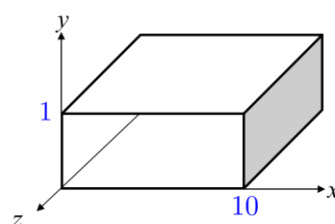
(e)  $R_s = \underline{\hspace{2cm}}$  [2]



### QUESTION 3 [25 Marks]

**3A** A rectangular air-filled waveguide supports TE propagation mode (shown below). Assume that the following magnetic field in the z direction is obtained:

$$H_z = E_0 \cos\left(\frac{\pi x}{2}\right) \cos(\pi y) e^{-j10z}$$



(a) What is the mode of operation? [2]

(b) What are the cutoff wave number  $k_c$  and the free-space wave number  $k$ ? [4]

(c) Find  $E_x$ ,  $E_y$ ,  $H_x$ , and  $H_y$ . [4]

**3B** A rectangular waveguide has a cross-section of width  $a = 5$  cm and height  $b = 3$  cm. Find:

(a) The cut-off frequency and the cut-off wavelength of the dominant  $TE_{10}$  mode. [2]

(b) The wavelength  $\lambda_g$  in the waveguide at the operating frequency of  $f = 4$  GHz. [3]

(c) The wave impedance  $\eta_{TE_{10}}$  at  $f = 4$  GHz. [2]

(d) The peak electric field value  $E_0$  if the power carried by the wave at 4 GHz is 1 W. [3]

**3C** Sketch the electric field patterns inside a rectangular waveguide for  $TE_{20}$  and  $TE_{01}$  modes in  $x - y$  plane. [5]

### QUESTION 4 [25 Marks]

**4A** Design a lossless T-junction divider with a  $30 \Omega$  source impedance to give a 3:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to  $30 \Omega$ . Determine the magnitude of the scattering parameters for this circuit, using a  $30 \Omega$  characteristic impedance. [8]

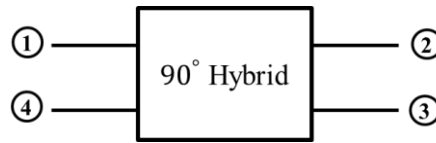
- 4B (a) If a quadrature coupler has the following input voltages,

$$V_1^+ = 1\angle 30^\circ \text{ Volts}$$

$$V_2^+ = 2\angle 60^\circ \text{ Volts}$$

$$V_3^+ = 3\angle 90^\circ \text{ Volts}$$

$$V_4^+ = 4\angle 120^\circ \text{ Volts}$$



$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

what is the output voltage on port 4? [4]

- (b) Does the result change when  $V_1^+$  is changed to  $1000\angle 30^\circ$  volts? Explain why or why not? [3]

- 4C (a) With neat diagram, explain the operation of a Faraday rotation isolator. [4]

- (b) What is the purpose of directional Coupler? Define coupling factor, directivity, isolation of Directional coupler and write expression for each. [6]

### QUESTION 5 [15 Marks]

- 5 Design a stepped-impedance, maximally flat low-pass filter having a cutoff frequency of 2.5 GHz and an attenuation of at least **12 dB** at 4 GHz. Use a reference impedance of  $Z_0 = 50 \Omega$ . The highest practical line impedance is  $110 \Omega$ , and the lowest is  $10 \Omega$ .

Use a high-impedance line for the first section (closest to the source) and use the minimum number of sections capable of meeting the attenuation requirement.

Sketch your design, and clearly specify the electrical lengths, widths of each section, and impedances of all lines. Assume FR4 fiberglass PCB with  $\epsilon_r = 4.2$  and  $d = 1.5$  mm.

### QUESTION 6 [15 Marks]

- 6 Using the provided Smith chart, design a shorted shunt, single-stub tuner to match the load  $Z_L = 60 - j80 \Omega$  to a T-Line with characteristic impedance  $Z_0 = 50 \Omega$ . Plot the resulting circuits with the location and length of stub shown terms of  $\lambda$ .

**\*END OF EXAMINATION\***

SEE OVER FOR SMITH CHART AND FORMULA SHEETS

## CONSTANTS

$$\begin{aligned}\varepsilon_o &= 8.854 \times 10^{-12} \text{ F/m}, & \mu_o &= 4\pi \times 10^{-7} \text{ H/m}, & \eta_o &= 377 \Omega, \\ c &= 3 \times 10^8 \text{ m/s}, & 1 \text{ Np} &= 8.686 \text{ dB}\end{aligned}$$

## USEFUL FORMULAS

$$\begin{aligned}H_z &= A_{mn} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta z} \\ E_x &= \frac{-j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y}, & H_x &= \frac{-j\beta}{k_c^2} \frac{\partial H_z}{\partial x} \\ E_y &= \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x}, & H_y &= \frac{-j\beta}{k_c^2} \frac{\partial H_z}{\partial y} \\ \varepsilon_e &= \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/W}}\end{aligned}$$

For FR4 PCB:

$W/d = 16$  @  $Z_0 = 10 \Omega$ ,  $W/d = 1.98$  @  $Z_0 = 50 \Omega$ , and  $W/d = 0.36$  @  $Z_0 = 110 \Omega$

Table 1 – Maximally flat low-pass prototype filter.

N	g1	g2	g3	g4	g5	g6	g7	g8	g9
1	2.0000	1.0000							
2	1.4142	1.4142	1.0000						
3	1.0000	2.0000	1.0000	1.0000					
4	0.7654	1.8478	1.8478	0.7654	1.0000				
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000			
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000		
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000	
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000