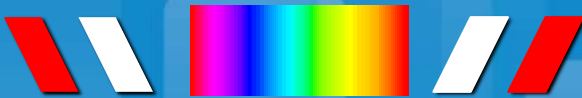


Lecture 1

Course Introduction

ECE 325
OPTOELECTRONICS



Kasap–1.1A, 1.2, and 1.3



February 13, 2019

Ahmed Farghal, Ph.D.

ECE, Menoufia University

Course Team

■ Course Instructor: Ahmed Farghal

- ➡ ahmedfarghal7080@gmail.com
- ➡ ahmed.farghal@el-eng.menofia.edu.eg
- ➡ <https://www.facebook.com/ahmed.farghal10>
- ➡ Office Hours: TBD
- ➡ Office: Commun. building – 2nd floor, **Room-309**.
- ➡ Open Door Policy: When my door is open, you are welcome to come in.



■ TA (Tutorials): Eman Salah Badr

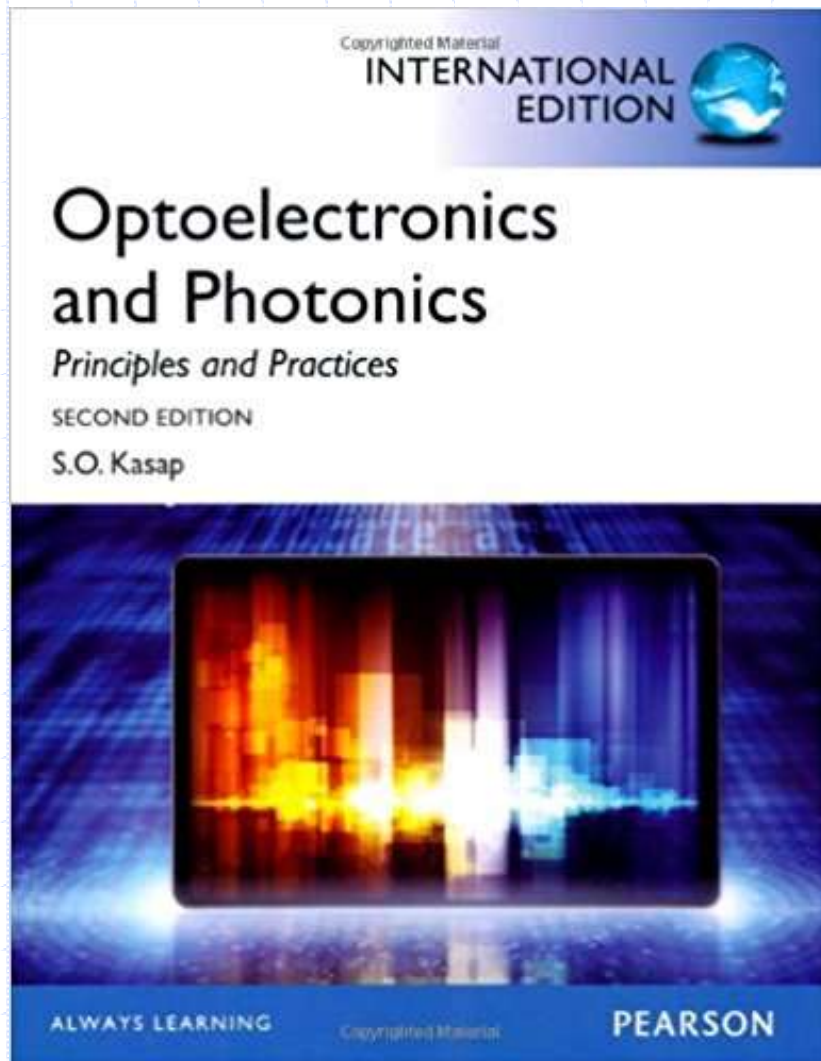
- ➡ Office Hours: TBD

Course Information

- **Goal:** Overview of optoelectronic device technology.
- **Emphasis:** device physics/operating principles (mainly concepts, less on math), along with some structural engineering.
- **Selected Topics:**
 - Introduction and Applications of Optoelectronics
 - Wave Nature of Light: Conceptual Overview
 - Dielectric Waveguides and Optical Fibers
 - LEDs, Lasers, and Optical Amplifiers
 - Photodetectors
 - Image Sensors: CCD and CMOS
 - Photovoltaic Devices: Solar Cells
 - Polarization and Modulation of Light



Course Textbook



**S. O. Kasap,
Optoelectronics and
Photonics: Principles
and Practices,
international ed.,
Prentice Hall,
2012.**



A very good book on
optoelectronics. Easier to read and
understand.
Recommended ★ ★ ★

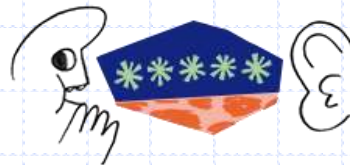
Reference: Mool C. Gupta, John Ballato, The Handbook of Photonics, 2nd ed, CRC Press, 2007.

ECE 325 Website

<https://ahmedfarghal.blogspot.com.eg/p/ece-325-optoelectronics.html>



- Will eventually contain:
- ➡ Syllabus/Schedule.
 - ➡ Lecture notes.
 - ➡ Problem sets.
 - ➡ Other useful material will be posted such as class announcements and useful links.

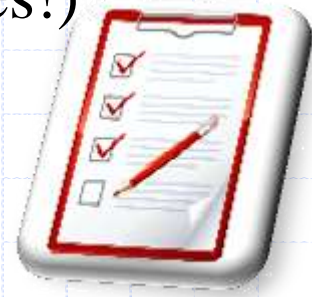


- Home
- ECE 325: Microwave Engineering
- ECE 325: Optoelectronics
- EEC 613: Optical Computer Networks
- EEC 713: Performance Analysis of Optical Networks
- Piazza ECE323
- Feedback

Access code for piazza: ece325

Lecture Style & Recommendations

- **Lecture notes** may have **gaps** in them that need to be **filled in** while you are in lecture (i.e., keep good notes!)
 - ➡ Goal is to facilitate learning
 - ➡ Consider using blank back-side of slides for notes
- If you **miss** a lecture, you will need to ask others for their notes
 - ➡ You can ask me **follow up questions** once you have gone through those notes
- **Be thorough** – The text and the handouts are **not** encyclopedia or manual! Each page builds on the previous one – you must read them **completely** and in **order**. When you come to a line, paragraph or page that you **don't understand**, do you **stop** and figure it out , or just skip it and go on?

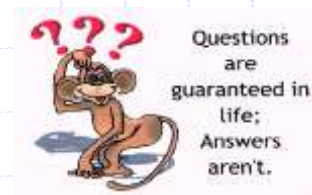


Lecture Style & Recommendations

- **Get help!** – **Office hours** are a great time to learn. All I ask is that you be **knowledgeable** of your **ignorance**!



- **Be prepared for each lecture** – Attend each lecture having **read the notes** from the previous lecture, and having read the relevant **text** for the **current** lecture. Come to lecture prepared to think and learn!
- **Ask Questions!** – I will make **an effort** to periodically stop and see if everyone understands the lecture material. However, you should stop me at any time if you have any questions.
 - If you are **confused** about something, chances are so is someone else.



Exams and Cheating



**Exams are closed-book.
HOWEVER**

**Formula sheet will be provided at midterm and final exams
(if needed)**

Old Exams and their solutions are on course website

**Cheating results in 0 grade and academic dishonesty
Cheating Policy: Just don't**



Answer Clarity

- You **must** present your answer clearly
 - ➡ Answers with **units** are to be boxed and **right justified**.
 - ➡ Show **supporting work** before the boxed answer with clearly shown steps of how you arrived at the answer.
 - ➡ Grade reduction will occur for **sloppy** work.

■ Example of preferred presentation

Problem 1:

Drawing

Equation(s)

Answer =



Let's enjoy discovering the secrets of **Optoelectronics**!



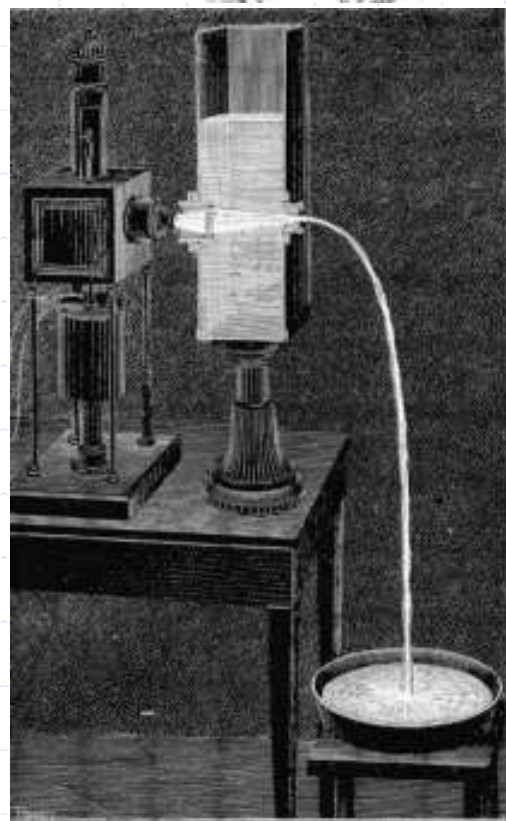
**I'm hoping to generate insight and interest – not pages of
equations!**

Jean-Daniel Colladon and the Light Guiding in a Water Jet

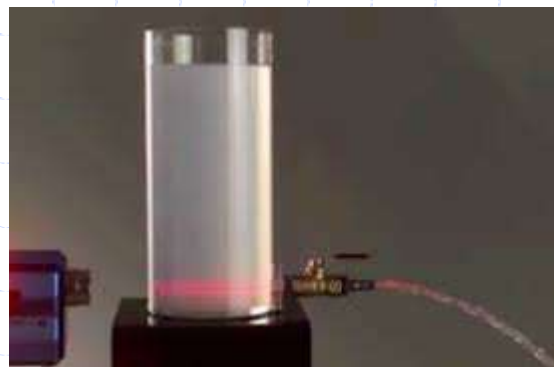


Light is guided along a water jet as demonstrated by **Jean-Daniel Colladon**. This illustration was published in *La Nature, Revue des Sciences*, in 1884 (p. 325). His first demonstration was **around 1841**. (*Comptes Rendes*, 15, 800-802, Oct. 24, 1842). A similar demonstration was done by **John Tyndall** for the Royal Institution in London in his **1854 lecture**.

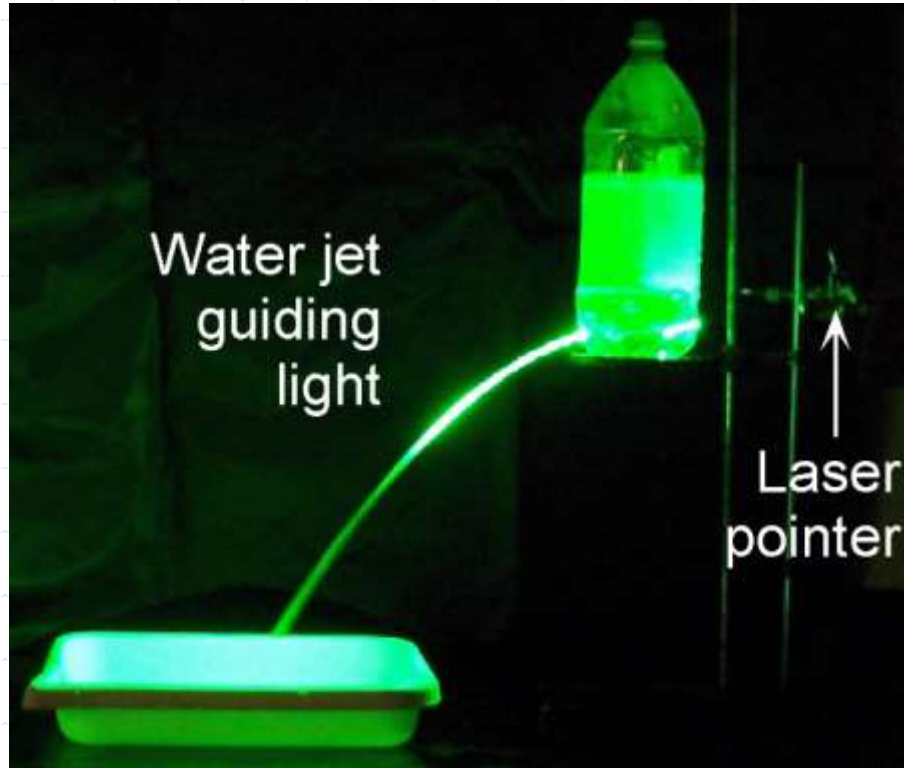
Although **John Tyndall** is often credited with the original discovery of a water-jet guiding light, Tyndall, himself, does not make that claim but neither does he attribute it to someone else.



1841



Water Jet Light Guide



A small hole is made in a plastic bottle full of water to generate a **water jet**. When the hole is illuminated with a laser beam (from a green laser pointer), the light is guided by total internal reflections along the jet to the tray. Light guiding by a water jet was demonstrated by **Jean-Daniel Colladon** in 1841 (Comptes Rendes, 15, 800-802, Oct. 24, 1842).

Optical Fibers

“The introduction of optical fiber systems will revolutionize the communications network. The low-transmission loss and the large bandwidth capability of the fiber systems allow signals to be transmitted for establishing communications contacts over large distances with few or no provisions of intermediate amplification.”

[Charles K. Kao (**one of the pioneers of glass fibers for optical communications**) Optical Fiber Systems: Technology, Design, and Applications (McGraw-Hill Book Company, New York, USA, 1982), p. 1]



Courtesy of the Chinese University of Hong Kong

Charles Kao and his colleagues carried out the early experiments on optical fibers at the Standard Telecommunications Laboratories Ltd at Harlow in the United Kingdom, during the 1960s. He shared the **Nobel Prize** in **2009** in Physics with Willard Boyle and George Smith for "groundbreaking achievements concerning the transmission of light in fibers for optical communication." In a milestone paper with George Hockam published in the IEE Proceedings in **1966** they predicted that the intrinsic losses of glass optical fibers could be much lower than **20 dB/km**, which would allow their use in long distance telecommunications. Today, optical fibers are used not only in telecommunications but also in various other technologies such as instrumentation and sensing.

Narinder Singh Kapany

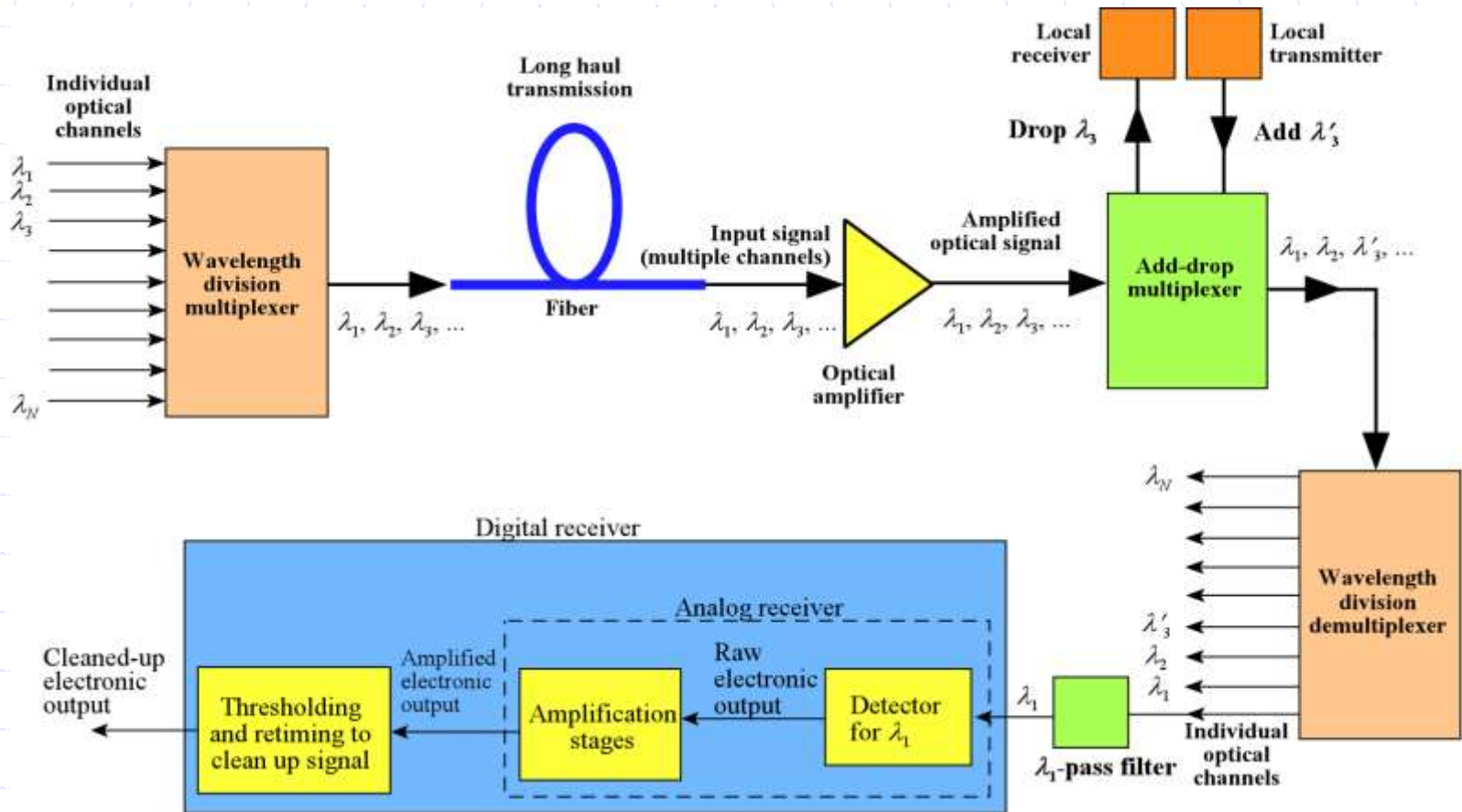


Dr. Narinder Singh Kapany, Father of Fiber Optics

Narinder Singh Kapany was born in India, studied at the Agra University and then obtained his PhD from the Imperial College of Science and Technology, University of London in 1955. He made significant contributions to optical glass fibers starting in 1950s, and essentially coined the term **fiber optics** in the **1960s**. His book **Fibre Optics: Principles and Applications**, published in 1967, was the first in optical fibers.

(Courtesy of Dr. Narinder S. Kapany)

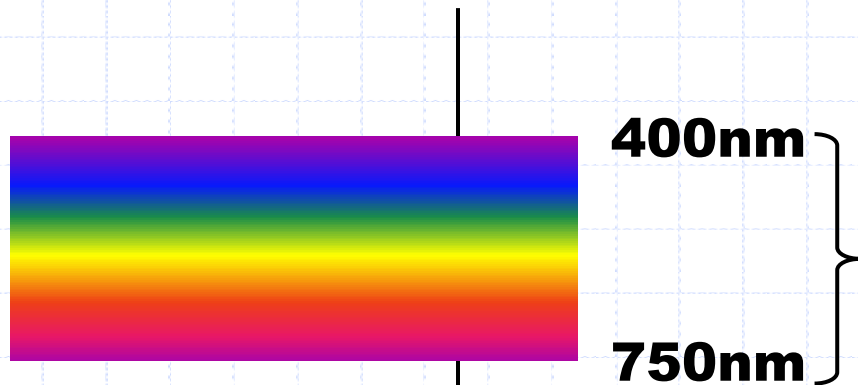
A Century and Half Later



- Fiber has replaced copper in communications.
- Photons have replaced electrons.
- Will “Photonics Engineering” replace “Electronics Engineering”?

Optical Wavelengths

UV



S (1460–1525 nm)

C (1525–1565 nm)

L (1565–1625 nm)

Visible Range

750nm

850nm

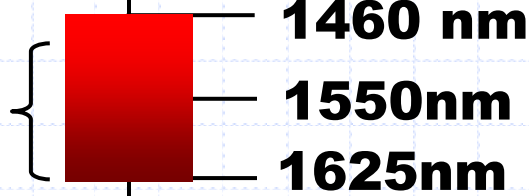
1st Window

IR

1310nm

2nd Window

DWDM Range



1460 nm

1550nm

1625nm

3rd Window

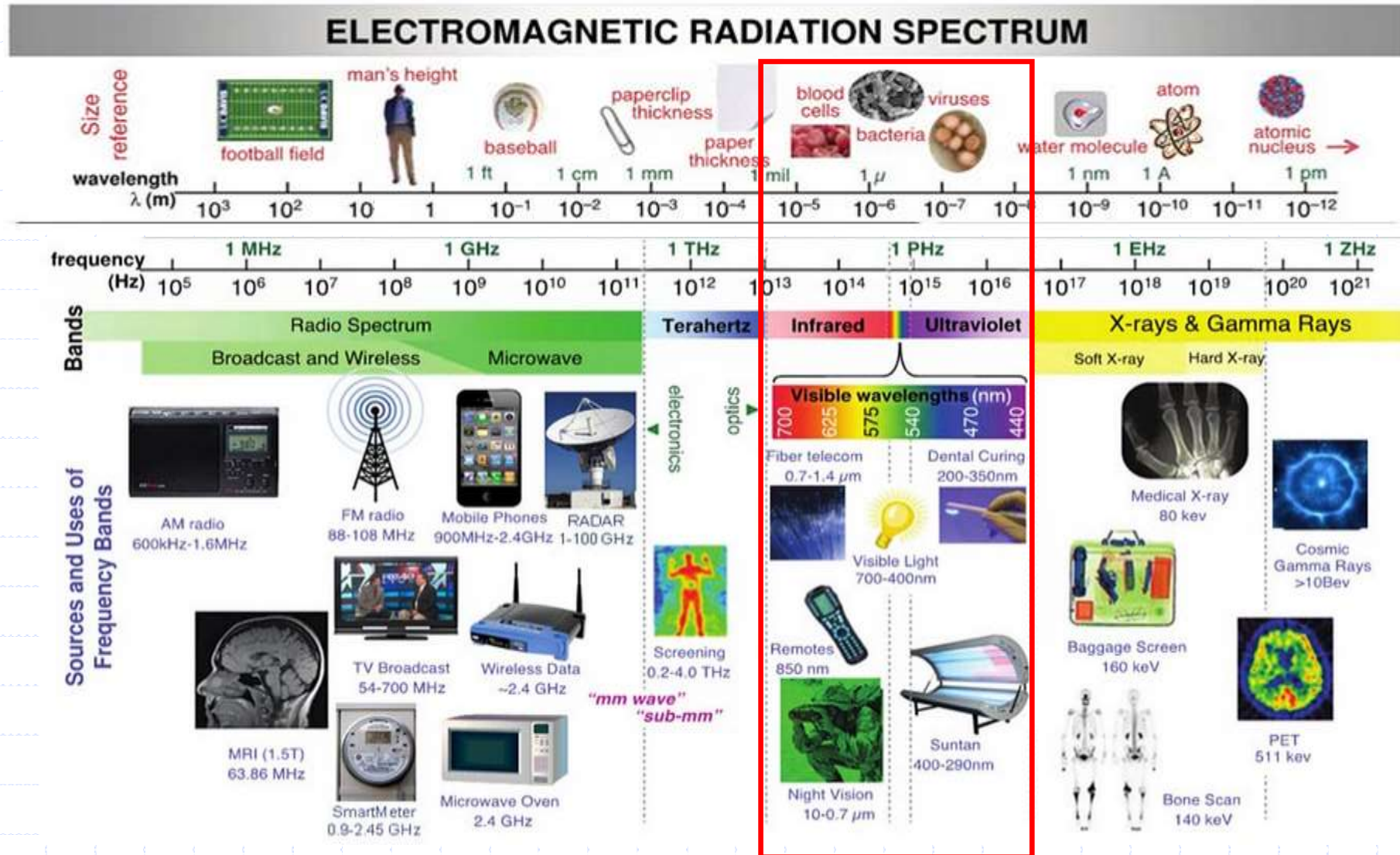


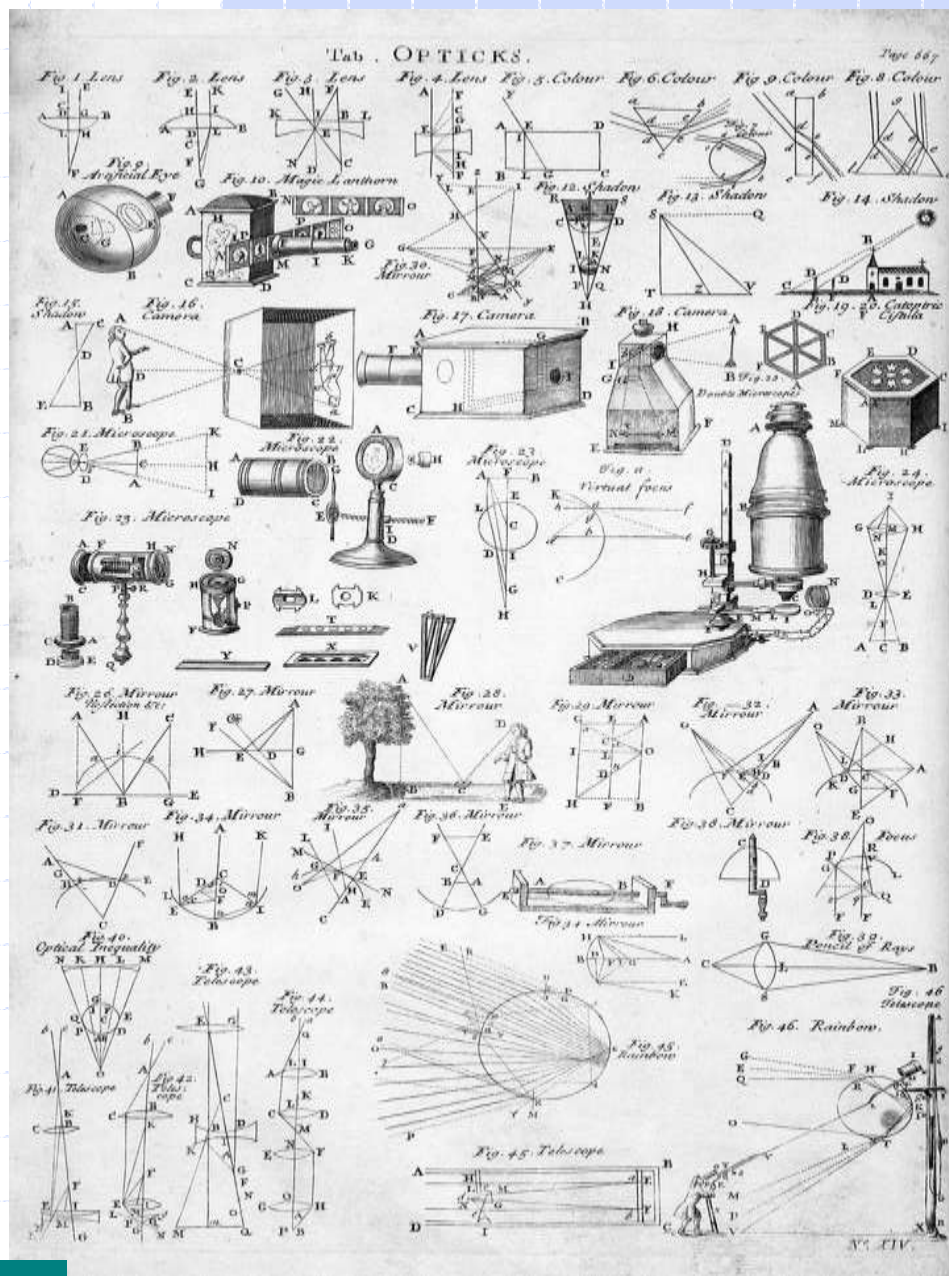
- Typical frequencies that are involved in optoelectronic devices are in the **infrared (including far infrared), visible, and UV**, and we generically refer to these frequencies as **optical frequencies**.

- Somewhat arbitrary range:

Roughly 10^{12} Hz to 10^{16} Hz

Electromagnetic Frequency Spectrum

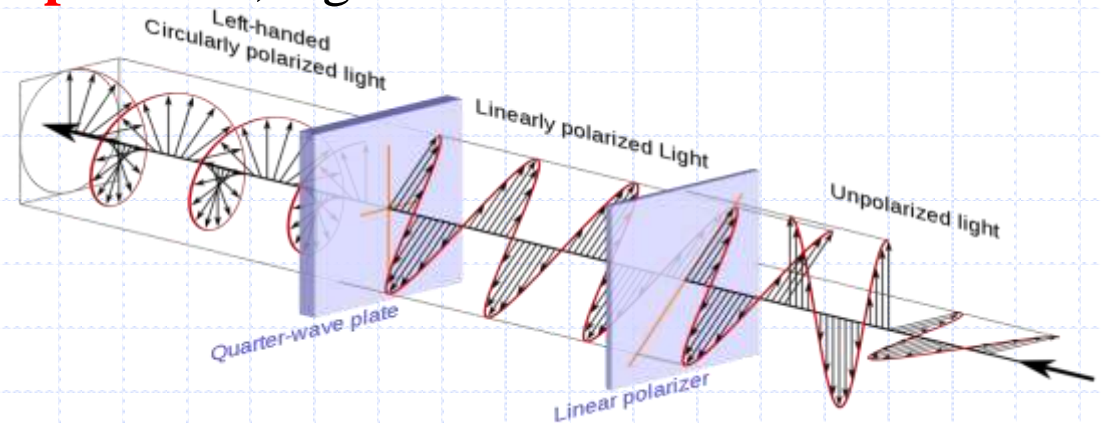
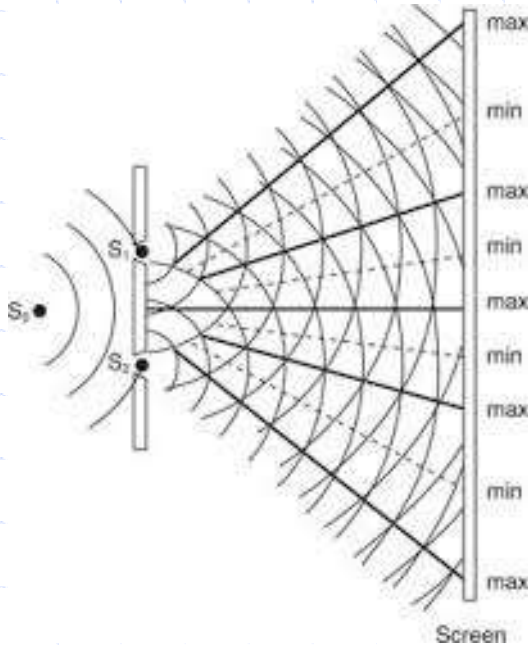




WHAT IS OPTICS?

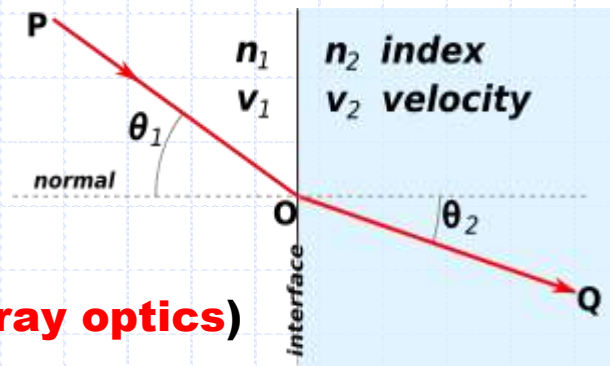
Optics

- **Optics** is light at work
- **Optics** is an old subject involving the generation, propagation & detection of light.
- For many centuries, the development of optical sources and optical detectors was very slow, hence progress was strongest in **studies of light propagation and light manipulation**, e.g.:



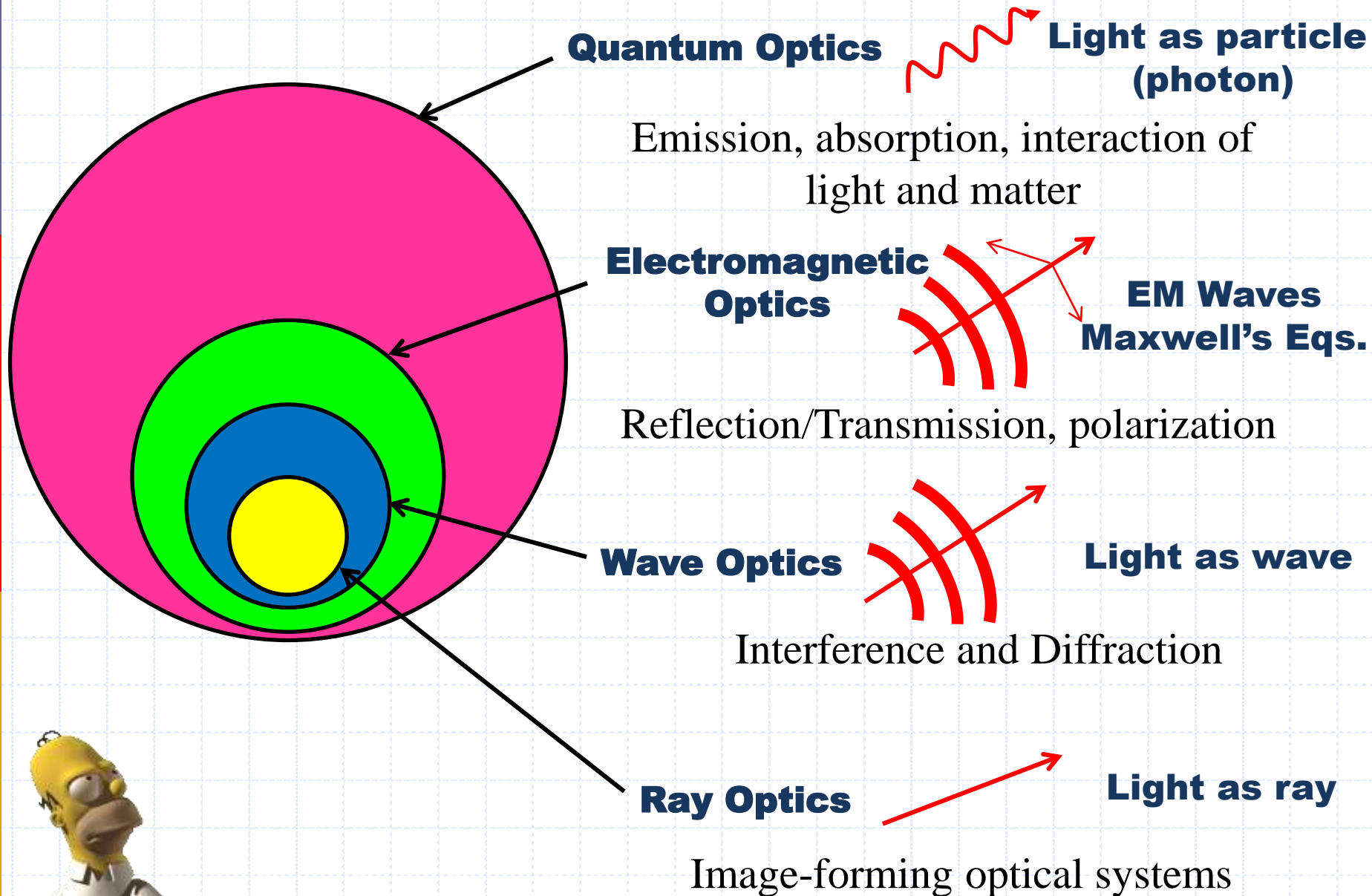
Polarization (electromagnetic optics)

Interference (wave optics)



Refraction (ray optics)

Hierarchy of Optical Theories



Optics

■ Three major developments are responsible for rejuvenation of optics & its application in modern technology:

- 1- **Invention of Laser (1960)**
- 2- **Fabrication of low-loss optical Fiber**
- 3- **Development of Semiconductor Optical Devices**



Corning scientists Dr. Peter Schultz, Dr. Donald Keck, and Dr. Robert Maurer invented the first low-loss optical fiber in 1970.

As a result, **new disciplines** have emerged & **new terms** describing them have come into use.



Photonics

- **Photonics** reflects the importance of the photon nature of light.
 - ➡ Photonics & electronics clearly overlap since electrons often control the flow of photons & conversely, photons control the flow of electrons.

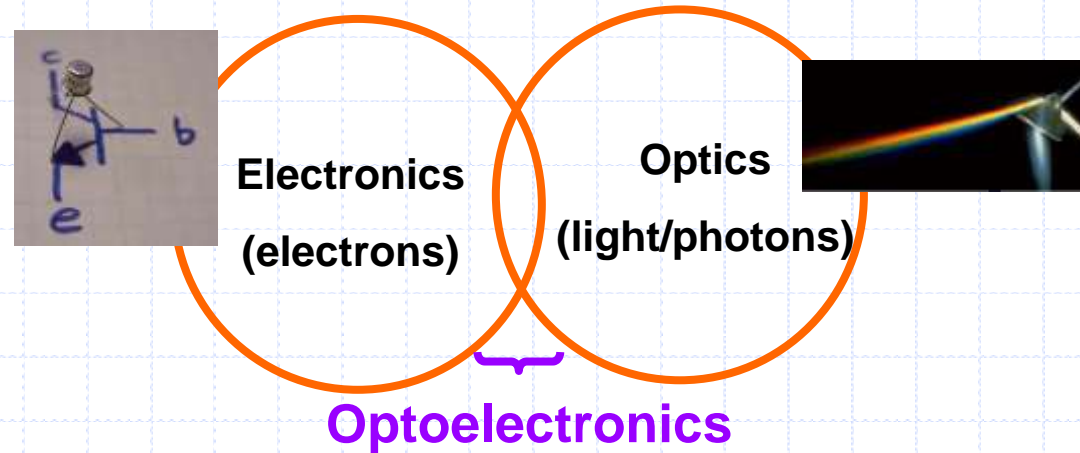
- **The scope of Photonics:**

1. **Generation of Light** (coherent & incoherent).
2. **Transmission of Light** (through free space, fibers, imaging systems, waveguides, ...).
3. **Processing of Light Signals** (modulation, switching, amplification, frequency conversion, ...).
4. **Detection of Light** (coherent & incoherent).

- **Photonic Communications:** describes the applications of photonic technology in communication devices & systems, such as transmitters, transmission media, receivers & signal processors.

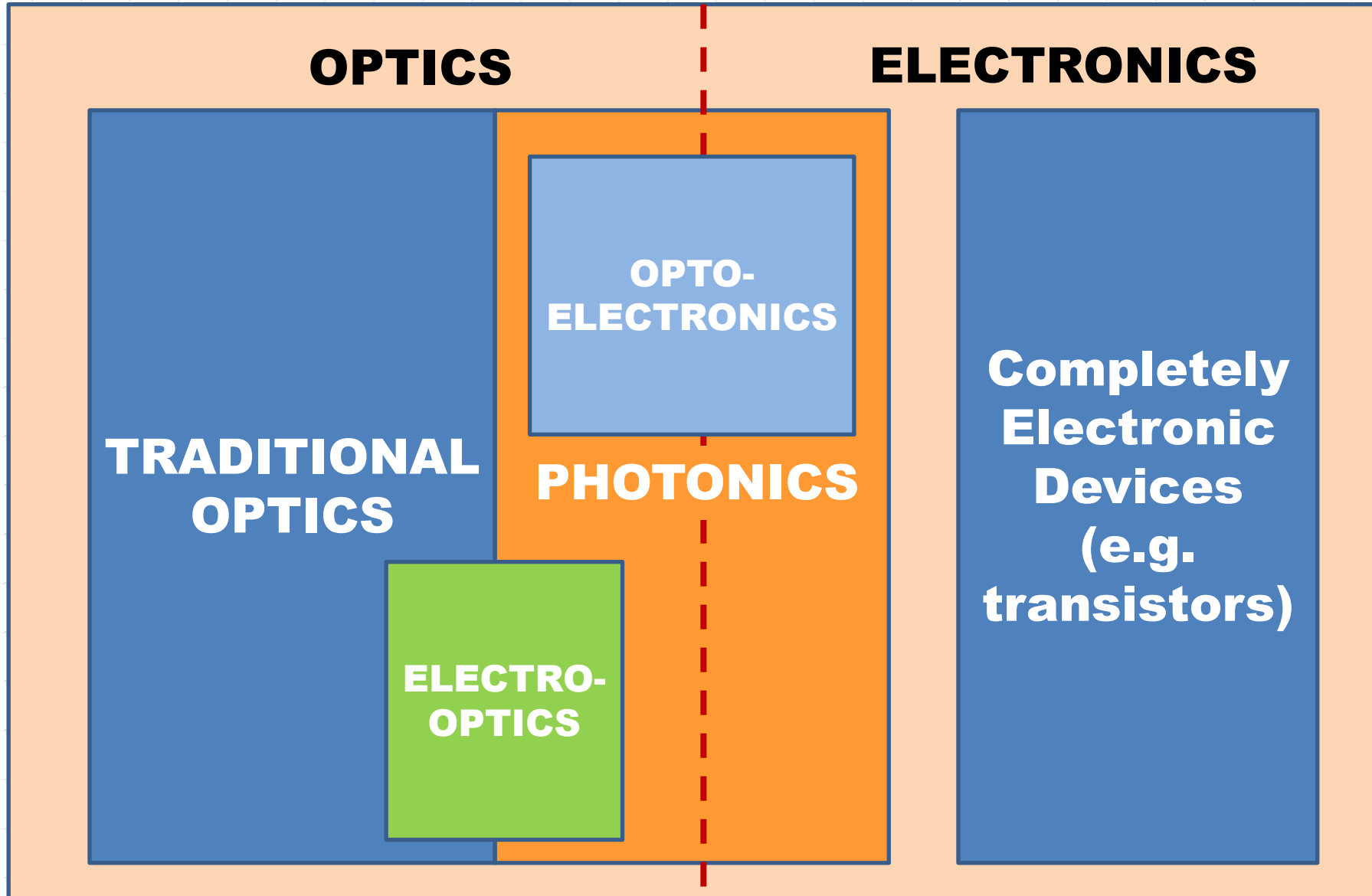
Areas Included in Photonics

- **Optoelectronics:** Devices & systems that are essentially electronic in nature but involve light (e.g. LEDs, LCDs, and array photodetectors).



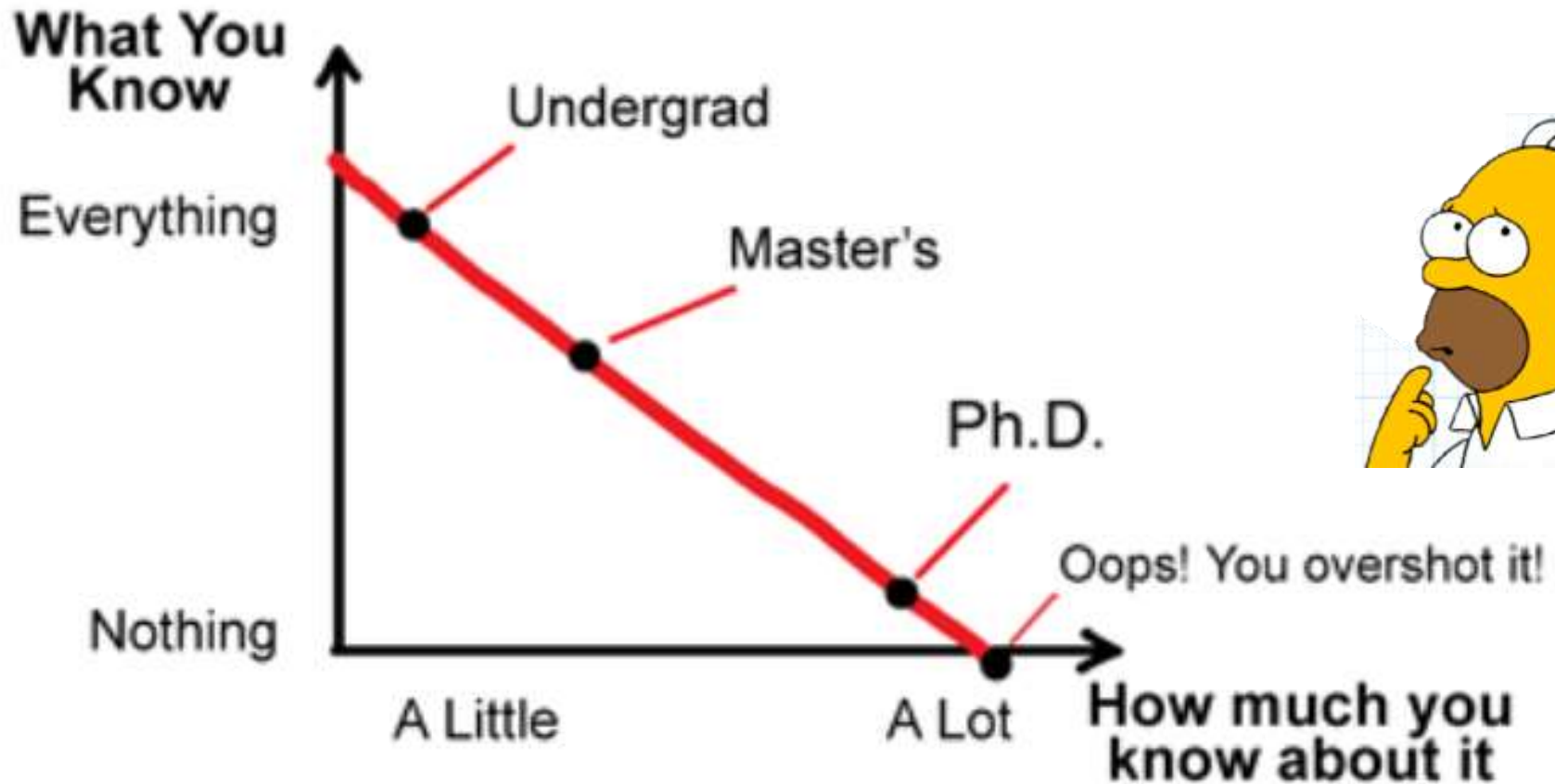
- **Electro-optics:** Optical devices in which electrical effects play a role (e.g. diode lasers, and electro-optic modulators and switches).
- **Quantum Electronics:** Devices & systems that rely principally on the interaction of light with matter (e.g. lasers and nonlinear optical devices used for optical amplification and wave-mixing).
- **Lightwave Technology:** Devices & systems that are used in optical communication and optical signal processing

Areas Included in Photonics



Photonics – is quite broad

What You Know vs How much you know about it

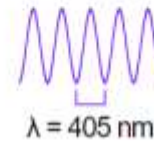
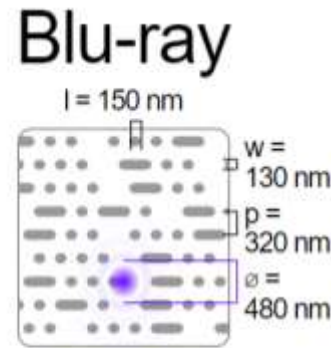
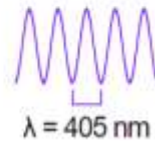
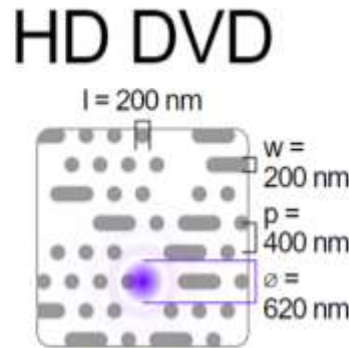
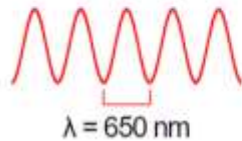
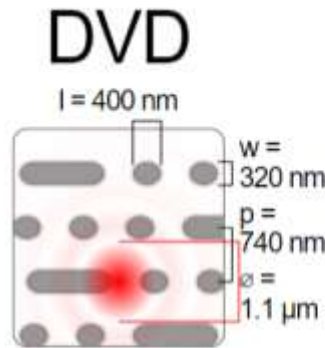
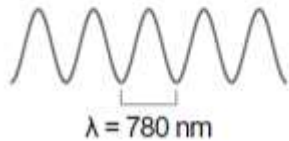
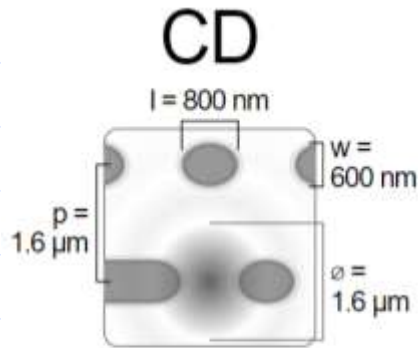


APPLICATIONS OF PHOTONICS

Photonics is now an important part of the world economy,
with multiple applications

Information Technology (IT)

Data Storage and Retrieval



optical scanner



photo copy machine

https://en.wikipedia.org/wiki/Comparison_of_high_definition_optical_disc_formats



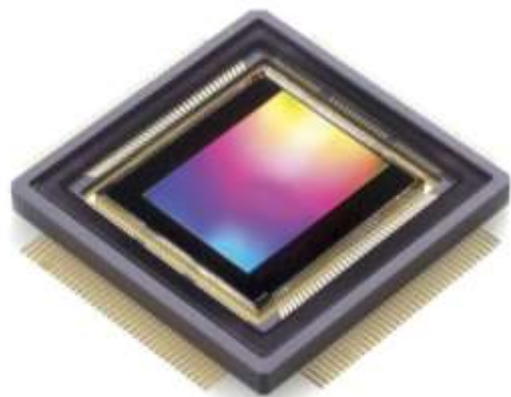
QR codes



Red LED

Information Technology

CCD and CMOS sensors for imaging



4 Megapixel CMOS image sensor
(Courtesy of Teledyne-DALSA)

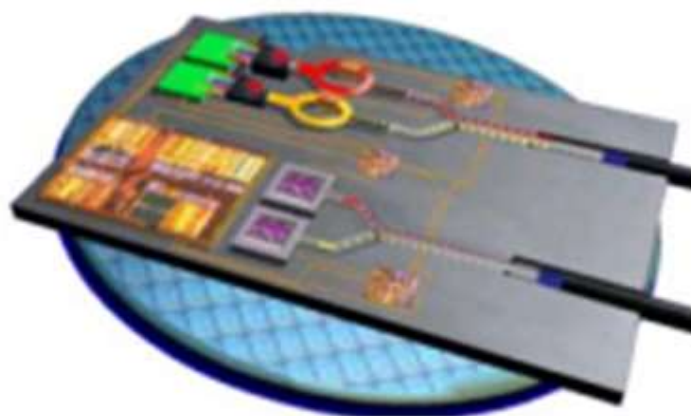


CCD Image Sensor



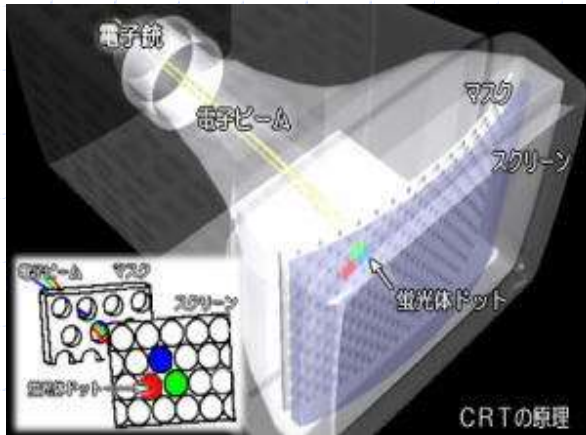
Optical interconnects

(mainly in high performance computing context today)



Displays

Can you tell how many kinds of displays are found in the market?



LED for displays



Solid State Lighting

➡ LEDs for indoor lighting

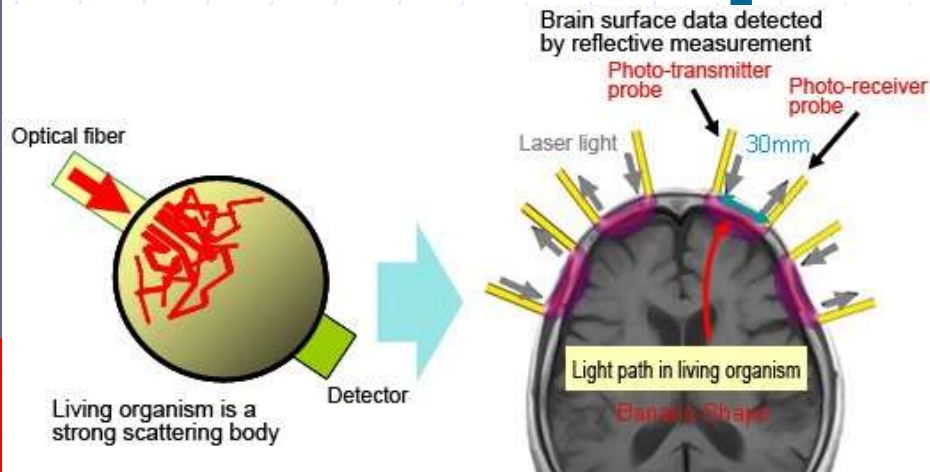


➡ LEDs and Lasers for artistic lighting



LED for traffic light

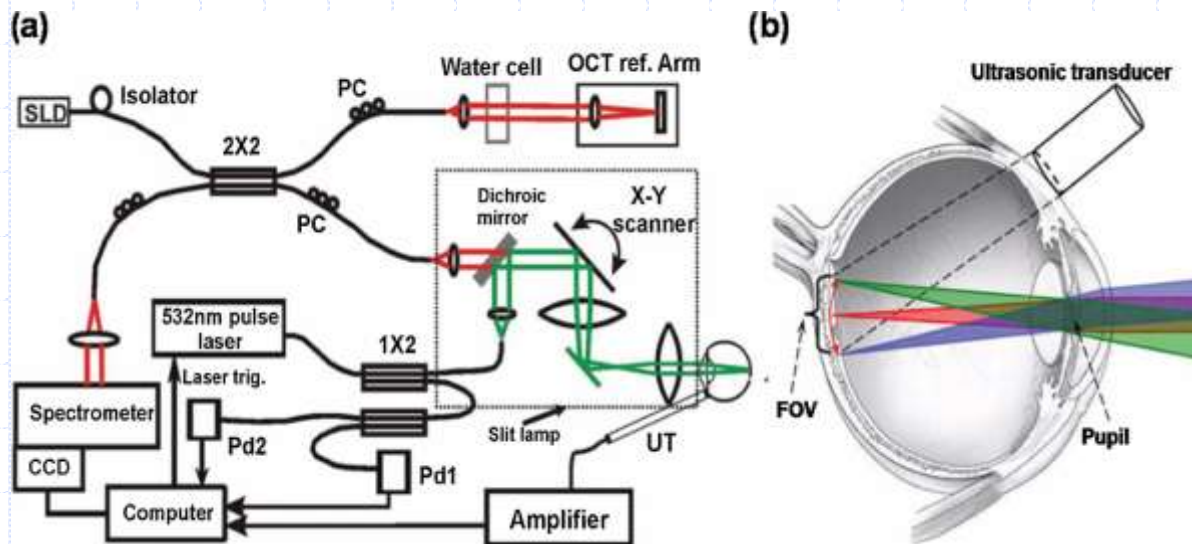
Biophotonics



Near-Infrared Spectroscopy

Optical tomography

➡ Optical tweezers, optical scalpels

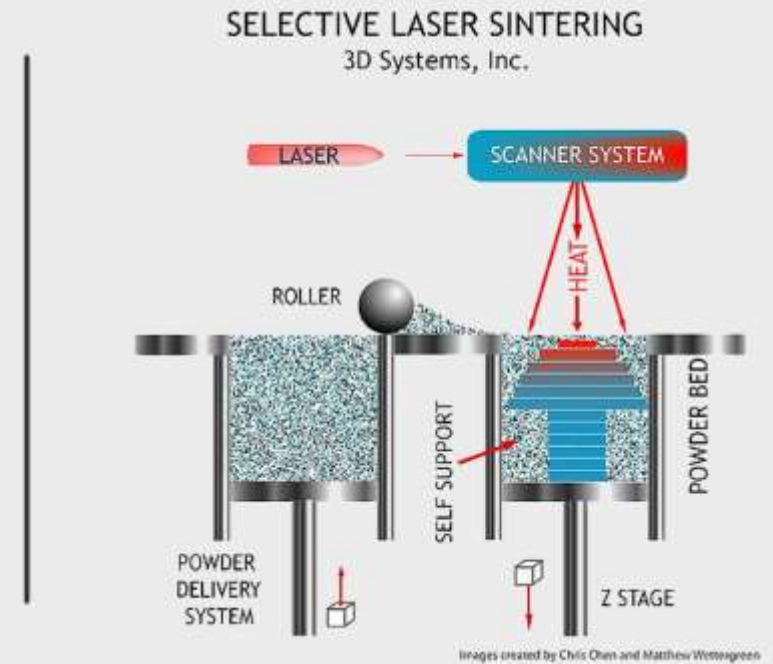
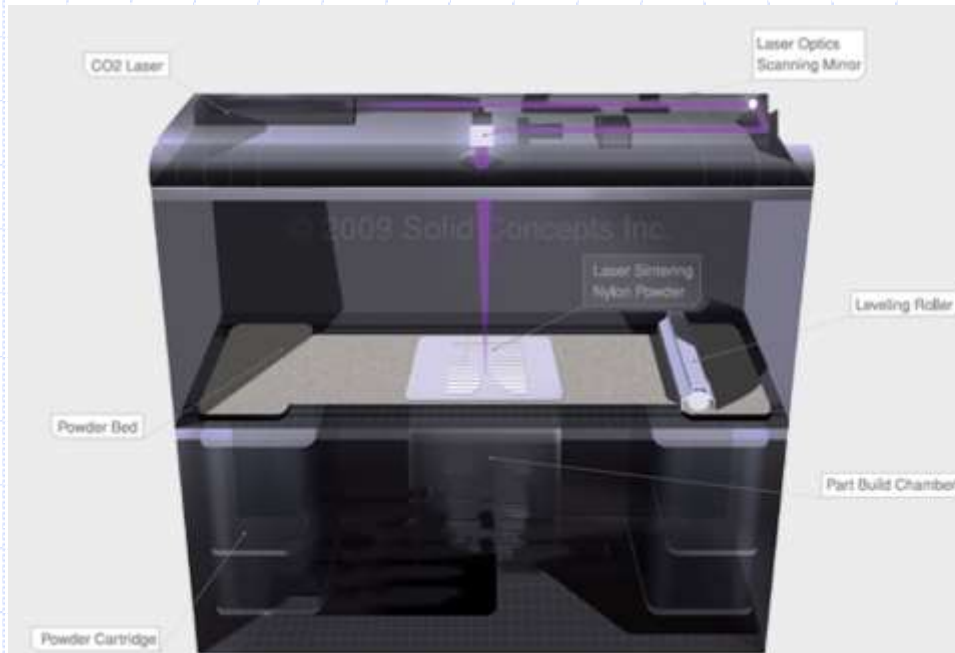


Photoacoustic imaging of the eye



Laser surgery

Production Technology



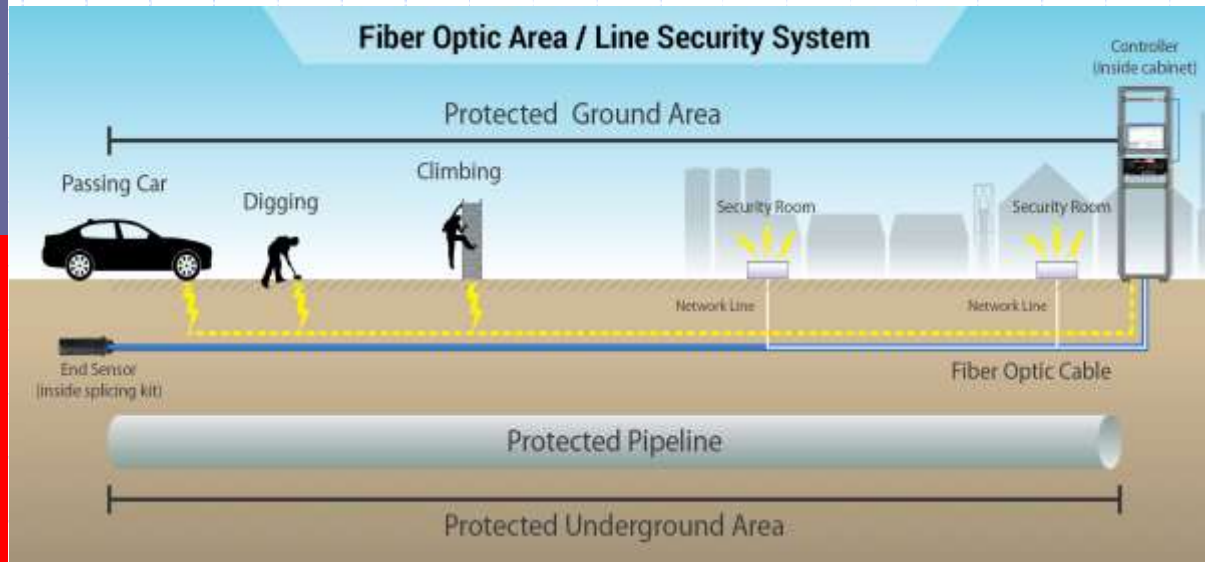
Laser Cutters



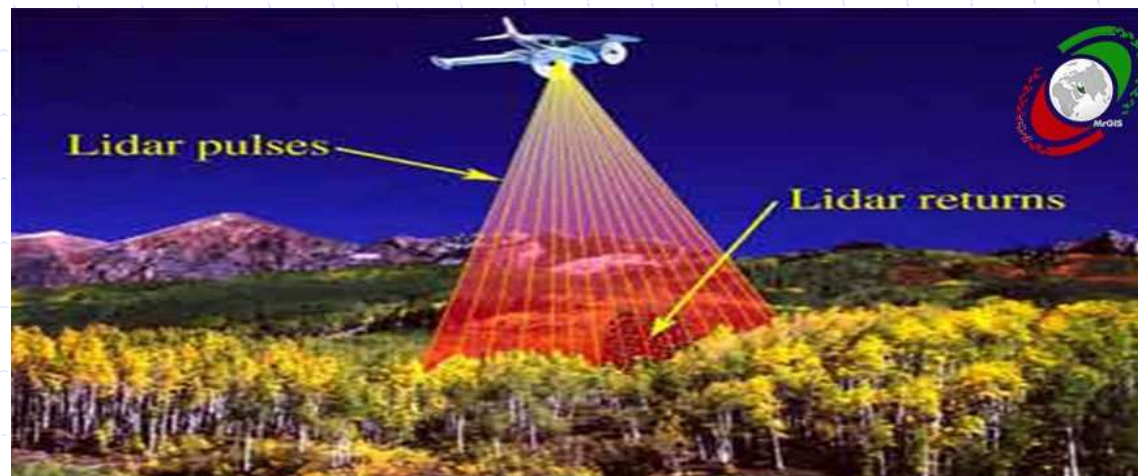
Laser Welding

Security

Intrusion detection



light detection and ranging (LIDAR)



Military

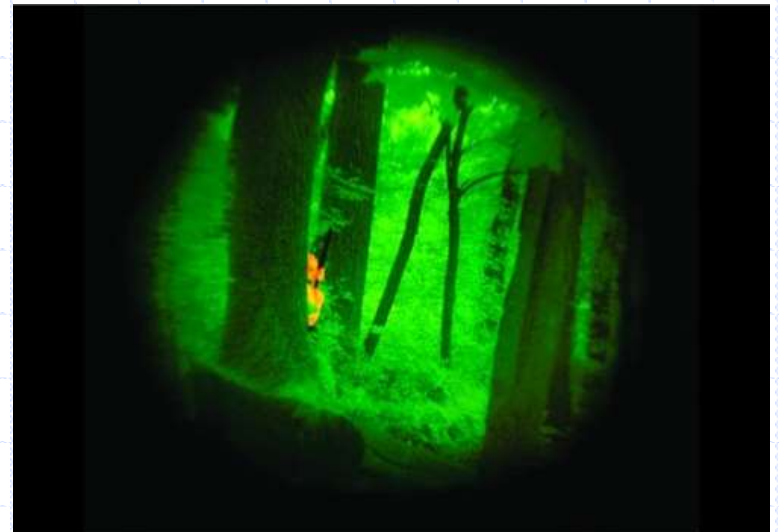
- ➡ Surveillance
- ➡ Weapon guidance
- ➡ Countermeasures and laser guns



ship-based infrared laser weapon system



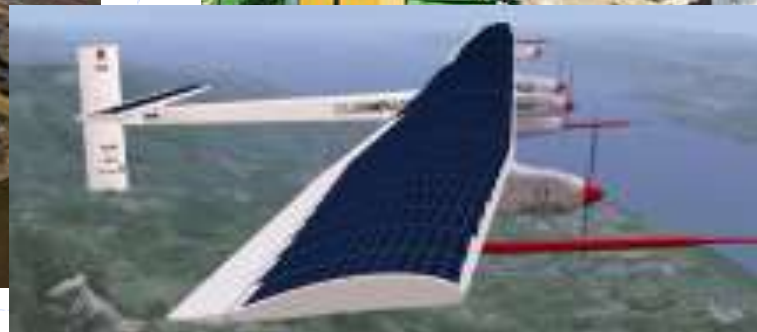
Laser Guns



Night Vision

Solar Cells

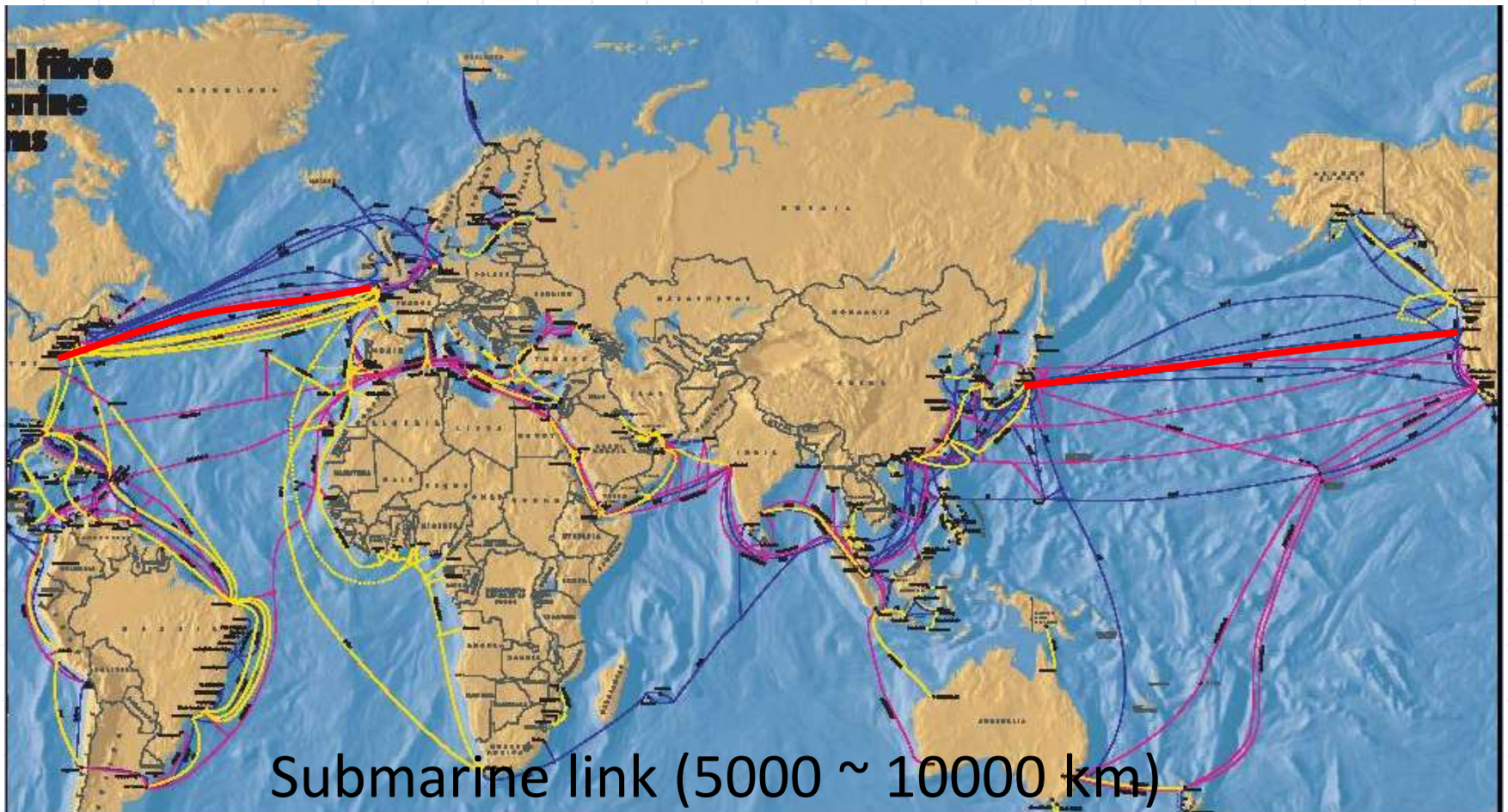
Energy



Telecommunications

Probably the **biggest application** for photonics is telecommunications, and more specifically the internet, which would not be possible without fiber-optic technology.

Lasers, modulators, fibers, detectors for communication systems



Worldwide Fiber Deployment

Deploying Fiber at the speed of Mach 3

Optical
Fiber



- In 2001, fiber was deployed at a rate of ~ 2000 miles every hour

2018 *This Is What Happens In An Internet Minute*



Telecommunications

Free-space optical links



laser air waveguides



PHOTONICS INDUSTRY

Because of its widespread, optics plays an important role in the global economy.

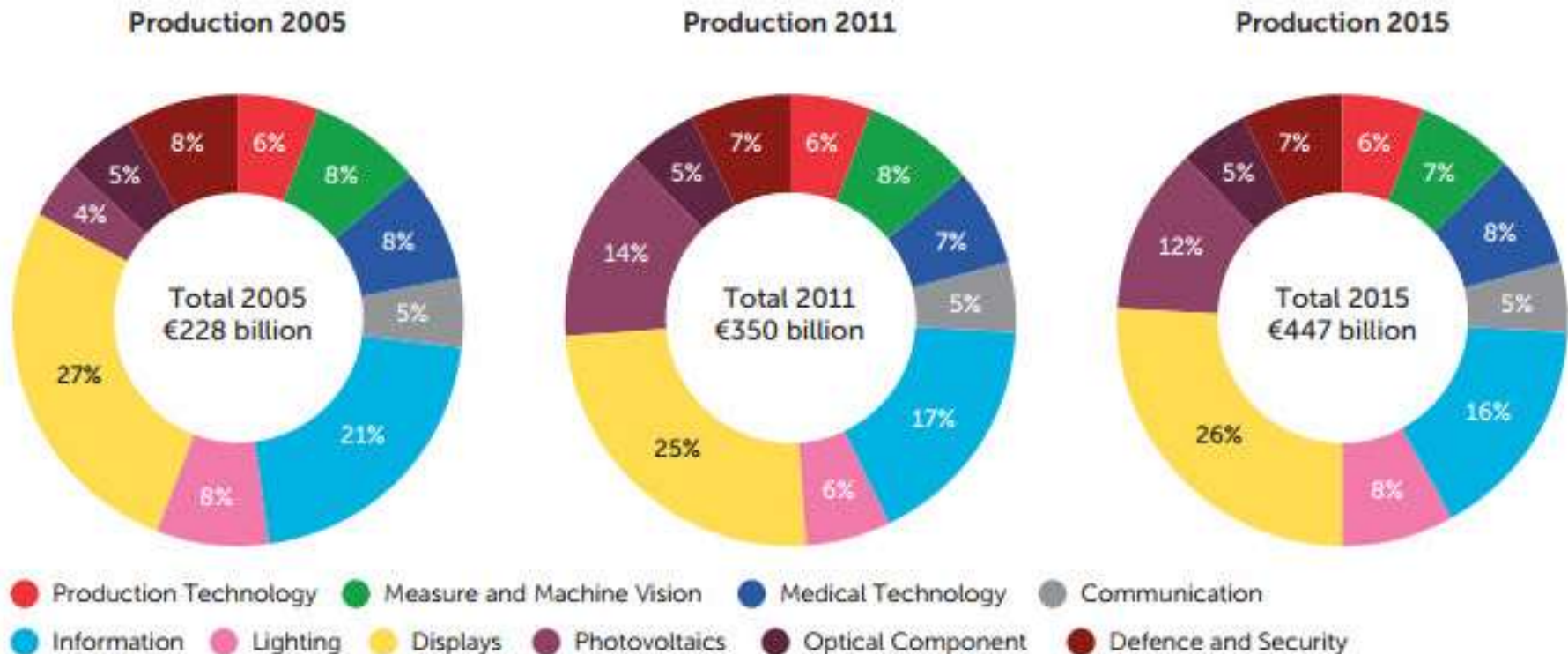
Innovation = Invention × Commercialization

The Photonics Industry

Global Photonics Industry

Development of the various Segments over Time – most Fluctuation came from Photovoltaics followed by Information Technology

Production Volume on Euro Basis*



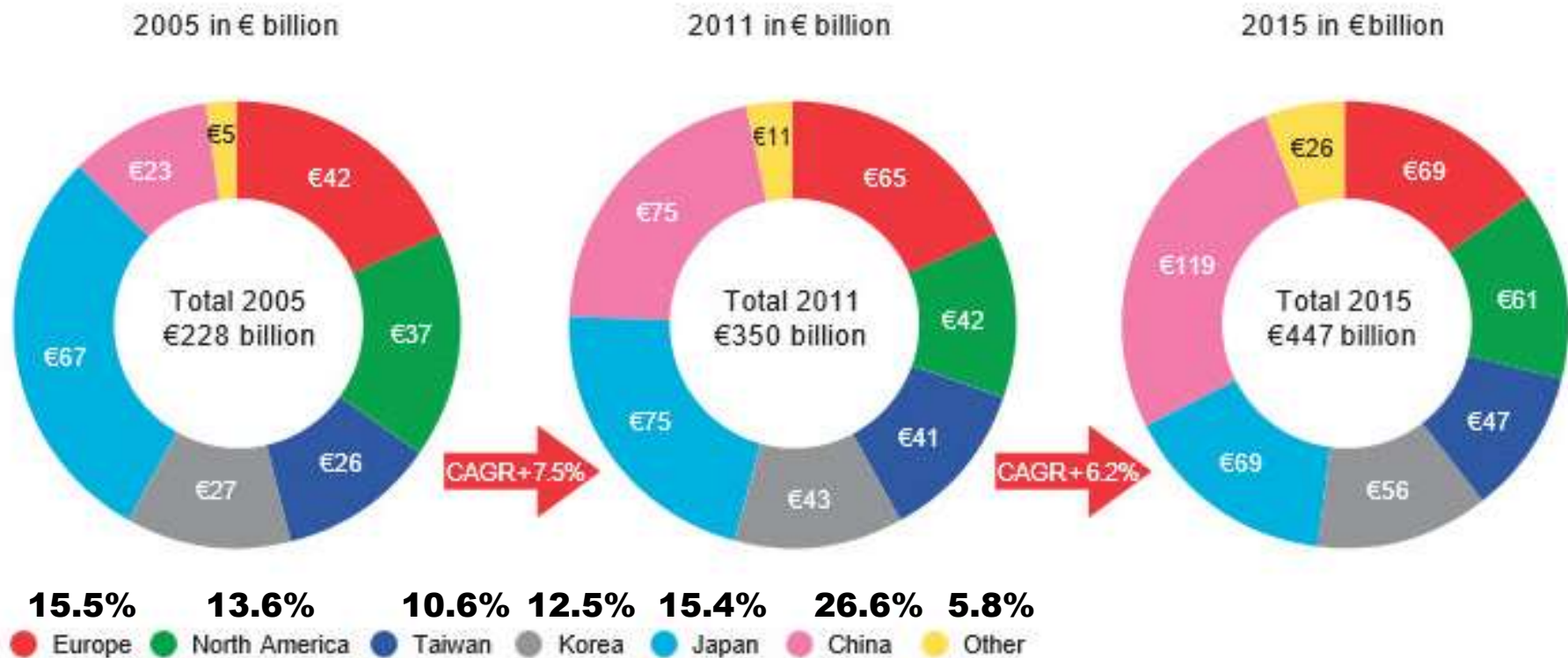
*including Photovoltaics

Source: Optech Consulting, Market Research Study 24.1.2017

The Photonics Industry

Solid growth above global GDP: Global Photonics Industry grew from a €228 billion industry in 2005 to a €447 billion industry in 2015.

Production Volume on Euro Basis*



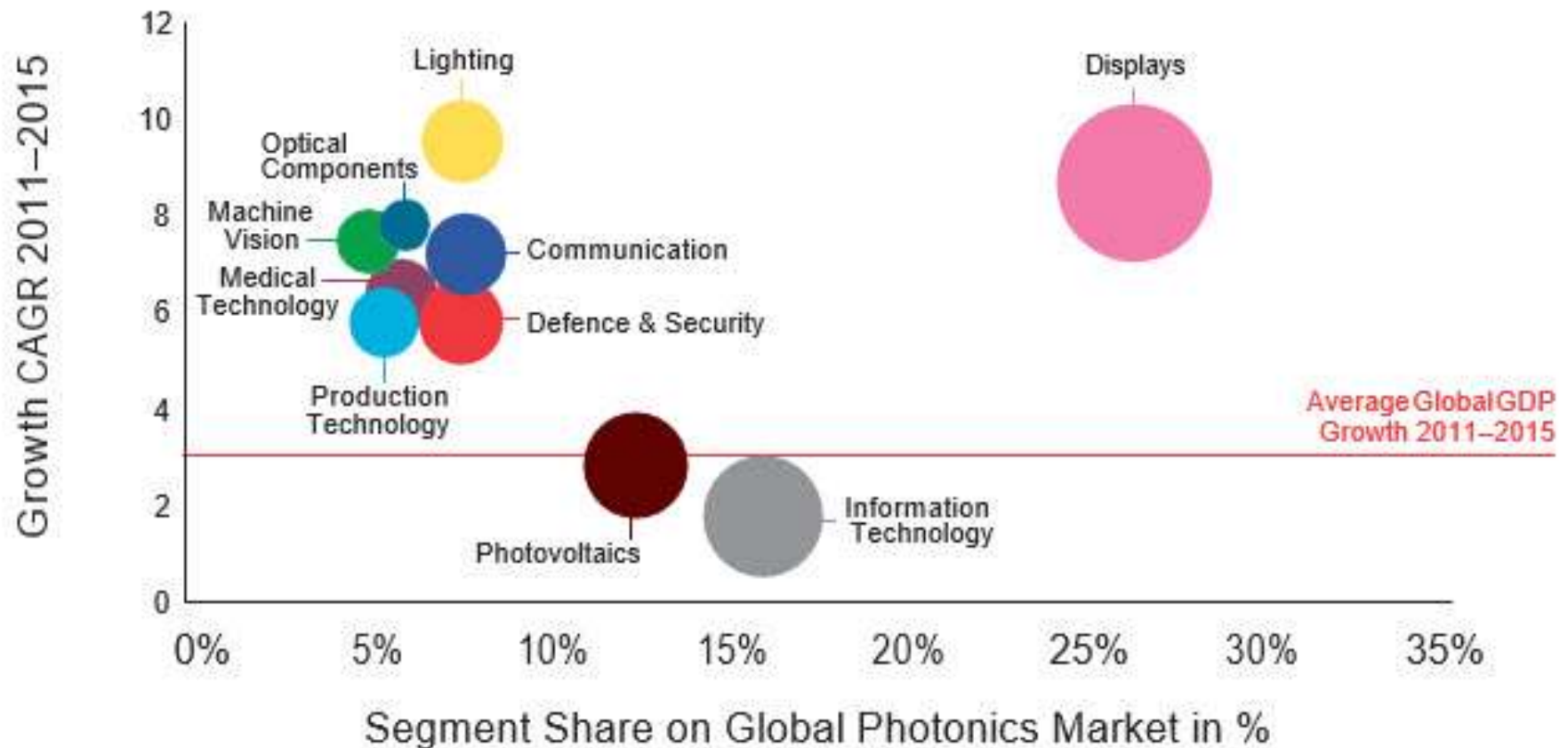
*including Photovoltaics, which is not subject of the PPP

Source: Optech Consulting, MarketResearch Study 24.1.2017

The Photonics Industry

Global Segments growth compared to global GDP growth – except for Information Technology and Photovoltaics all segments grow faster

Global Photonics Segments 2015 Data on Euro basis

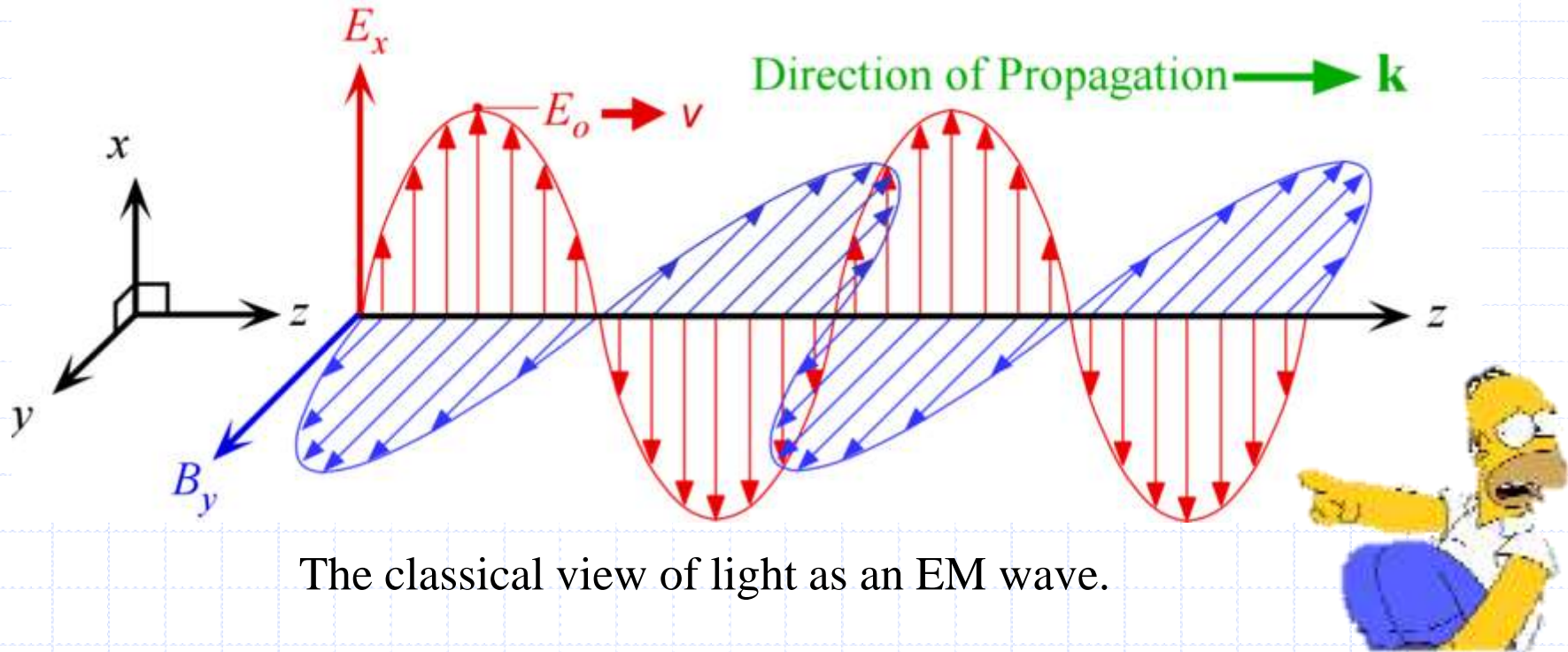


Question For Thoughts

- What will happen in the field of Photonics in 10, 20, 50 years?
- What is the area of Photonics that you are going to work in?
- Can we replace Electronics with Photonics?
- Can you create a new research area in Photonics? New Industry?
- What are the photonics devices that you think have improved your life?
- What is the Photonics device that you think would be great to have but does not exist?

WAVE NATURE OF LIGHT

Light as an EM Wave



The classical view of light as an EM wave.

- By the late 19th century, the theoretical work of **Maxwell** and the experiments of **Hertz** had resulted in the EM view of light, in which it holds that light consists of coupled time-varying electric and magnetic fields E_x and B_y that are perpendicular to each other and to the direction of propagation z and satisfy a wave equation (which itself can be derived from Maxwell's equations).

Light as an EM Wave

Traveling wave description

$$E_x(z, t) = E_o \cos(\omega t - kz + \phi_o)$$

E_x = electric field along x at position z at time t ,

k = **propagation constant, or wavenumber** = $2\pi/\lambda$

λ = wavelength

ω = angular frequency = $2\pi\nu$ (ν = frequency)

E_o = amplitude of the wave

ϕ_o = Phase constant; at $t = 0$ and $z = 0$, E_x may or may not necessarily be zero depending on the choice of origin.

$(\omega t - kz + \phi_o) = \phi$ = **phase** of the wave

This equation describes a **monochromatic plane wave** of infinite extent in the x and y directions traveling in the positive z direction

Wavefronts in Plane EM Wave

A surface over which the phase of a wave is constant is referred to as a **wavefront**.

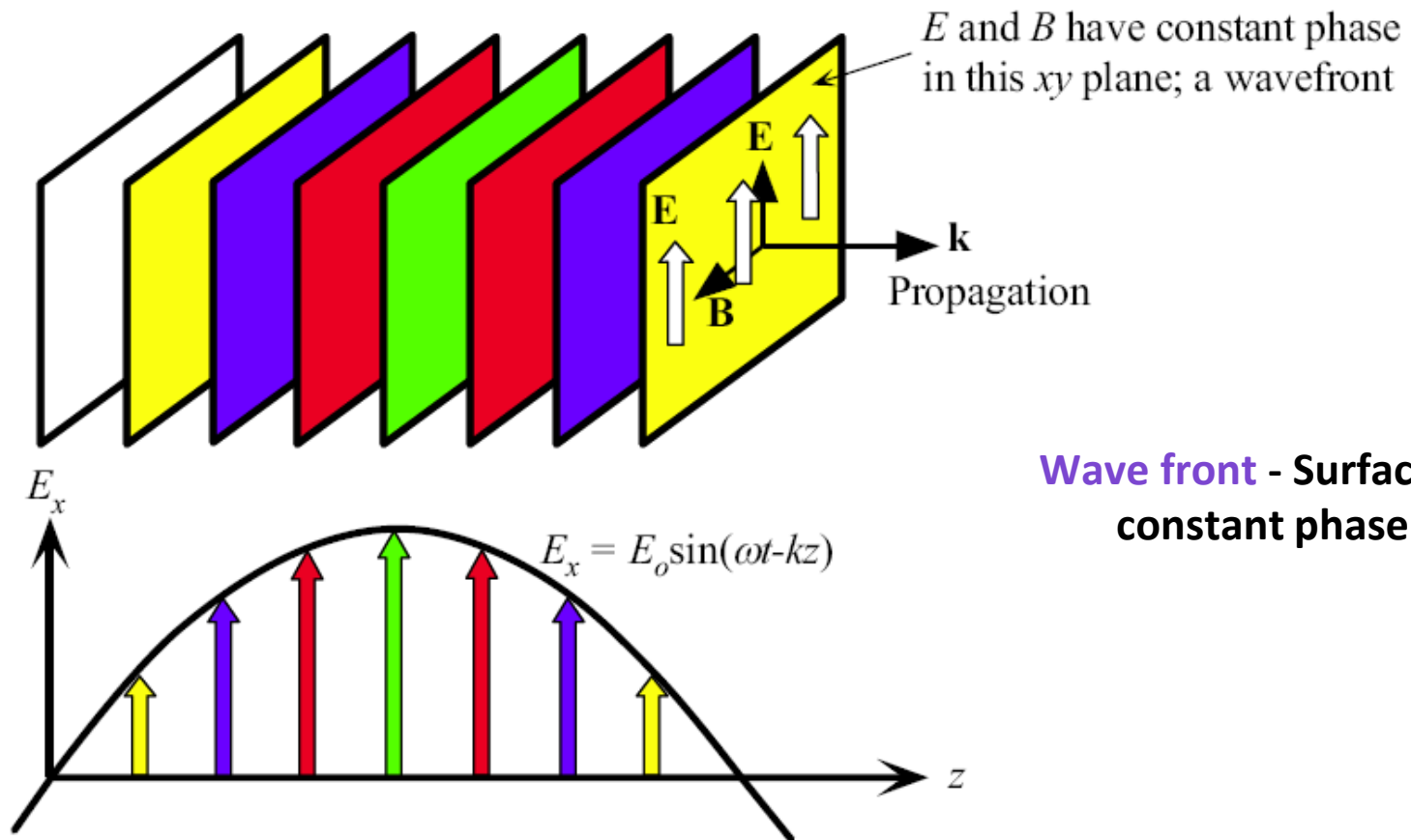
A **wavefront** of a plane wave is a plane **perpendicular** to the **direction of propagation**.

The interaction of a light wave with a nonconducting medium (conductivity, $\sigma = 0$) uses the electric field component E_x rather than B_y .

It is the **electric field** that displaces the electrons in molecules or ions in the crystal and thereby gives rise to the polarization of matter.

Optical field refers to the electric field E_x .

Wavefronts in Plane EM Wave



Wave front - Surface of constant phase

A plane EM wave traveling along z , has **the same E_x** (or B_y) at any point in a given xy plane.

All electric field vectors in a given xy plane are **therefore in phase**. The xy planes are of infinite extent in the x and y directions.

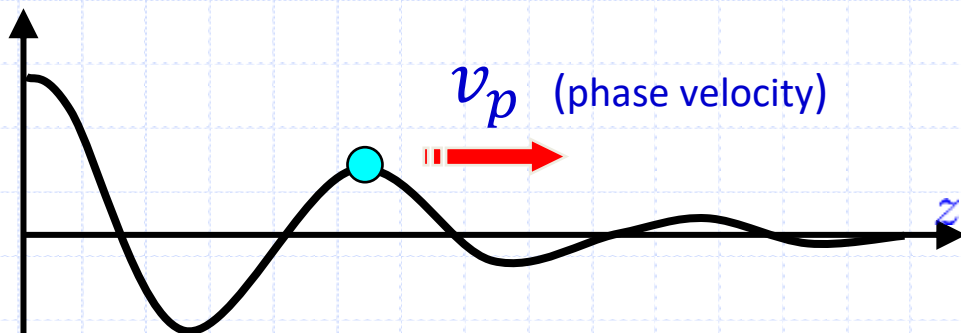
Phase Velocity

The time and space evolution of a given phase ϕ , for example that corresponding to a maximum field is described by

$$\phi = \omega t - kz + \phi_o = \text{constant}$$

During a time interval δt , this constant phase (and hence the maximum field) moves a distance δz . The phase velocity of this wave is therefore $\delta z / \delta t$. The **phase velocity** v is

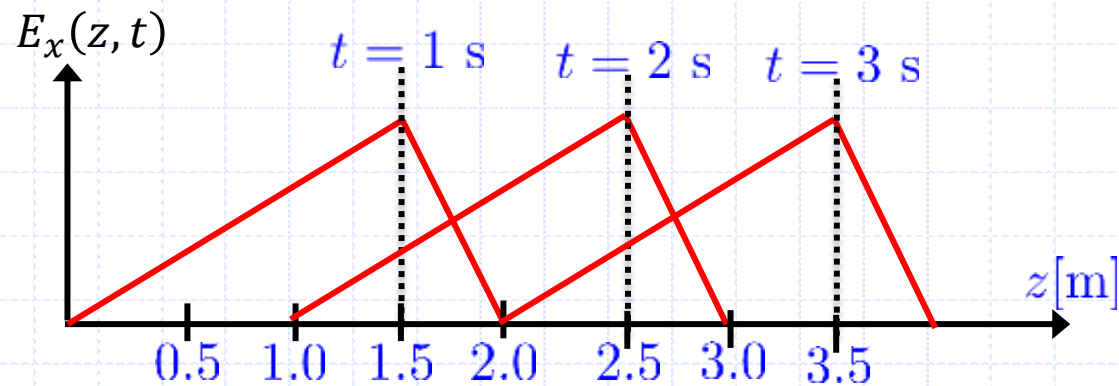
$$v = \frac{\delta z}{\delta t} = \frac{\omega}{k} = v\lambda$$



Wave Motion

- $E_x(z, t) = E_o \cos(\omega t - kz + \phi_o) = E_o \cos\left(\omega\left(t - \frac{k}{\omega}z\right) + \phi_o\right)$
- $E_x(z, t) = f\left(t - \frac{z}{V}\right)$

Let $V = 1 \text{ m/s}$



At $t = 1 \text{ s}$, focus on the peak located at $z = 1.5 \text{ m}$

$$\Rightarrow S = t - \frac{z}{V} = 1 - \frac{1.5}{1} = -0.5$$

The argument S stays constant for varying t & $z \Rightarrow$ **at $t = 2 \text{ s}$** , for example:

$$S = -0.5 = t - \frac{z}{V} = 2 - \frac{z}{1} \Rightarrow z = 2.5 \text{ m.}$$

So, the peak has now moved to position $z = 2.5 \text{ m}$ at $t = 2 \text{ s}$.

Likewise, every point on this function moves the same distance (1 m) in this time (1 s). \Rightarrow This is called **wave motion**.

The speed of this movement is: $\frac{\delta z}{\delta t} = \frac{1 \text{ m}}{1 \text{ s}} = 1 \text{ m/s} = V$



Phase change over a distance Δz

$$\phi = \omega t - kz + \phi_o$$

The **phase difference** between two points separated by Δz is simply

$$D\phi = kDz$$

since ωt is the same for each point

If this phase difference is 0 or multiples of 2π then the two points are in phase. Thus, the phase difference $\Delta\phi$ can be expressed as

$$\Delta\phi = k\Delta z = 2\pi\Delta z/\lambda$$

Exponential Notation

Recall that

$$\cos\phi = \text{Re}[\exp(j\phi)]$$

where Re refers to the real part. We then need to take the real part of any complex result at the end of calculations. Thus,

$$E_x(z,t) = \text{Re}[E_o \exp(j\phi_o) \exp j(\omega t - kz)]$$

or

$$E_x(z,t) = \text{Re}[E_c \exp j(\omega t - kz)]$$

where $E_c = E_o \exp(j\phi_o)$ is a complex number that represents the amplitude of the wave and includes the constant phase information ϕ_o .

Wave Vector \mathbf{k}

Direction of propagation is indicated with a vector \mathbf{k} , called the **wave vector**, whose magnitude is the **propagation constant**, $k = 2\pi/\lambda$.

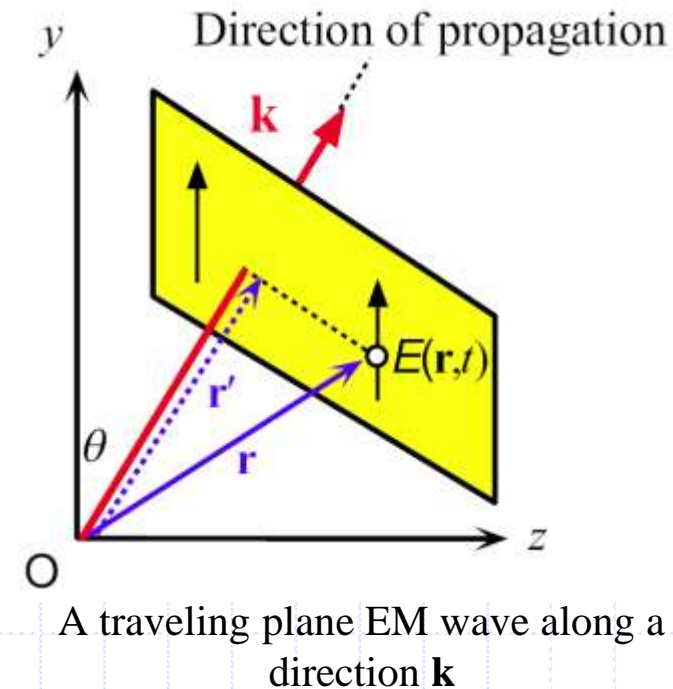
\mathbf{k} is **perpendicular** to constant phase planes.

When the EM wave is propagating along some arbitrary direction \mathbf{k} , then the electric field $E(\mathbf{r},t)$ at a point \mathbf{r} on a plane perpendicular to \mathbf{k} is

$$E(\mathbf{r},t) = E_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_0)$$

If propagation is along z , $\mathbf{k} \cdot \mathbf{r}$ becomes kz .

In general, if \mathbf{k} has components k_x , k_y and k_z along x , y and z , then from the definition of the dot product, $\mathbf{k} \cdot \mathbf{r} = k_x x + k_y y + k_z z$.



Refractive Index

When an EM wave is traveling in a dielectric medium, the oscillating electric field **polarizes** the molecules of the medium at the frequency of the wave.

The **stronger** is the interaction between the field and the dipoles, the **slower** is the propagation of the wave.

Maxwell's Wave Equation in an isotropic medium

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \epsilon_o \epsilon_r \mu_o \frac{\partial^2 E}{\partial t^2} = 0$$

A plane wave is a solution of Maxwell's wave equation

$$E_x = E_o \cos(\omega t - kz + \phi_o)$$

The phase velocity of this plane wave in the medium is given by

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_o \epsilon_r \mu_o}}$$

Phase Velocity and ϵ_r

The **relative permittivity** ϵ_r measures the ease with which the medium becomes polarized and hence it indicates the extent of interaction between the field and the induced dipoles.

For an EM wave traveling in a nonmagnetic dielectric medium of relative permittivity ϵ_r , the **phase velocity** v is given by

$$v = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_0}}$$

The phase velocity in vacuum is

$$c = \frac{\omega}{k_0} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Refractive Index

The ratio of the speed of light in free space to its speed in a medium is called the **refractive index n** of the medium,

$$n = \frac{c}{v} = \sqrt{\epsilon_r}$$

Refractive Index and Propagation Constant

k_o	Free-space propagation constant (wave vector)
k_o	$2\pi/\lambda_o$
λ_o	Free-space wavelength
k_{medium}	Propagation constant (wave vector) in the medium
λ_{medium}	Wavelength in the medium

$$n = \frac{k_{\text{medium}}}{k_o} = \frac{\lambda_o}{\lambda_{\text{medium}}}$$

$$k_{\text{medium}} = nk_o$$

$$\lambda_{\text{medium}} = \lambda_o / n$$

Refractive Index and Wavelength

It is customary to drop the subscript *o* on *k* and *λ*

$$k_{\text{medium}} = nk$$

In free space

$$\lambda_{\text{medium}} = \lambda / n$$

n depends on the wavelength λ

Dispersion relation: $n = n(\lambda)$

Cauchy dispersion relation

$$n = n_{-2}(h\nu)^{-2} + n_0 + n_2(h\nu)^2 + n_4(h\nu)^4$$

where $h\nu$ is the photon energy, and n_0 , n_{-2} , n_2 , and n_4 are constants

Sellmeier Equation

$$n^2 = 1 + \frac{A_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3\lambda^2}{\lambda^2 - \lambda_3^2}$$

where A_1 , A_2 , A_3 and λ_1 , λ_2 , λ_3 are constants, called **Sellmeier coefficients**.

n depends on the wavelength λ

TABLE 1.2 Sellmeier and Cauchy coefficients

Sellmeier	A_1	A_2	A_3	λ_1 (μm)	λ_2 (μm)	λ_3 (μm)
SiO ₂ (fused silica)	0.696749	0.408218	0.890815	0.0690660	0.115662	9.900559
86.5%SiO ₂ -13.5%GeO ₂	0.711040	0.451885	0.704048	0.0642700	0.129408	9.425478
GeO ₂	0.80686642	0.71815848	0.85416831	0.068972606	0.15396605	11.841931
Sapphire	1.023798	1.058264	5.280792	0.0614482	0.110700	17.92656
Diamond	0.3306	4.3356	–	0.1750	0.1060	–

Cauchy	Range of $h\nu$ (eV)	n_{-2} (eV ²)	n_0	n_2 (eV ⁻²)	n_4 (eV ⁻⁴)
Diamond	0.05–5.47	-1.07×10^{-5}	2.378	8.01×10^{-3}	1.04×10^{-4}
Silicon	0.002–1.08	-2.04×10^{-8}	3.4189	8.15×10^{-2}	1.25×10^{-2}
Germanium	0.002–0.75	-1.0×10^{-8}	4.003	2.2×10^{-1}	1.4×10^{-1}

Source: Sellmeier coefficients combined from various sources. Cauchy coefficients from D. Y. Smith *et al.*, *J. Phys. CM*, 13, 3883, 2001.

Group Velocity

Consider two sinusoidal waves that are close in frequency, that is, they have frequencies $\omega - \delta\omega$ and $\omega + \delta\omega$

Their wavevectors will be $k - \delta k$ and $k + \delta k$. The resultant wave is

$$E_x(z,t) = E_o \cos[(\omega - \delta\omega)t - (k - \delta k)z] \\ + E_o \cos[(\omega + \delta\omega)t - (k + \delta k)z]$$

By using the trigonometric identity

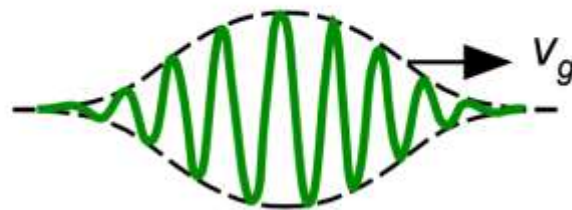
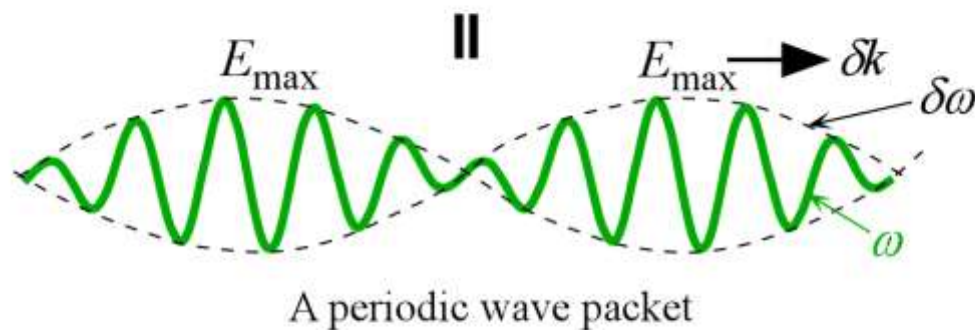
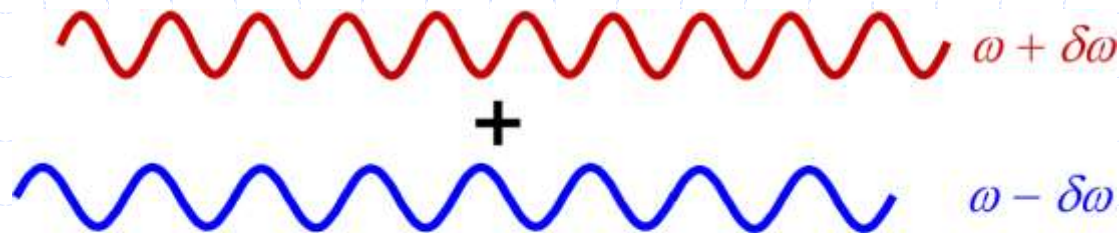
$$\cos A + \cos B = 2\cos[1/2(A - B)]\cos[1/2(A + B)]$$

we arrive at

$$E_x(z,t) = 2E_o \cos[(\delta\omega)t - (\delta k)z] [\cos(\omega t - kz)]$$

They generate a **wave packet** which contains an oscillating field at the mean frequency ω that is amplitude **modulated** by a slowly varying field of frequency $\delta\omega$

Group Velocity



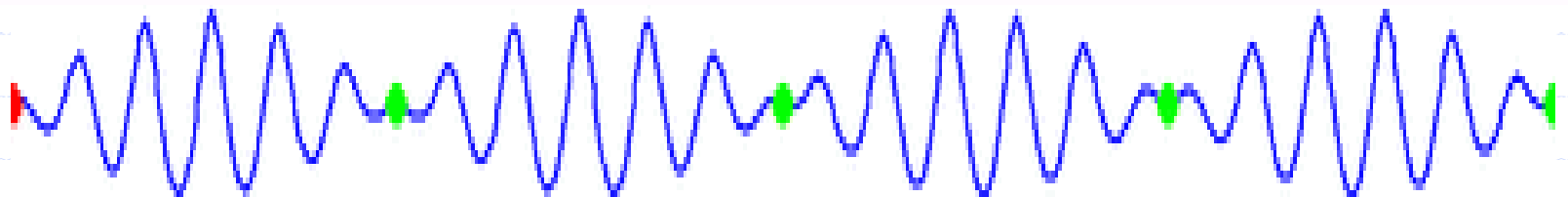
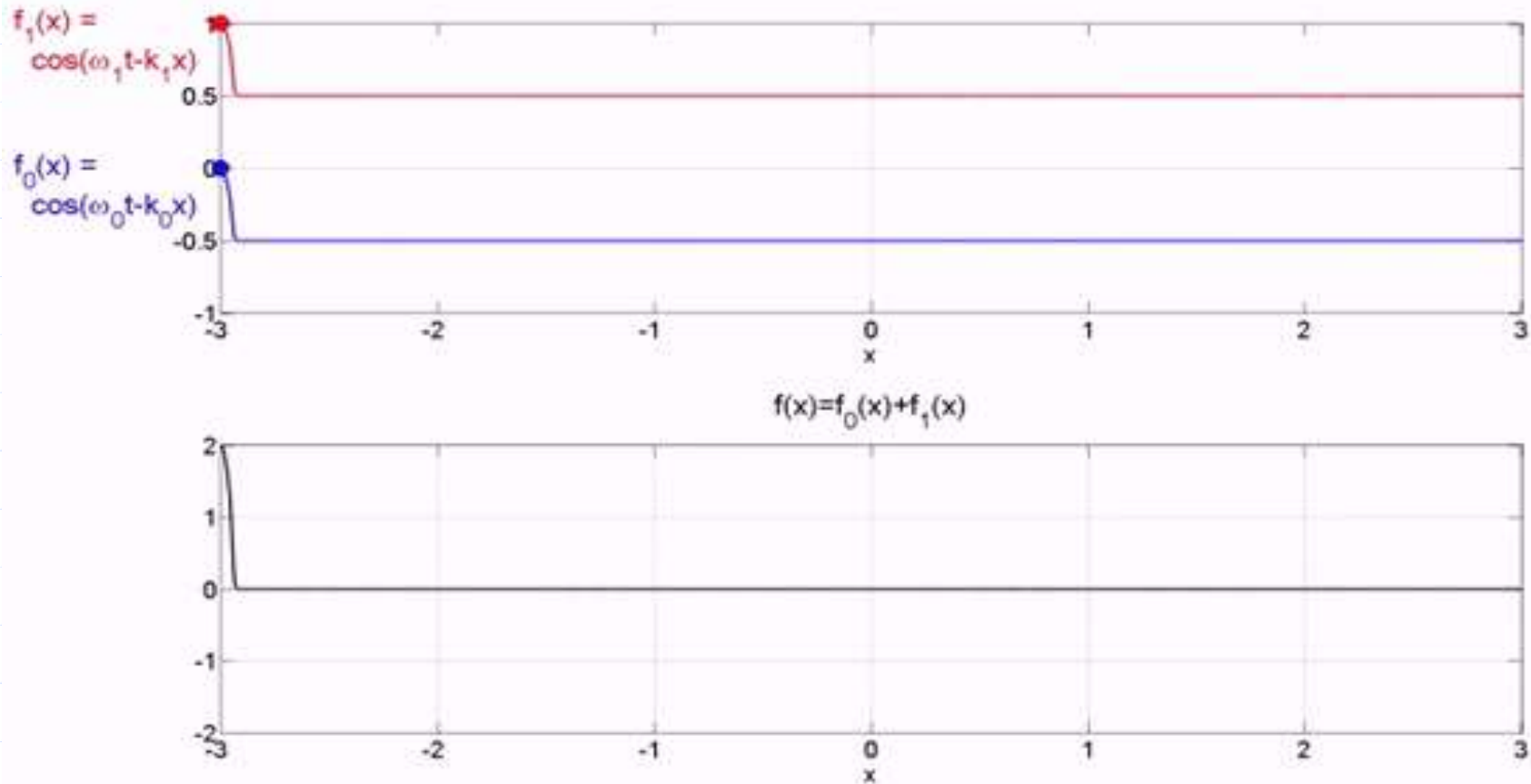
A single wave packet

$$V_g = \frac{d\omega}{dk}$$

Two slightly different wavelength waves traveling in the same direction result in a **wave packet** that has an amplitude variation that travels at the **group velocity**.



Adding up waves of different frequencies



● : phase velocity

● : group velocity

Group Velocity

$$E_x(z,t) = 2E_o \cos[(\delta\omega)t - (\delta k)z] [\cos(\omega t - kz)]$$

This represents a sinusoidal wave of frequency ω . This is amplitude modulated by a very slowly varying sinusoidal of frequency $\delta\omega$. This system of waves, *i.e.* the modulation, travels along z at a speed determined by the modulating term, $\cos[(\delta\omega)t - (\delta k)z]$.

The maximum in the field occurs when $[(\delta\omega)t - (\delta k)z] = 2m\pi = \text{constant}$ (m is an integer), which travels with a velocity

$$\frac{dz}{dt} = \frac{\delta\omega}{\delta k}$$

or

$$V_g = \frac{d\omega}{dk}$$

The group velocity therefore defines the speed with which **energy or information** is propagated.

Group Velocity

$$V_g = \frac{d\omega}{dk_{\text{medium}}}$$

$\omega = 2\pi c / \lambda_o$ and $k_{\text{medium}} = 2\pi n / \lambda_o$, λ_o is the free space wavelength.

where $n = n(\lambda)$ is a function of the wavelength.

Differentiate the above equations in red

$$d\omega = -(2\pi c / \lambda_o^2) d\lambda_o$$

$$dk_{\text{medium}} = 2\pi n (-1 / \lambda_o^2) d\lambda_o + (2\pi / \lambda_o) \left(\frac{dn}{d\lambda_o} \right) d\lambda_o$$

$$dk = -(2\pi / \lambda_o^2) \left(n - \lambda_o \frac{dn}{d\lambda_o} \right) d\lambda_o$$

$$\therefore V_g = \frac{d\omega}{dk_{\text{medium}}} = \frac{-(2\pi c / \lambda_o^2) d\lambda_o}{-(2\pi / \lambda_o^2) \left(n - \lambda_o \frac{dn}{d\lambda_o} \right) d\lambda_o} = \frac{c}{n - \lambda_o \frac{dn}{d\lambda_o}}$$

Group Velocity and Group Index

The group velocity v_g in a medium is given by,

$$v_g(\text{medium}) = \frac{d\omega}{dk_{\text{medium}}} = \frac{c}{n - \lambda_o \left(\frac{dn}{d\lambda_o} \right)}$$

This can be written as

$$v_g(\text{medium}) = \frac{c}{N_g}$$

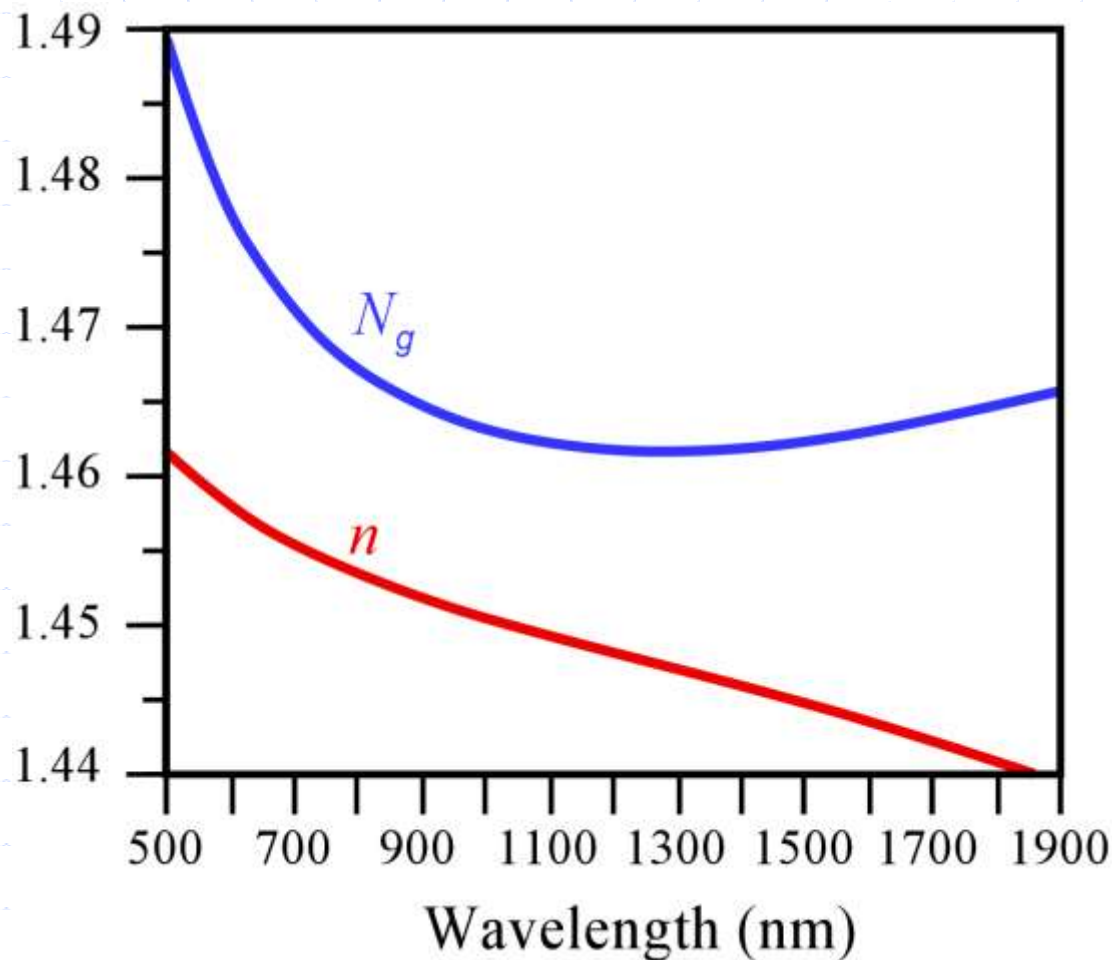
$$N_g = n - \lambda_o \left(\frac{dn}{d\lambda_o} \right)$$



is defined as the **group index of the medium**

In general, for many materials the refractive index n and hence the group index N_g depend on the wavelength of light. Such materials are called **dispersive**

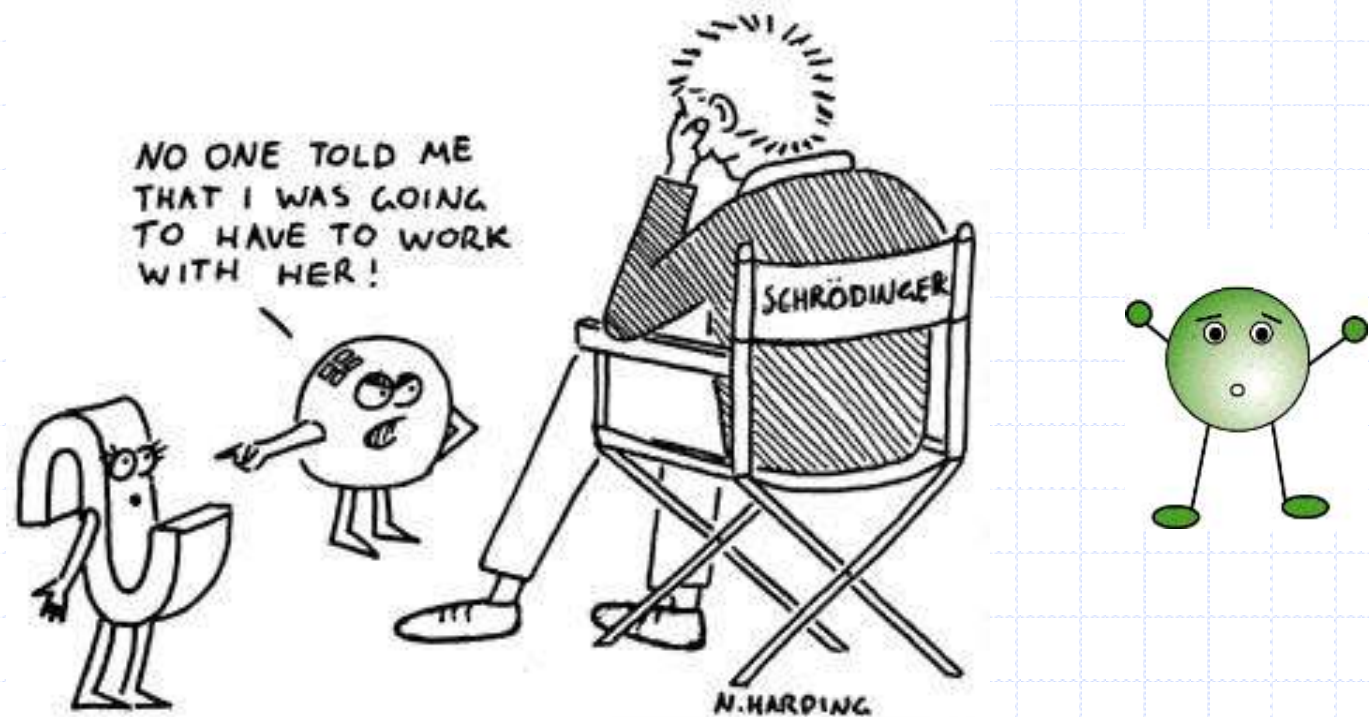
Refractive Index and Group Index



Refractive index n and the group index N_g of pure SiO_2 (silica) glass as a function of wavelength.

Light and Photons

However, the development of modern physics (and especially the work of **Planck** and **Einstein**) led to the photon view of light.



Photon energy

$$E_{\text{ph}} = h\nu = hc/\lambda$$

λ = wavelength

h = Planck's constant (6.63×10^{-34} J.s)

c = speed of light (3×10^8 m/s)

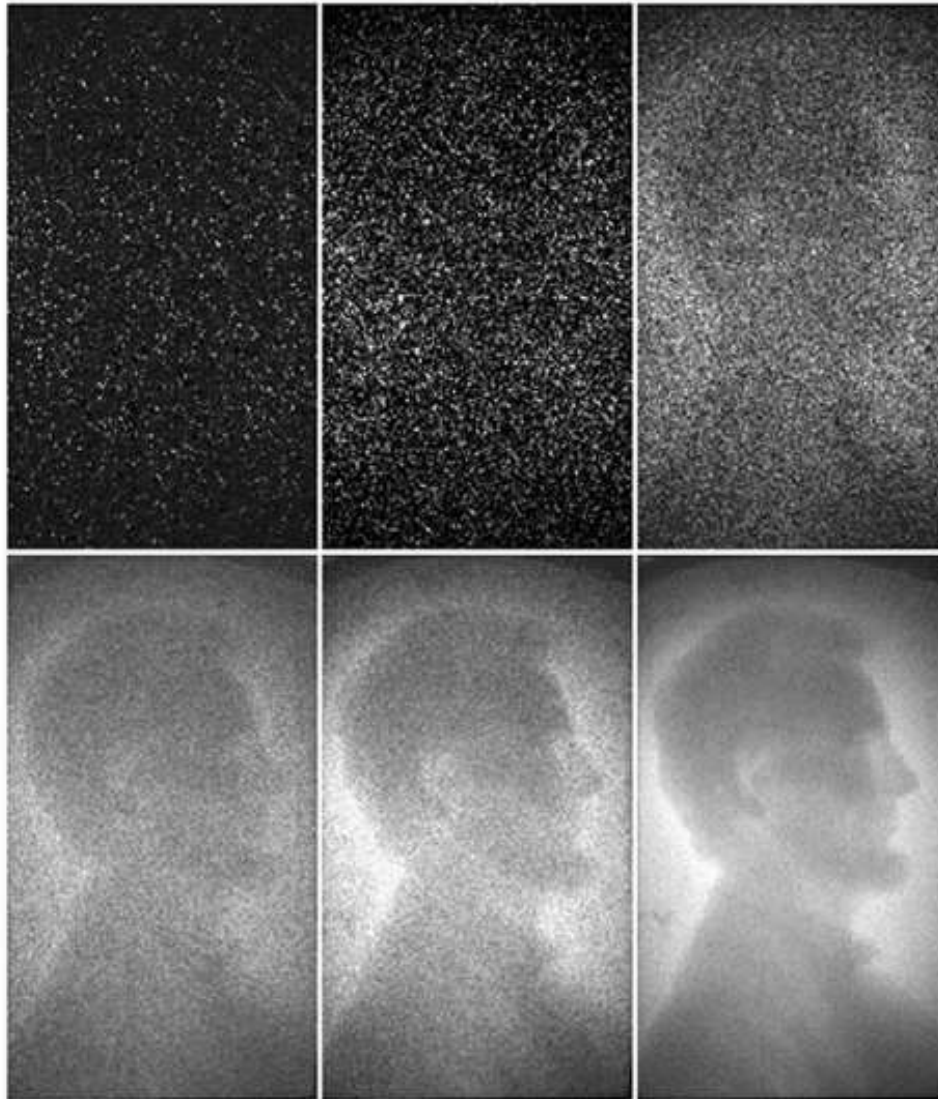
 3×10^3 photons 1.2×10^4 photons 9.3×10^4 photons 7.6×10^5 photons 3.6×10^6 photons 2.8×10^7 photons

Light consists of photons

These electronic images were made with the number of photons indicated. The discrete nature of photons means that a large number of photons are needed to constitute an image with satisfactorily discernable details.

SOURCE: A. Rose, "Quantum and noise limitations of the visual process" J. Opt. Soc. of America, vol. 43, 715, 1953.

X-rays are photons

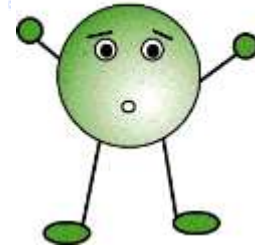
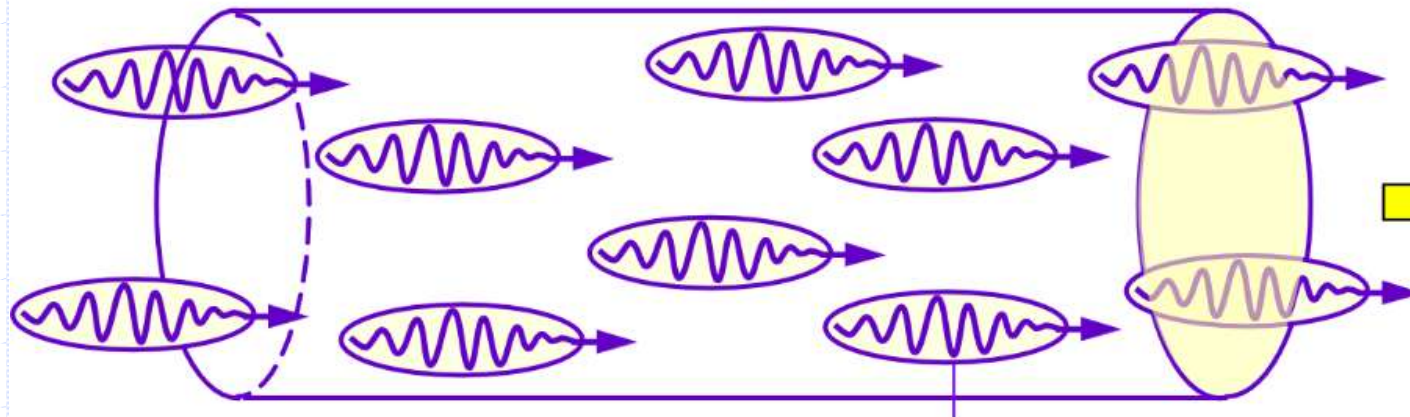


X-ray image of an American one-cent coin captured using an x-ray a-Se HARP camera. The first image at the top left is obtained under extremely low exposure and the subsequent images are obtained with increasing exposure of approximately one order of magnitude between each image. The slight attenuation of the X-ray photons by Lincoln provides the image. The **image sequence clearly shows the discrete nature of x-rays**, and hence their description in terms of photons.

SOURCE: Brian J. M. Lui, D. C. Hunt, A. Reznik, K. Tanioka, and J. A. Rowlands, "X-ray imaging with amorphous selenium: Pulse height measurements of avalanche gain fluctuations", Medical Physics, 33, 3183-3192 (2006); Figure 3.

Light and Photons

Stream of photons



Flux of photons

$$E_{ph} = h\nu = hc/\lambda \text{ and } p = h/\lambda$$

Intuitive visualization of light consisting
of a stream of photons

λ = wavelength

h = Planck's constant

(6.63×10^{-34} J.s)

c = speed of light (3×10^8 m/s)

p = Momentum

Light Intensity (Irradiance): the amount of energy
flowing through a unit area per unit time

$$I = \Gamma_{ph} h\nu$$

Photon flux density $\Gamma_{ph} = \frac{\Delta N_{ph}}{A \Delta t}$

ΔN_{ph} is the net number of photons crossing an
area A in time Δt .

Example

- **ENERGY OF A BLUE PHOTON** What is the energy of a blue photon that has a wavelength of 450 nm?

Solution

- The energy of the photon is given by

$$E_{\text{ph}} = hv = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J.s})(3 \times 10^8 \text{ m s}^{-1})}{450 \times 10^{-9} \text{ m}} = 4.4 \times 10^{-19} \text{ J}$$

- Generally, with such small energy values, we prefer **electron-volts** (eV), so the energy of the photon is

$$E_{\text{ph}} = \frac{4.4 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 2.75 \text{ eV}$$

- Useful relations:

$$E_{\text{ph}} = \frac{hc}{\lambda} = \frac{(4.143 \times 10^{-15} \text{ eV.s})(3 \times 10^8 \text{ m s}^{-1})}{\lambda} = \frac{1240 \text{ (eV.nm)}}{\lambda \text{ (nm)}}$$

$$\lambda \text{ (nm)} = \frac{1240 \text{ (eV.nm)}}{E_{\text{ph}}}$$

Thank you



Have a nice day!