Lecture 2

Wave Nature of Light

ECE 325

OPTOELECTRONICS

Menoufia University

Kasap–1.5, 1.6, and 1.7

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Snell's Law or Descartes's Law?

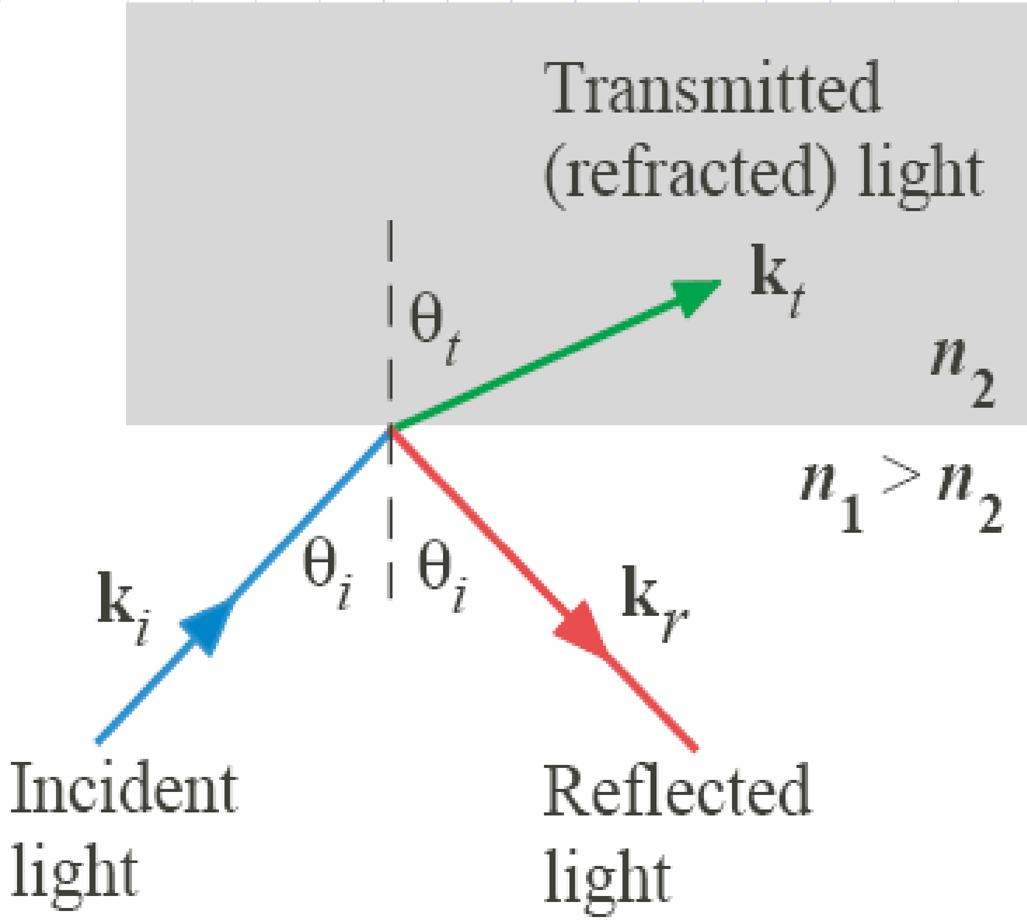
WILLEBRORD SNELL

Willebrord Snellius (Willebrord Snel van Royen, 1580–1626) was a Dutch astronomer and a mathematician, who was a professor at the University of Leiden. He discovered his law of refraction in 1621 which was published by Réne Descartes in France 1637; it is not known whether Descartes knew of Snell's law or formulated it independently. *(Courtesy of AIP Emilio Segre Visual Archives, Brittle Books Collection.)*

René Descartes (1596–1650) was a French philosopher who was also involved with mathematics and sciences. He has been called the "Father of Modern Philosophy." Descartes was responsible for the development of Cartesian coordinates and analytical geometry. He also made significant contributions to optics, including reflection and refraction. *(Courtesy of Georgios Kollidas/Shutterstock.com.)*

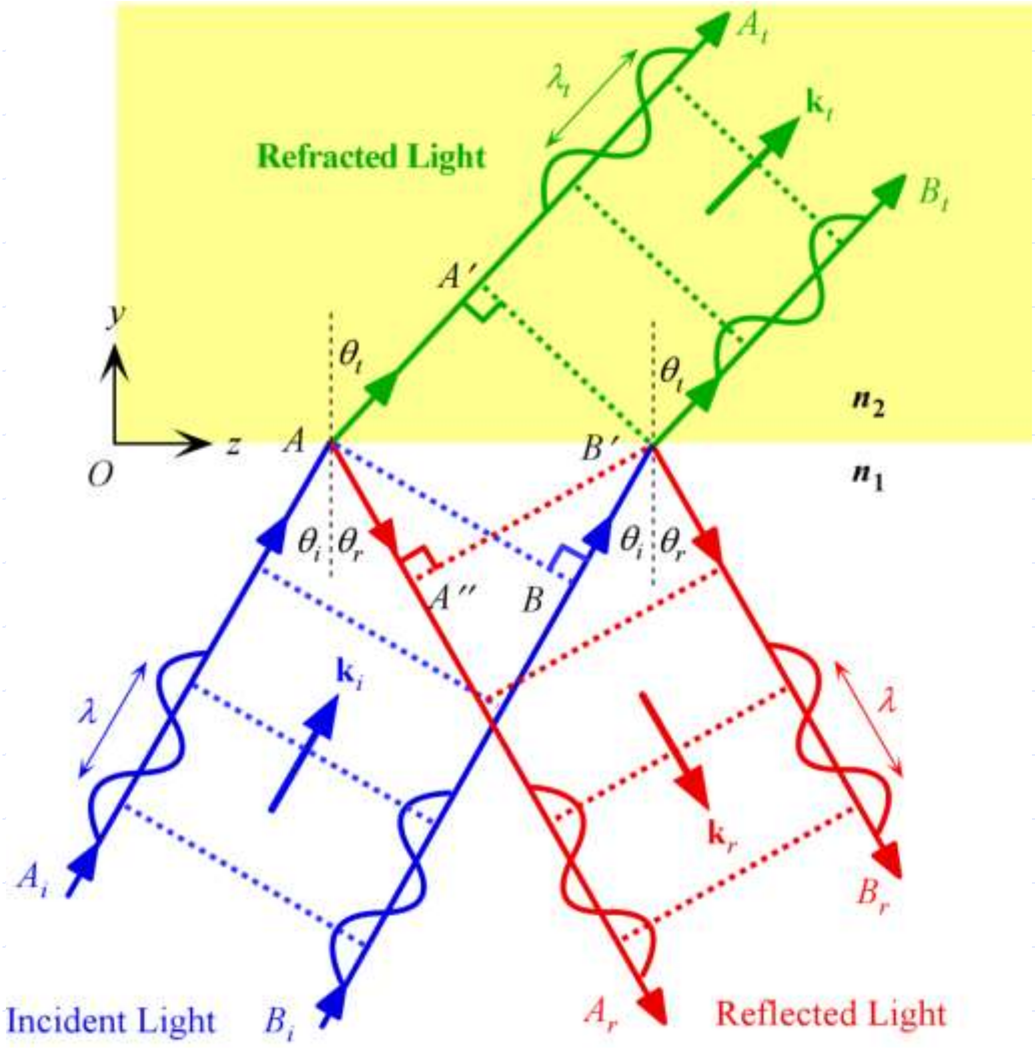
Snell's Law

- Neglect absorption and emission
- Light interfacing with a surface boundary will reflect back into the medium and transmit through the second medium
- Transmitted wave is called refracted light
- The angles θ_i, θ_r, and θ_t define the direction of the waves w.r.t. the normal to the interface



$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1}$$

Derivation of Snell's Law



A light wave traveling in a medium with a greater refractive index ($n_1 > n_2$) suffers reflection and refraction at the boundary. (Notice that λ_t is slightly longer than λ.)

Snell's Law

It takes time t for the phase at B on wave B_i to reach B'

$$BB' = v_1 t = ct/n_1$$

During this time t, the phase A has progressed to A'

$$AA' = v_2 t = ct/n_2$$

A' and B' belong to the same front just like A and B so that AB is perpendicular to $\mathbf{k}_i$ in medium 1 and $A'B'$ is perpendicular to $\mathbf{k}_t$ in medium 2. From geometrical considerations,

$AB' = BB'/\sin\theta_i$ and $AB' = AA'/\sin\theta_t$ so that

$$AB' = \frac{v_1 t}{\sin\theta_i} = \frac{v_2 t}{\sin\theta_t} \qquad \frac{\sin\theta_i}{\sin\theta_t} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

or

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

This is **Snell's law** which relates the angles of incidence and refraction to the refractive indices of the media.

Snell's Law

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

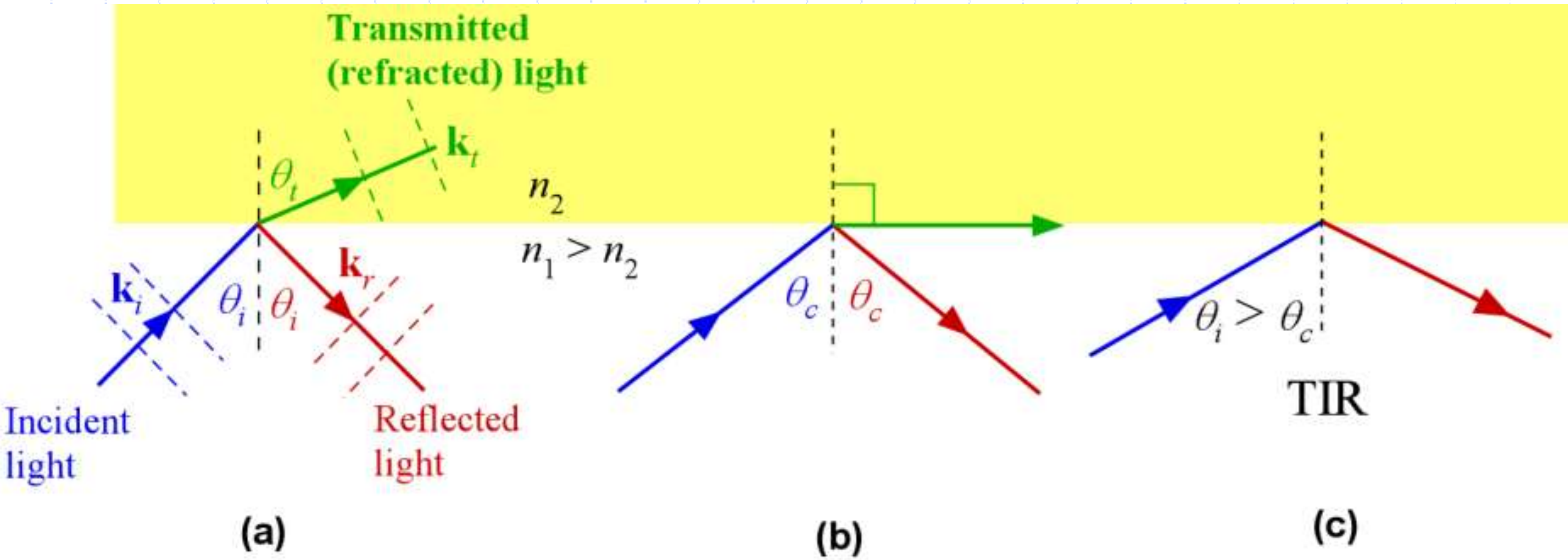
If $n_1 > n_2$, then obviously the transmitted angle > the incidence angle. When $\theta_t = 90°\theta_t$, the incidence angle is called the **critical angle** θ_c

$$\sin\theta_c = \frac{n_2}{n_1}$$

When $\theta_i > \theta_c$ then there is no transmitted wave but only a reflected wave. The latter phenomenon is called **total internal reflection** (TIR). TIR phenomenon that leads to the propagation of waves in a dielectric medium surrounded by a medium of smaller refractive index as in **optical waveguides**, *e.g.* **optical fibers**.

Although Snell's law for $\theta_i > \theta_c$ shows that $\sin\theta_t > 1$ and hence θ_t is an **"imaginary"** angle of refraction, there is however an attenuated wave called the **evanescent wave**.

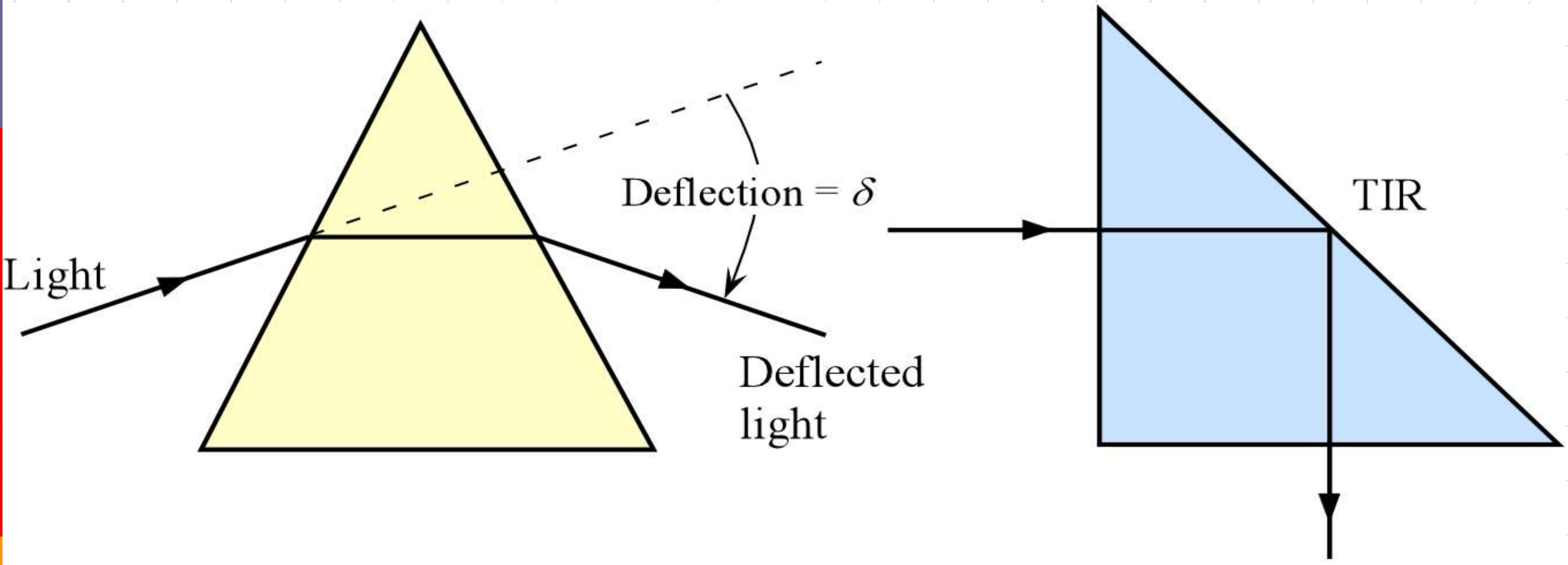
Total Internal Reflection



Light wave traveling in a more dense medium strikes a less dense medium. Depending on the incidence angle with respect to θ_c, which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected.

(a) $\theta_i < \theta_c$ (b) $\theta_i = \theta_c$ (c) $\theta_i > \theta_c$ and TIR.

Prisms



Refracting prism

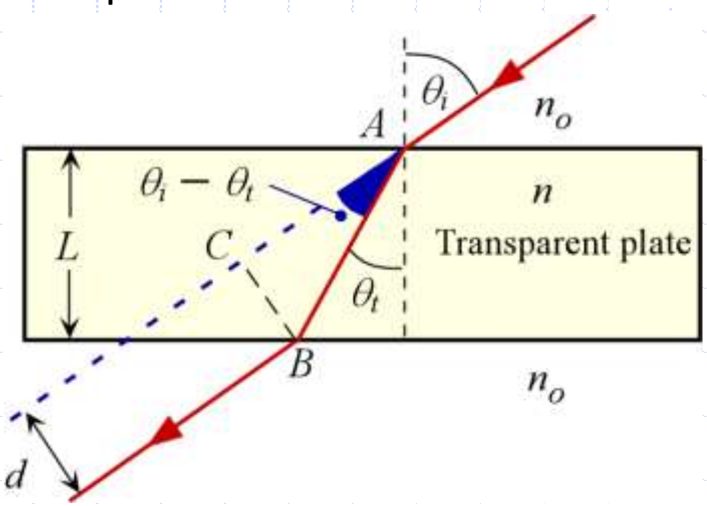
Reflecting prism

Example: Lateral Displacement

Lateral displacement of light, or, **beam displacement,** occurs when a beam if light passes obliquely through a plate of transparent material, such as a glass plate. When a light beam is incident on a plate of transparent material of refractive index *n*, it emerges from the other side traveling parallel to the incident light but displaced from it by a distance *d*, called *lateral displacement.* Find the displacement *d* in terms of the incidence angle the plate thickness *L*. What is *d* for a glass of *n* = 1.600, *L* = 10 mm if the incidence angle is 45°

Solution

The displacement $d = BC = AB\sin(q_i - q_t)$. Further, $L/AB = \cos q_t$ so that combining these two equation we find

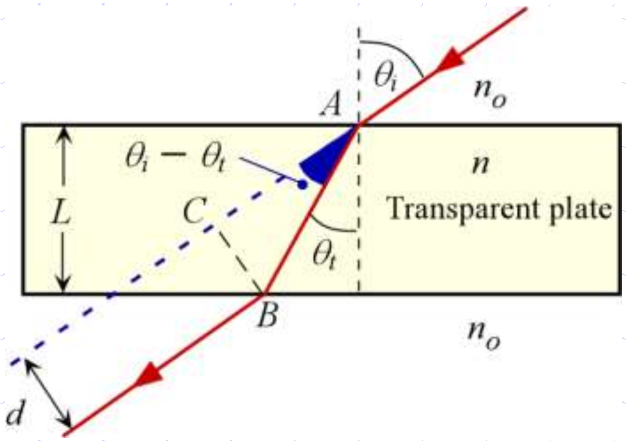


$$d = L\left[\frac{\sin(\theta_i - \theta_t)}{\cos\theta_t}\right]$$

Example: Lateral Displacement (Continued)

Solution (Continued)

Expand $\sin(\theta_i - \theta_t)$ and eliminate $\sin\theta_t$ and $\sin\theta_t$



$$d = L\left[\frac{\sin(\theta_i - \theta_t)}{\cos\theta_t}\right]$$

$$\sin(\theta_i - \theta_t) = \sin\theta_i \cos\theta_t - \cos\theta_i \sin\theta_t$$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t}$$

Snell's law $n\sin\theta_t = n_o \sin\theta_i$

$$\frac{d}{L} = \sin\theta_i\left[1 - \frac{\cos\theta_i}{\sqrt{(n/n_o)^2 - \sin^2\theta_i}}\right]$$

Example: Lateral Displacement (Continued)

Solution (Continued)

$$\frac{d}{L} = \sin\theta_i \left[1 - \frac{\cos\theta_i}{\sqrt{(n/n_o)^2 - \sin^2\theta_i}} \right]$$

$L = 10$ mm

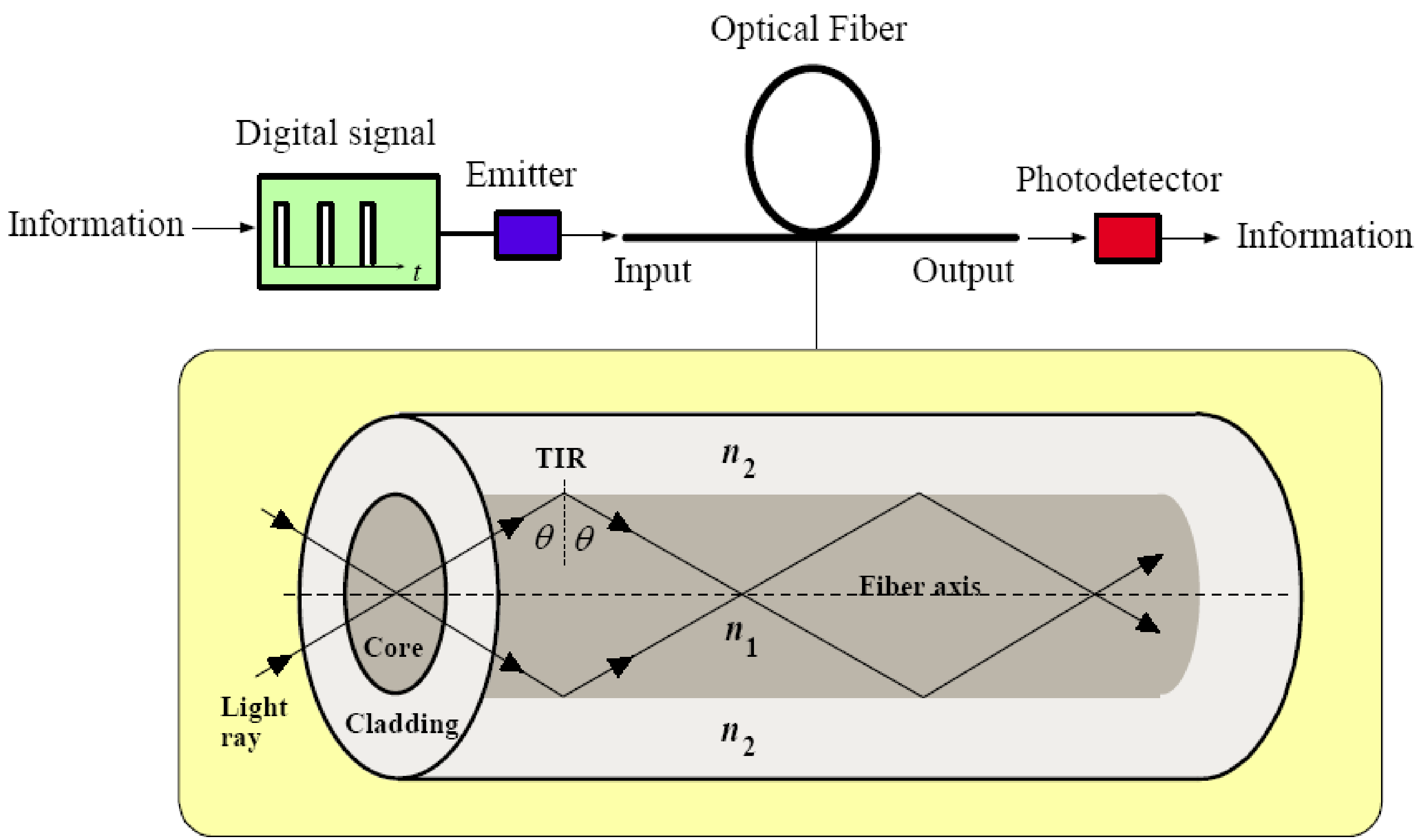
$q_i = 45°$

$n = 1.600$

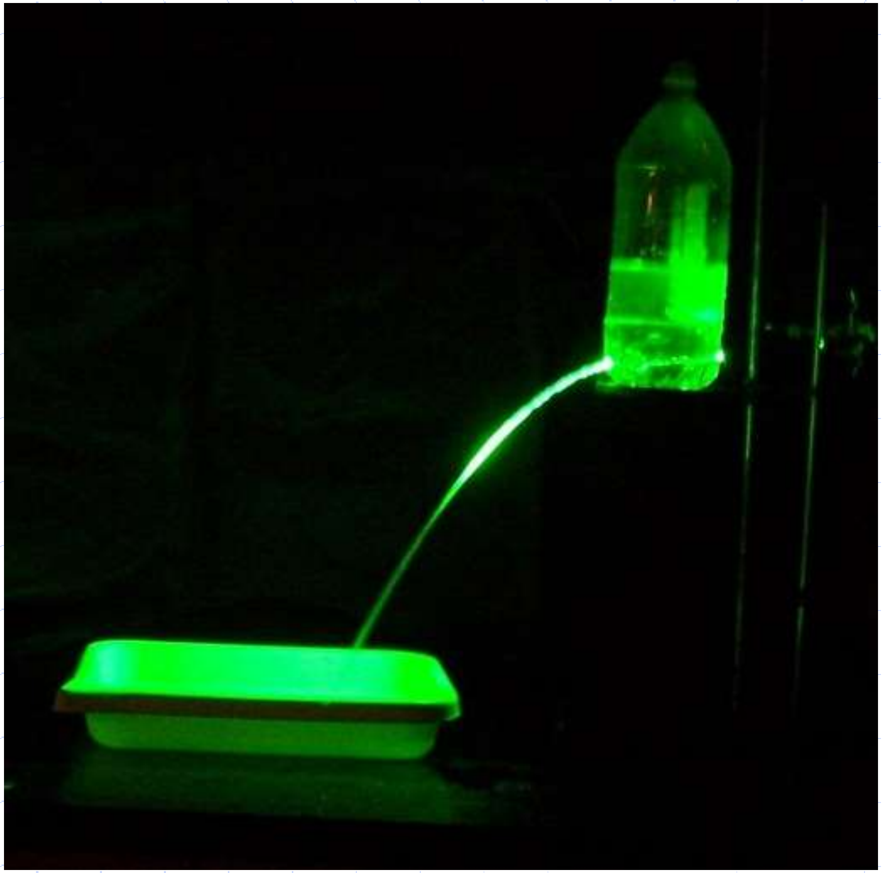
$n_o = 1$

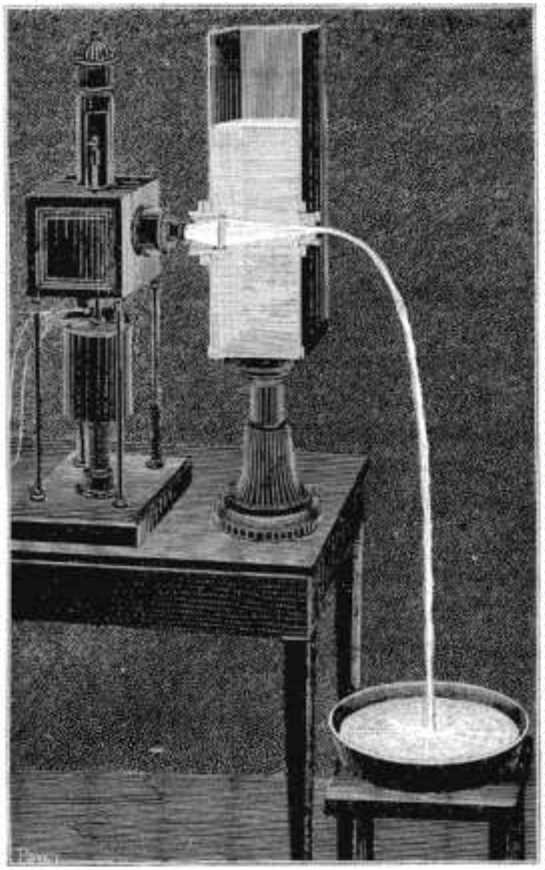
$\boldsymbol{d = 3.587}$ **mm**

Light travels by total internal reflection in optical fibers



An optical fiber link for transmitting digital information in communications. The fiber core has a higher refractive index so that the light travels along the fiber inside the fiber core by total internal reflection at the core-cladding interface.





A small hole is made in a plastic bottle full of water to generate a water jet. When the hole is illuminated with a laser beam (from a green laser pointer), the light is guided by total internal reflections along the jet to the tray. The light guiding by a water jet was first demonstrated by Jean-Daniel Colladan, a Swiss scientist (Water with air bubbles was used to increase the visibility of light. Air bubbles scatter light.) [Left: Copyright: S.O. Kasap, 2005] [Right: *Comptes Rendes*, *15*, 800–802, October 24, 1842; *Cnum, Conservatoire Numérique des Arts et Métiers, France*

Physicists use the wave theory on Mondays, Wednesdays and Fridays and the particle theory on Tuesdays, Thursdays and Saturdays.

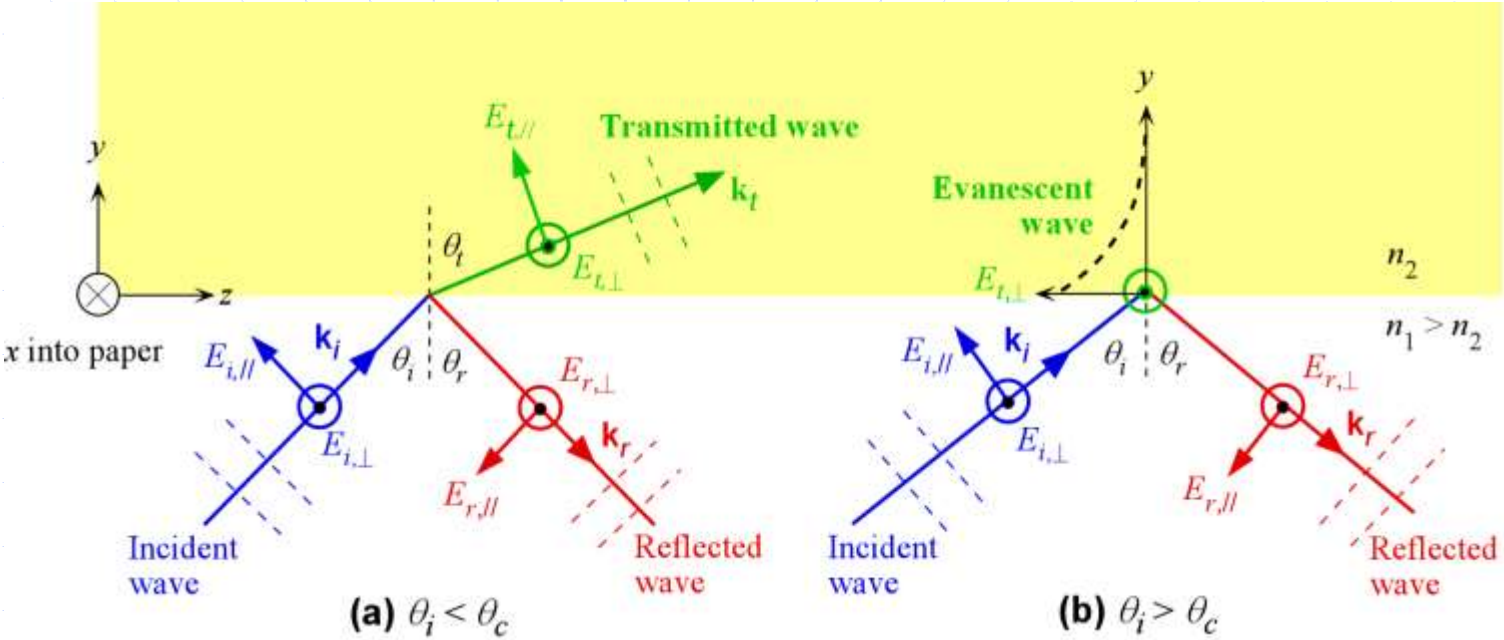
—Sir William Henry Bragg[1]

Augustin Jean Fresnel (1788–1827) was a French physicist and a civil engineer for the French government who was one of the principal proponents of the wave theory of light. He made a number of distinct contributions to optics including the well-known Fresnel lens that was used in lighthouses in the nineteenth century. He fell out with Napoleon in 1815 and was subsequently put under house arrest until the end of Napoleon's reign. During his enforced leisure time he formulated his wave ideas of light into a mathematical theory. *(© INTERFOTO/Alamy.)*

If you cannot saw with a file or file with a saw, then you will be no good as an experimentalist.

—Attributed to Augustin Fresnel

Fresnel's Equations



Light wave traveling in a more dense medium strikes a less dense medium. The plane of incidence is the plane of the paper and is perpendicular to the flat interface between the two media. The electric field is normal to the direction of propagation. It can be resolved into perpendicular (⊥) and parallel components (∥).

- Transverse Electric Field (TE) wave if $E_{i\perp}$, $E_{r\perp}$, and $E_{t\perp}$
- Transverse Magnetic Field (TM) wave if $E_{i\parallel}$, $E_{r\parallel}$, and $E_{t\parallel}$

Fresnel's Equations

Describe the incident, reflected and refracted waves by the exponential representation of a **traveling plane wave**, *i.e.*

$$E_i = E_{io}\exp j(\omega t - \mathbf{k}_i \cdot \mathbf{r})$$ **Incident wave**

$$E_r = E_{ro}\exp j(\omega t - \mathbf{k}_r \cdot \mathbf{r})$$ **Reflected wave**

$$E_t = E_{to}\exp j(\omega t - \mathbf{k}_t \cdot \mathbf{r})$$ **Transmitted wave**

where **r** is the **position vector**, the wave vectors $\mathbf{k}_i$, $\mathbf{k}_r$ and $\mathbf{k}_t$ describe the directions of the incident, reflected and transmitted waves and E_{io}, E_{ro} and E_{to} are the respective amplitudes.

Any **phase changes** such as ϕ_r and ϕ_t in the reflected and transmitted waves with respect to the phase of the incident wave are incorporated into the **complex amplitudes**, E_{ro} and E_{to}.

Our objective is to find E_{ro} and E_{to} with respect to E_{io}, i.e., amplitude reflection and transmission coefficients

Fresnel's Equations

The electric and magnetic fields anywhere on the wave must be perpendicular to each other as a requirement of EM wave theory.

$$B_{\perp} = (n/c)E$$

$$B_{//} = (n/c)E_{\perp}$$

We use **boundary conditions**

$$E_{\text{tangential}}(1) = E_{\text{tangential}}(2)$$

$$\boldsymbol{B}_{\textbf{tangential}}\mathbf{(1)} = \boldsymbol{B}_{\textbf{tangential}}\mathbf{(2)}$$

Applying the boundary conditions to the EM wave going from medium 1 to 2, the amplitudes of the reflected and transmitted waves can be readily obtained in terms of n_1, n_2 and the incidence angle θ_i alone. These relationships are called **Fresnel's equations**.

The boundary conditions can only be satisfied if the reflection and incidence angles are equal, $\theta_r = \theta_i$ and the angles for the transmitted and incident wave obey Snell's law, $n_1\sin\theta_1 = n_2\sin\theta_2$

Fresnel's Equations

Define $n = n_2/n_1$, as the **relative refractive index** of the system, then the **reflection and transmission coefficients** for $E_\perp$ are,

$$r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$t_\perp = \frac{E_{t0,\perp}}{E_{i0,\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

If we let E_{io} be real, then phase angles of $r_\perp$ and $t_\perp$ correspond to the **phase changes** measured w.r.t the incident wave.

The **reflection and transmission coefficients** for the $E_{//}$ are

$$r_{//} = \frac{E_{r0,//}}{E_{i0,//}} = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

$$t_{//} = \frac{E_{t0,//}}{E_{i0,//}} = \frac{2n\cos\theta_i}{n^2\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

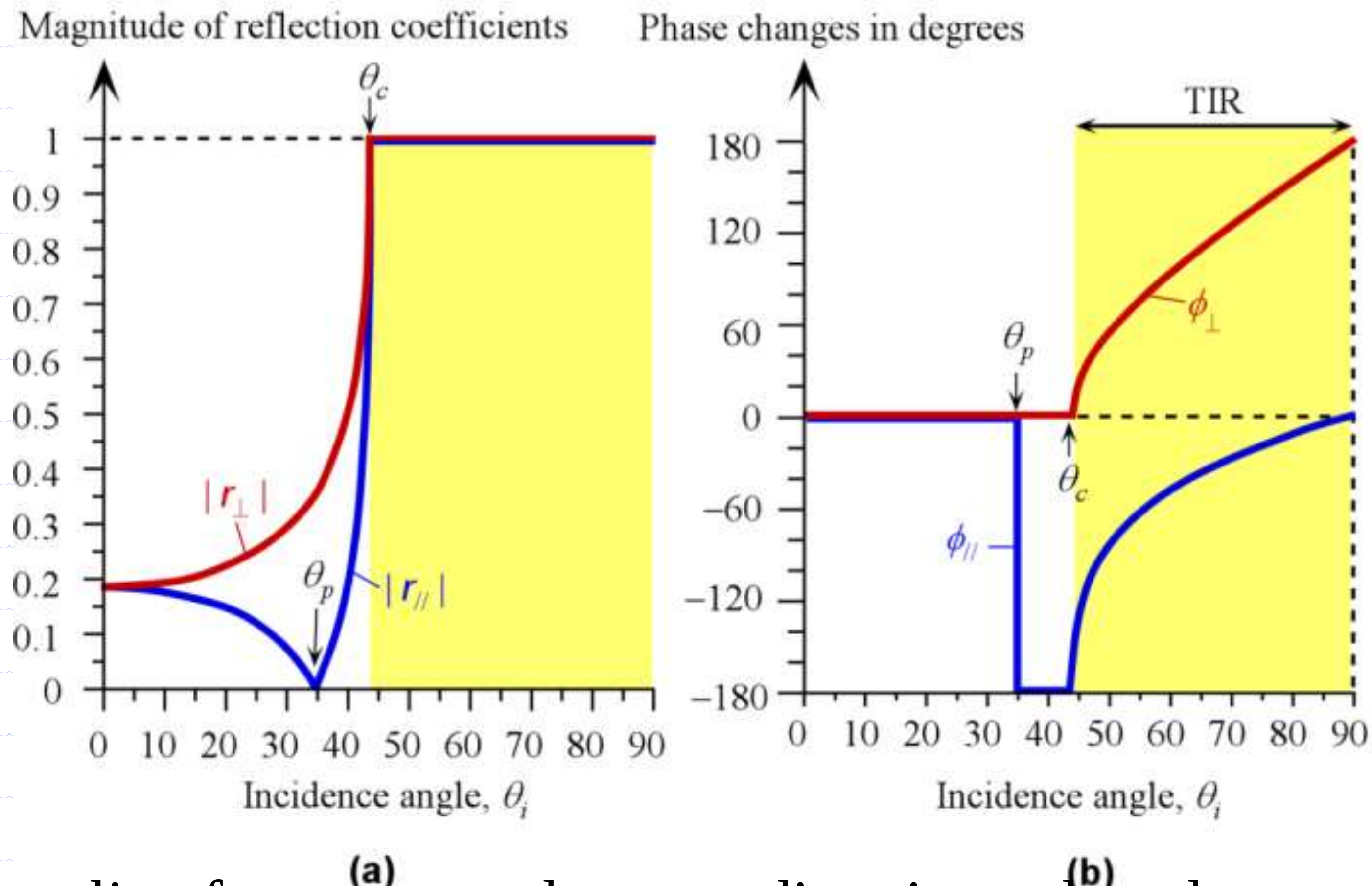
For normal incidence ($\theta_i = 0$):

$$r_{//} = r_\perp = \frac{n_1 - n_2}{n_1 + n_2}$$

Further, the above coefficients are related by

$$r_{//} + nt_{//} = 1 \quad \text{and} \quad r_\perp + 1 = t_\perp$$

Internal reflection



Light traveling from a more dense medium into a less dense one $(n_2 < n_1)$

(a) Magnitude of the reflection coefficients $r_{//}$ and $r_\perp$ vs. angle of incidence θ_i for $n_1 = 1.44$ and $n_2 = 1.00$. The critical angle is 44°.

(b) The corresponding changes $\phi_{//}$ and $\phi_\perp$ vs. incidence angle.

Total Internal Reflection

In linearly polarized light, however, the field oscillations are contained within a well defined plane. Light emitted from many light sources such as a tungsten light bulb or an LED diode is unpolarized and the field is randomly oriented in a direction that is perpendicular to the direction of propagation.

At the critical angle and beyond (past 44° in the figure), *i.e.* when $\theta_i \geq \theta_c$, the magnitudes of both $r_{//}$ and $r_{\perp}$ go to unity so that the reflected wave has the same amplitude as the incident wave. The incident wave has suffered **total internal reflection**, TIR.

Reflection and Polarization Angle

Brewster's angle = polarization angle = θ_p is the angle at which $r_{//}$ becomes zero and the field in the reflected wave is then always perpendicular to the plane of incidence and hence well-defined.

Solving the Fresnel equation for $r_{//} = 0$

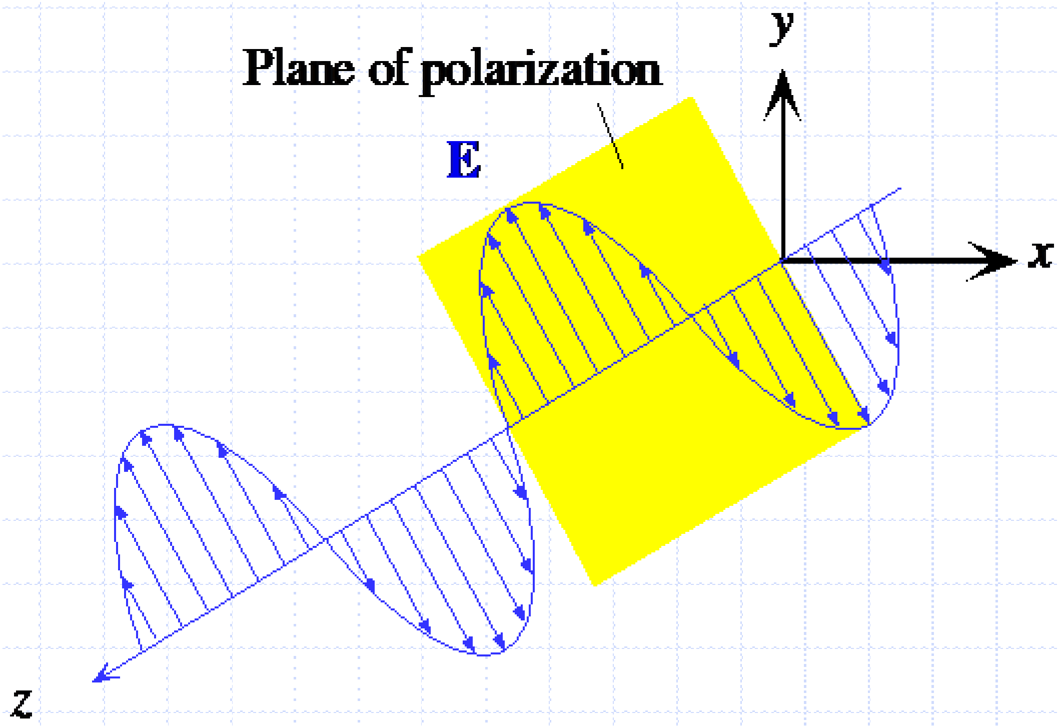
$$r_{//} = \frac{E_{r0,//}}{E_{i0,//}} = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i} = 0$$

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{1}{1.44}$$

For both $n_1 > n_2$ or $n_1 < n_2$.

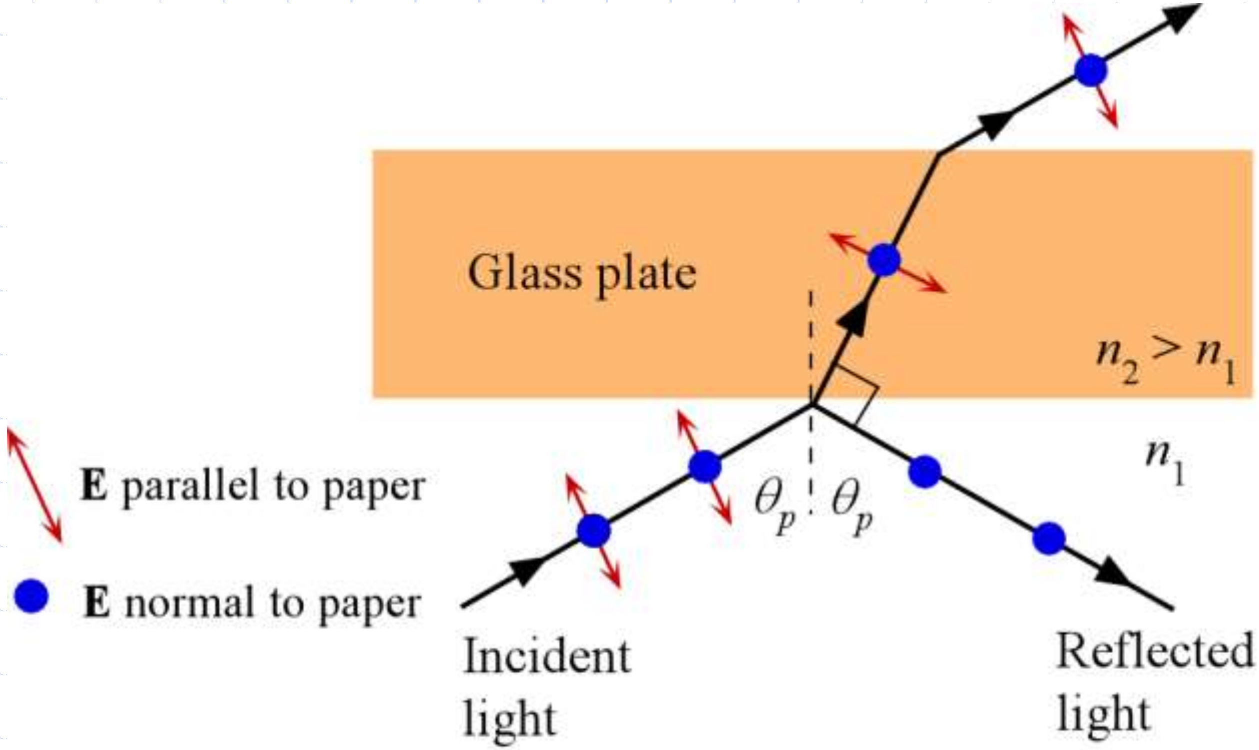
$$\theta_p = 35^\circ$$

Polarized Light



A **linearly polarized** wave has its electric field oscillations defined along a line perpendicular to the direction of propagation, *z*. The field vector **E** and *z* define a **plane of polarization**.

Brewster's angle



Reflected light at $q_i = q_p$ has only **E**^

for both $n_1 > n_2$ or $n_1 < n_2$.

Reflected waves at angles $> \theta_p$ are **linearly polarized** because they contain field oscillations that are contained within a well defined plane perpendicular to the plane of incidence and the plane of propagation.

Phase Change in TIR

When $\theta_i > \theta_c$, in the presence of TIR, the reflection coefficients become complex quantities of the type

$$\boldsymbol{r_\perp = 1 \cdot \exp(-j\phi_\perp) \text{ and } r_{//} = 1 \cdot \exp(-j\phi_{//})}$$

with the phase angles $\phi_\perp$ and $\phi_{//}$ being other than zero or 180°. The reflected wave therefore suffers phase changes, $\phi_\perp$ and $\phi_{//}$, in the components $E_\perp$ and $E_{//}$. These phase changes depend on the incidence angle, and on n_1 and n_2.

The phase change $\phi_\perp$ is given by

$$\tan\left(\frac{1}{2}\phi_\perp\right) = \frac{\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i}$$

For the $E_{//}$ component, the phase change $\phi_{//}$ is given by

$$\tan\left(\tfrac{1}{2}\phi_{//} + \tfrac{\pi}{2}\right) = \frac{\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i}$$

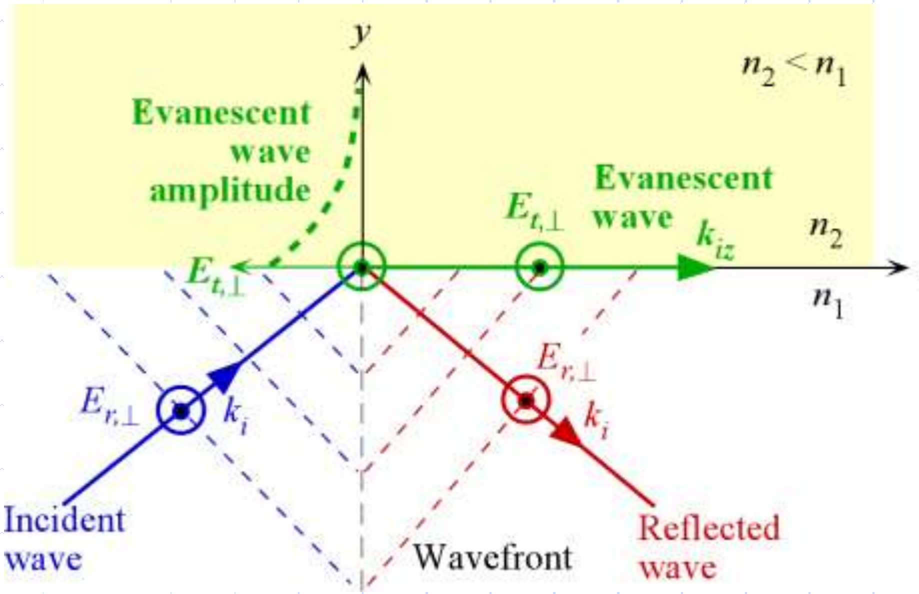
Evanescent Wave

In internal reflection ($n_1 > n_2$), the amplitude of the reflected wave from TIR is equal to the amplitude of the incident wave but its phase has shifted.

What happens to the transmitted wave when $\theta_i > \theta_c$?

According to the boundary conditions, there must still be an electric field in medium 2, otherwise, the boundary conditions cannot be satisfied. When $\theta_i > \theta_c$, the field in medium 2 is attenuated (decreases with y, and is called the **evanescent wave**.

When $\theta_i > \theta_c$, for a plane wave that is reflected, there is an evanescent wave at the boundary propagating along z.



Evanescent Wave

$$E_{t,\perp}(y,z,t) \propto \boldsymbol{e}^{-\alpha_2 y} \exp j(\omega t - k_{iz} z)$$

where $k_{iz} = k_i \sin\theta_i$ is the wavevector of the incident wave along the z-axis, and α_2 is an **attenuation coefficient** for the electric field penetrating into medium 2

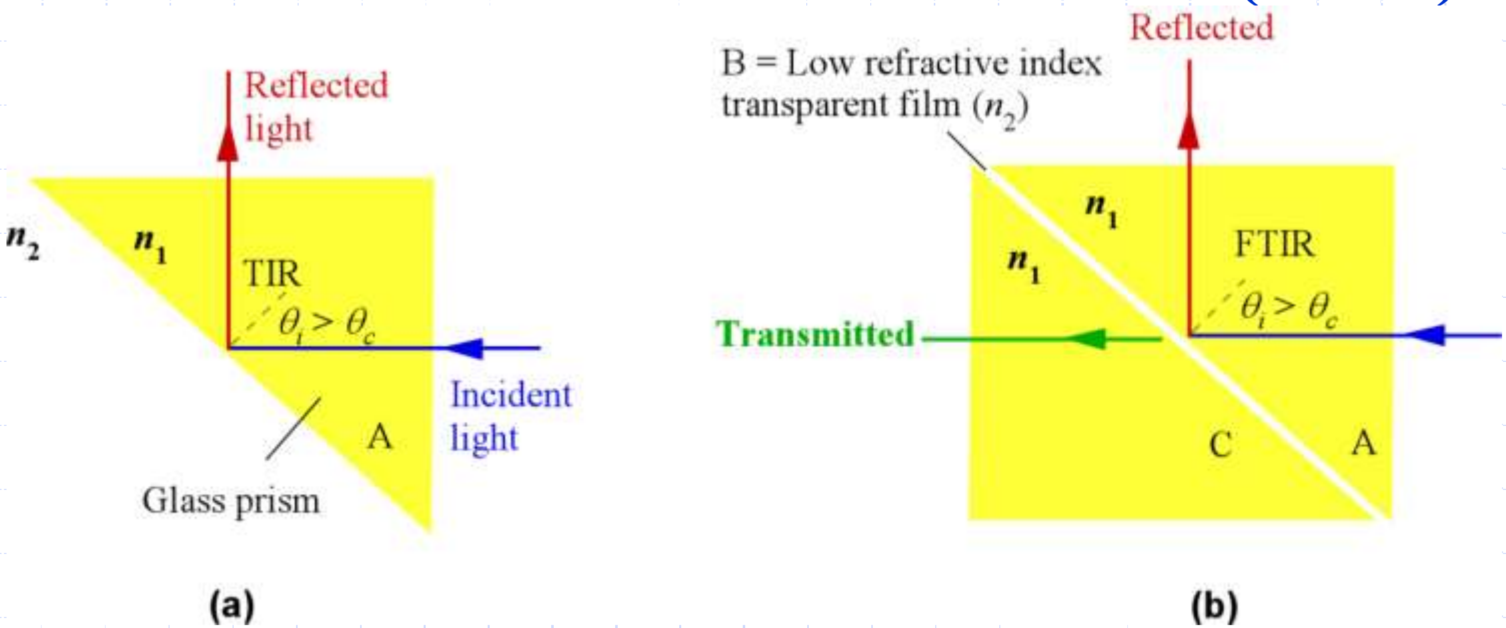
$$\alpha_2 = \frac{2\pi n_2}{\lambda} \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1}$$

The **penetration depth** of the electric filed in medium 2 (field of the evanescent wave) is

$$\delta = 1/\alpha_2 \;\; \rightarrow E_{t\perp} = e^{-1}$$

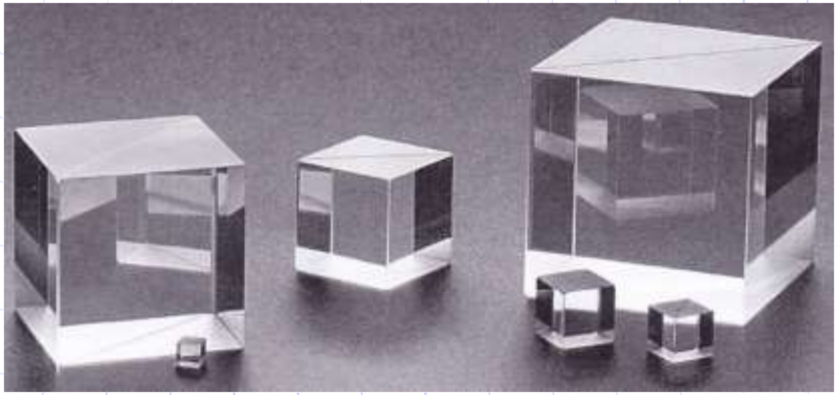
Beam Splitters

Frustrated Total Internal Reflection (FTIR)



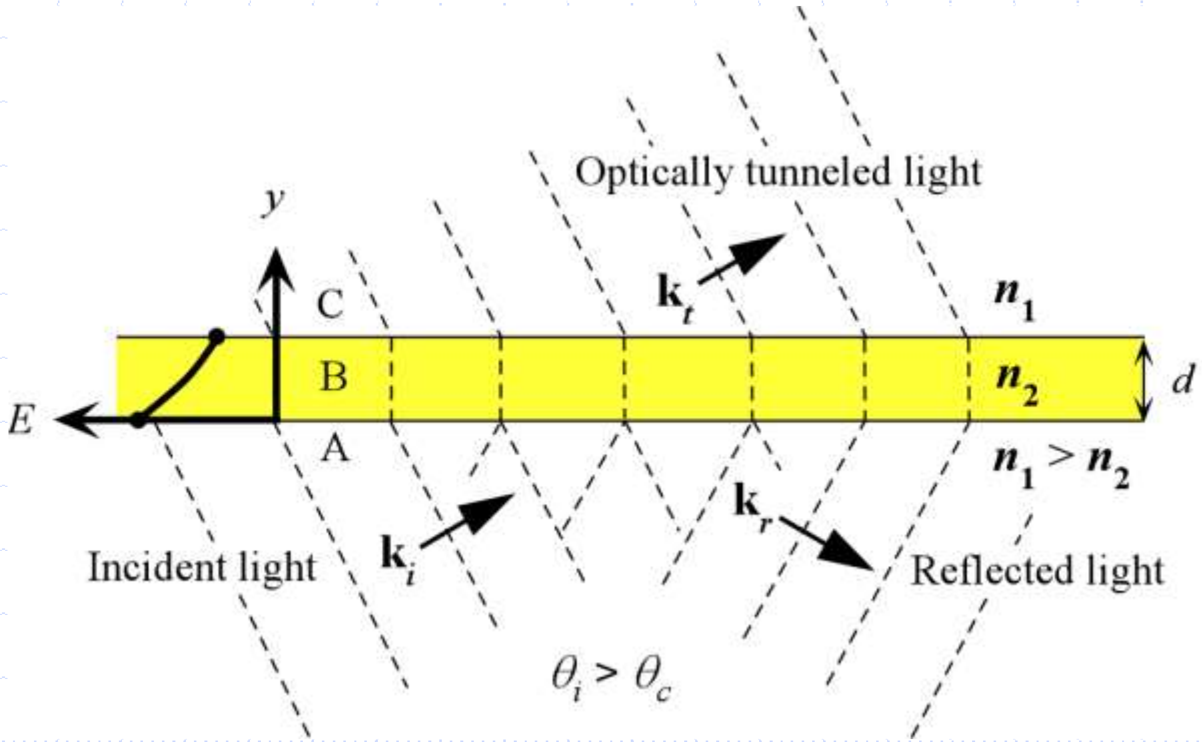
(a) A light incident at the long face of a glass prism suffers TIR; the prism deflects the light.

(b) Two prisms separated by a thin low refractive index film forming a beam-splitter cube. The incident beam is split into two beams by FTIR.



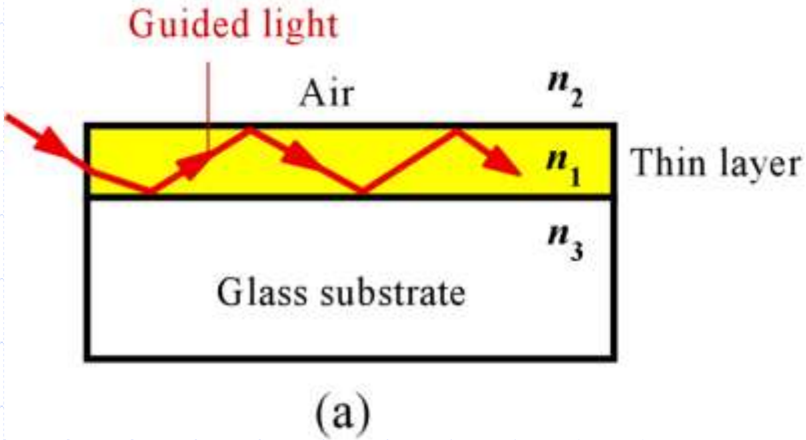
Beam splitter cubes (Courtesy of CVI Melles Griot)

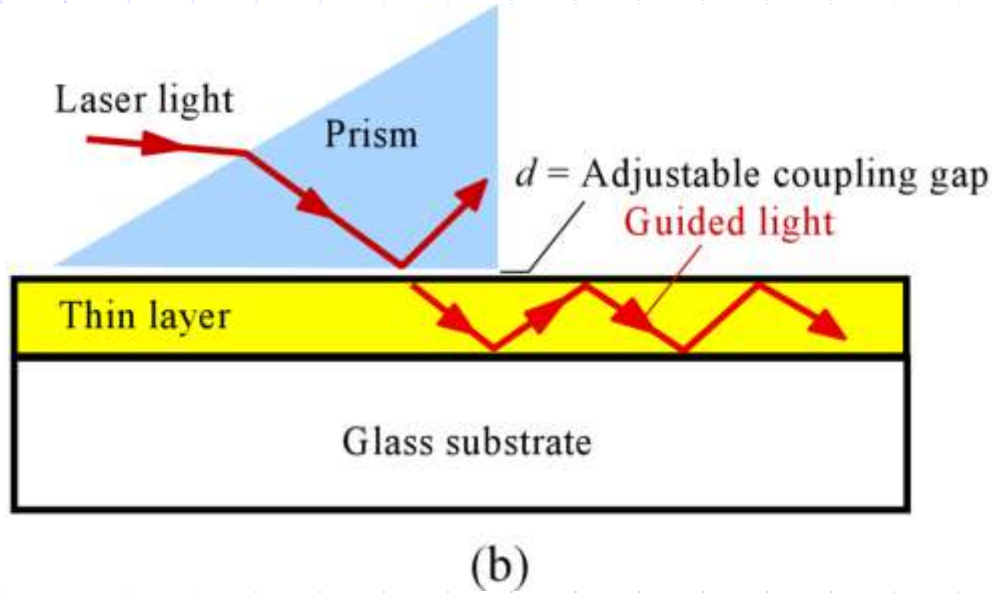
Optical Tunneling



When medium B is thin (thickness d is small), the field penetrates from the AB interface into medium B and reaches BC interface, and gives rise to a transmitted wave in medium C. The effect is the tunneling of the incident beam in A through B to C. The maximum field E_{max} of the evanescent wave in B decays in B along y and but is finite at the BC boundary and excites the transmitted wave.

Optical Tunneling





Light propagation along an optical guide by total internal reflections

Coupling of laser light into a thin layer - optical guide - using a prism. The light propagates along the thin layer.

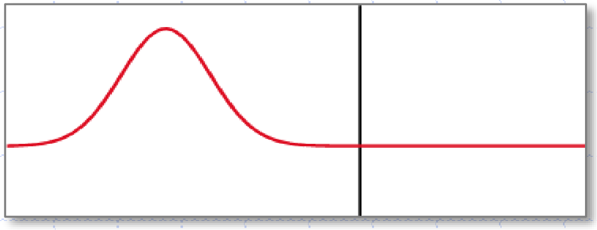
External Reflection

Light traveling from a less dense medium into a more dense one,

$$n_1 < n_2$$

This is **external reflection**.

Light becomes reflected by the surface of an optically denser (higher refractive index) medium.



$r_\perp$ and $r_{//}$ depend on the incidence angle θ_i.

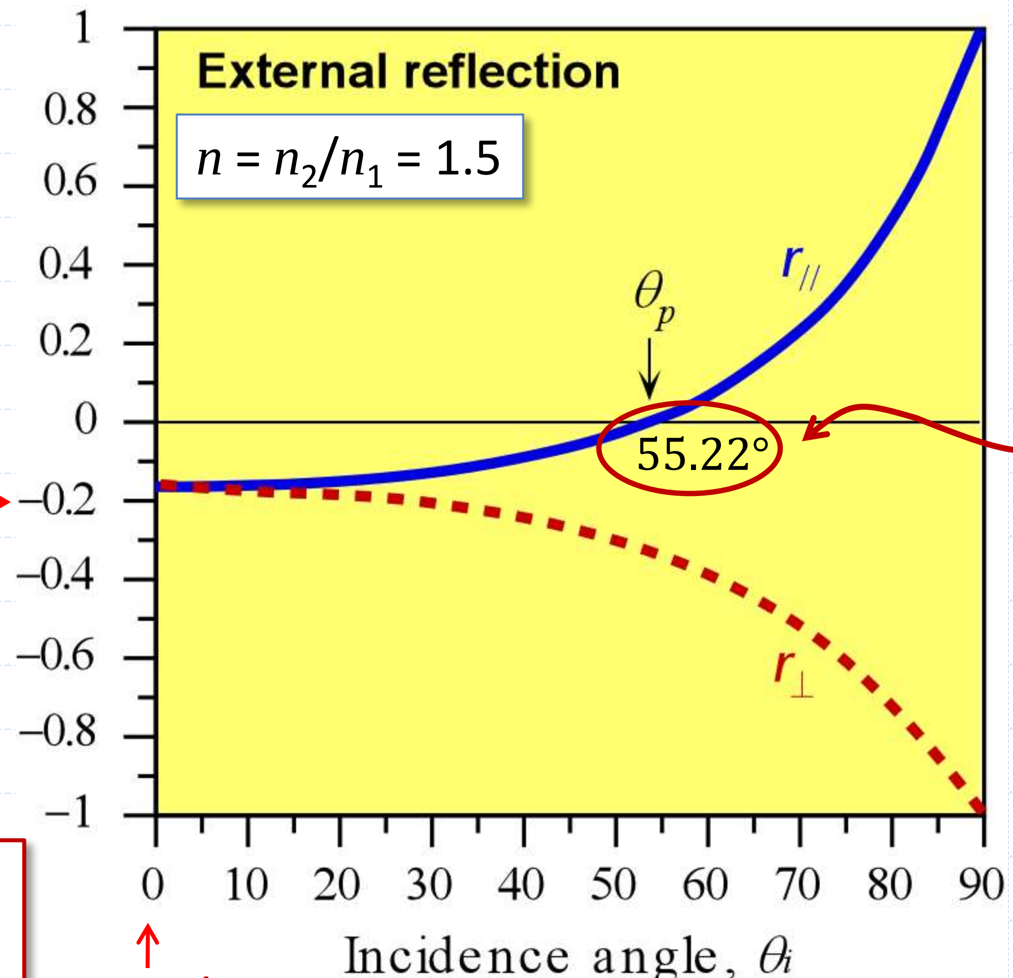
At **normal incidence**, $r_\perp$ and $r_{//}$ are **negative. In external reflection at normal incidence there is a phase shift of 180°.**

$r_{//}$ goes through zero at the **Brewster angle**, θ_p.

At θ_p, the reflected wave is polarized in the $E_\perp$ component only.

Transmitted light in both internal reflection (when $\theta_i < \theta_c$) and external reflection does **NOT** experience a phase shift.

External Reflection



negative reflection coefficients

here, reflected light is linearly polarized;

$R_{\parallel} = 0$

$R_{TE} = 15\%$

at normal : 3.25%

$$R = r^2 = \left(\frac{1-n}{1+n}\right)^2$$

normal

The reflection coefficients $r_{//}$ and $r_{\perp}$ versus angle of incidence θ_i for $n_1 = 1.00$ and $n_2 = 1.44$.

Intensity, Reflectance and Transmittance

Reflectance R measures the intensity of the reflected light w.r.t. that of the incident light.

The reflectances $R_\perp$ and $R_{//}$ are defined by

$$R_\perp = \frac{|E_{ro,\perp}|^2}{|E_{io,\perp}|^2} = |r_\perp|^2$$

and

$$R_{//} = \frac{|E_{ro,//}|^2}{|E_{io,//}|^2} = |r_{//}|^2$$

At normal incidence

$$R = R_\perp = R_{//} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

Since a glass medium has a refractive index of around 1.5 this means that typically 4% of the incident radiation on an air-glass surface will be reflected back.

Example: Internal and external reflection

Consider the reflection of light at normal incidence on a boundary between a glass medium of refractive index 1.5 and air of refractive index 1.

(a) If light is traveling from air to glass, what is the reflection coefficient and the intensity of the reflected light w.r.t. that of the incident light?

(b) If light is traveling from glass to air, what is the reflection coefficient and the intensity of the reflected light with respect to that of the incident light?

(c) What is the polarization angle in the external reflection in **a** above? How would you make a polaroid from this?

Solution

(a) The light travels in air and becomes partially reflected at the surface of the glass which corresponds to external reflection. Thus $n_1 = 1$ and $n_2 = 1.5$. Then,

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1-1.5}{1+1.5} = -0.2$$

This is negative which means that there is a 180° phase shift. The reflectance (*R*), which gives the fractional reflected power, is

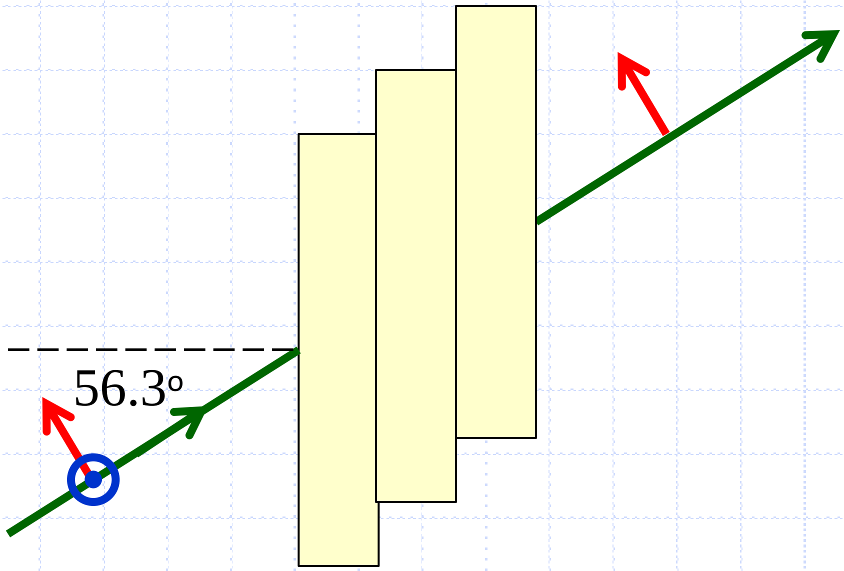
$$R = r_{//}^2 = 0.04 \text{ or } 4\%.$$

(b) The light travels in glass and becomes partially reflected at the glass-air interface which corresponds to internal reflection. $n_1 = 1.5$ and $n_2 = 1$. Then,

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1.5-1}{1.5+1} = 0.2$$

There is no phase shift. The reflectance is again 0.04 or 4%. In both cases **(a)** and **(b)** the amount of reflected light is the same.

(c) Light is traveling in air and is incident on the glass surface at the polarization angle. Here $n_1 = 1$, $n_2 = 1.5$ and $\tan\theta_p = (n_2/n_1) = 1.5$ so that $\theta_p = 56.3°$.



This type of *pile-of-plates polarizer* was invented by Dominique F.J. Arago in 1812

Transmittance

Transmittance *T* relates the intensity of the transmitted wave to that of the incident wave in a similar fashion to the reflectance.

However the transmitted wave is in a **different medium** and further its direction with respect to the boundary is also different due to refraction.

For **normal incidence**, the incident and transmitted beams are normal so that the equations are simple:

$$T_{\perp} = \frac{n_2 |E_{to,\perp}|^2}{n_1 |E_{io,\perp}|^2} = \left(\frac{n_2}{n_1}\right) |t_{\perp}|^2$$

$$T_{//} = \frac{n_2 |E_{to,//}|^2}{n_1 |E_{io,//}|^2} = \left(\frac{n_2}{n_1}\right) |t_{//}|^2$$

or

$$T = T_{\perp} = T_{//} = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

Further, the fraction of light reflected and fraction transmitted must add to unity.

Thus $R + T = 1$.

Reflection and Transmission – An Example

Question A light beam traveling in air is incident on a glass plate of refractive index 1.50 . What is the Brester or polarization angle? What are the relative intensities of the reflected and transmitted light for the polarization perpendicular and parallel to the plane of incidence at the Brestwer angle of incidence?

Solution Light is traveling in air and is incident on the glass surface at the polarization angle q_p. Here $n_1 = 1$, $n_2 = 1.5$ and $\tan q_p = (n_2/n_1) = 1.5$ so that $q_p = 56.31°$. We now have to use Fresnel's equations to find the reflected and transmitted amplitudes. For the perpendicular polarization

$$r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos\theta_i - \sqrt{n^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n^2 - \sin^2\theta_i}}$$

$$r_\perp = \frac{\cos(56.31^\circ) - \sqrt{1.5^2 - \sin^2(56.31^\circ)}}{\cos(56.31^\circ) + \sqrt{1.5^2 - \sin^2(56.31^\circ)}} = -0.385$$

On the other hand, $r_{//} = 0$. The reflectances $R_\perp = |r_\perp|^2 = 0.148$ and $R_{//} = |r_{//}|^2 = 0$ so that $R = 0.074$, and has no parallel polarization in the plane of incidence. Notice the negative sign in $r_\perp$, which indicates a phase change of p.

Reflection and Transmission – An Example

$$t_{\perp} = \frac{E_{t0,\perp}}{E_{i0,\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$t_{\perp} = \frac{2\cos(56.31^{o})}{\cos(56.31^{o}) + [1.5^2 - \sin^2(56.31^{o})]^{1/2}} = 0.615$$

$$t_{//} = \frac{E_{t0,//}}{E_{i0,//}} = \frac{2n\cos\theta_i}{n^2\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$t_{//} = \frac{2(1.5)\cos(56.31^{o})}{(1.5)^2\cos(56.31^{o}) + [1.5^2 - \sin^2(56.31^{o})]^{1/2}} = 0.667$$

Notice that $r_{//} + nt_{//} = 1$ and $r_{\perp} + 1 = t_{\perp}$, as we expect.

Reflection and Transmission – An Example

To find the transmittance for each polarization, we need the refraction angle θ_t.
From Snell's law, $n_1 \sin\theta_i = n_t \sin\theta_t$ *i.e.* (1)sin(56.31°) = (1.5)sinθ_t, we find **θ_t = 33.69°**.

$$T_{//} = \frac{n_2 |E_{to,//}|^2}{n_1 |E_{io,//}|^2} = \left(\frac{n_2}{n_1}\right) |t_{//}|^2 \qquad T_{\perp} = \frac{n_2 |E_{to,\perp}|^2}{n_1 |E_{io,\perp}|^2} = \left(\frac{n_2}{n_1}\right) |t_{\perp}|^2$$

$$T_{//} = \left[\frac{(1.5)\cos(33.69^{\circ})}{(1)\cos(56.31^{\circ})}\right](0.667)^2 = 1 \qquad T_{\perp} = \left[\frac{(1.5)\cos(33.69^{\circ})}{(1)\cos(56.31^{\circ})}\right](0.615)^2 = 0.852$$

Clearly, light with polarization parallel to the plane of incidence has greater intensity.

If we were to reflect light from a glass plate, keeping the angle of incidence at 56.3°, then the reflected light will be polarized with an electric field component perpendicular to the plane of incidence. The transmitted light will have the field greater in the plane of incidence, that is, it will be partially polarized. By using a stack of glass plates one can increase the polarization of the transmitted light. (This type of *pile-of-plates polarizer* was invented by Dominique F.J. Arago in 1812.)

Example: Reflection of light from a less dense medium (internal reflection)

A ray of light which is traveling in a glass medium of refractive index $n_1 = 1.460$ becomes incident on a less dense glass medium of refractive index $n_2 = 1.440$. The free space wavelength (λ) of the light ray is 1300 nm.

(a) What should be the minimum incidence angle for TIR?
(b) What is the phase change in the reflected wave when $\theta_i = 87°$ and when $\theta_i = 90°$?
(c) What is the penetration depth of the evanescent wave into medium 2 when $\theta_i = 87°$ and when $\theta_i = 90°$?

Solution

(a) The critical angle θ_c for TIR is given by

$\sin\theta_c = n_2/n_1 = 1.440/1.460$ so that $\boldsymbol{\theta_c = 80.51°}$

(b) Since the incidence angle $\theta_i > \theta_c$ there is a phase shift in the reflected wave. The phase change in $E_{r,\perp}$ is given by $\phi_\perp$.

Using $n_1 = 1.460$, $n_2 = 1.440$ and $\theta_i = 87°$,

$$\tan\left(\tfrac{1}{2}\phi_\perp\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{\cos\theta_i} = \frac{\left[\sin^2(87^\circ) - \left(\frac{1.440}{1.460}\right)^2\right]^{1/2}}{\cos(87^\circ)}$$

$$= 2.989 = \tan[{}^1/_2(143.0°)]$$

so that the phase change $\boldsymbol{\phi_\perp = 143°}$.

For the $E_{r//}$ component, the phase change is

$$\tan\left(\tfrac{1}{2}\phi_{//} + \tfrac{1}{2}\pi\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{n^2\cos\theta_i} = \frac{1}{n^2}\tan\left(\tfrac{1}{2}\phi_{\perp}\right)$$

so that

$$\tan({}^1/_2\phi_{//} + {}^1/_2\pi) = (n_1/n_2)^2\tan(\phi_{\perp}/2) =$$

$$(1.460/1.440)^2\tan({}^1/_2 143°)$$

which gives $\boldsymbol{\phi_{//}}$ **= 143.95° – 180°** or **–36.05°**

Repeat with θ_i = 90° to find $\boldsymbol{\phi_{\perp}}$ **= 180°** and $\boldsymbol{\phi_{//}}$ **= 0°**.

Note that as long as $\theta_i > \theta_c$, the magnitude of the reflection coefficients are unity. Only the phase changes.

(c) The amplitude of the evanescent wave as it penetrates into medium 2 is

$$E_{t,\perp}(y,t) \propto E_{to,\perp}\exp(-\alpha_2 y)$$

The field strength drops to $\boldsymbol{e}^{-1}$ when $y = 1/\alpha_2 = \delta$, which is called the **penetration depth**. The attenuation constant α_2 is

$$\alpha_2 = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

i.e.

$$\alpha_2 = \frac{2\pi(1.440)}{(1300\times10^{-9}\text{ m})}\left[\left(\frac{1.460}{1.440}\right)^2 \sin^2(87^\circ) - 1\right]^{1/2}$$

= 1.10×10^6 m^{-1}.

The penetration depth is,

$\delta = 1/\alpha_2 = 1/(1.104\times10^6\text{ m}) = 9.06\times10^{-7}$ m, or **0.906 μm**

For 90°, repeating the calculation, $\alpha_2 = 1.164\times10^6$ m^{-1}, so that

$\delta = 1/\alpha_2 =$ **0.859 μm**

The penetration is greater for smaller incidence angles

ECE 325 Spring 2019 Dr. Ahmed Farghal Lecture 2

AR Coatings on Solar Cells

When light is incident on the surface of a semiconductor it becomes partially reflected. The refractive index of Si is about 3.5 at wavelengths around 700 - 800 nm. Thus the reflectance with n_1(air) = 1 and n_2(Si) ≈ 3.5 is

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{1 - 3.5}{1 + 3.5}\right)^2 = 0.309$$

30% of the light is reflected and is not available for conversion to electrical energy; a considerable reduction in the efficiency of the solar cell.

AR Coatings on Solar Cells

We can coat the surface of the semiconductor device with a thin layer of a dielectric material such as Si_3N_4 (silicon nitride) that has an intermediate refractive index. In this case n_1(air) = 1, n_2(coating) ≈ 1.9 and n_3(Si) = 3.5

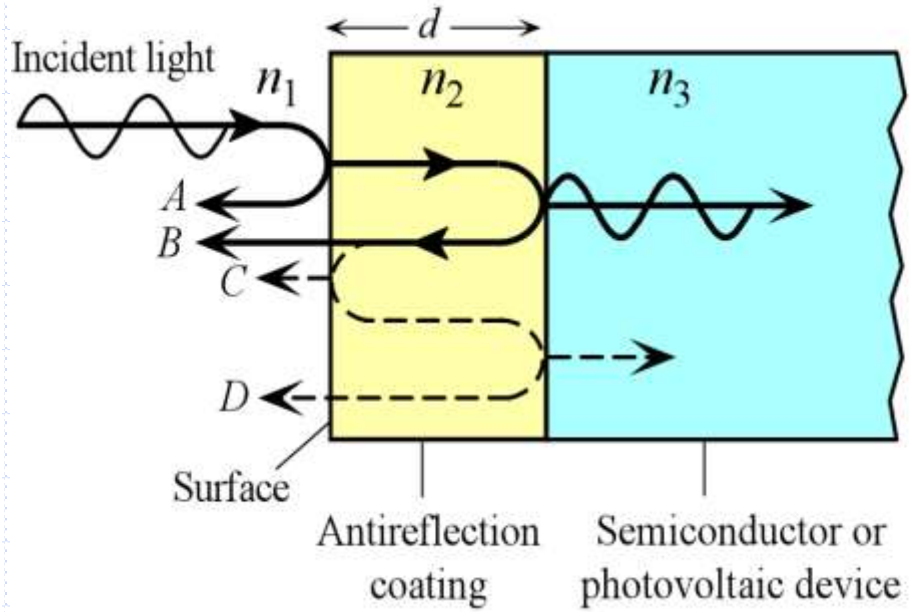


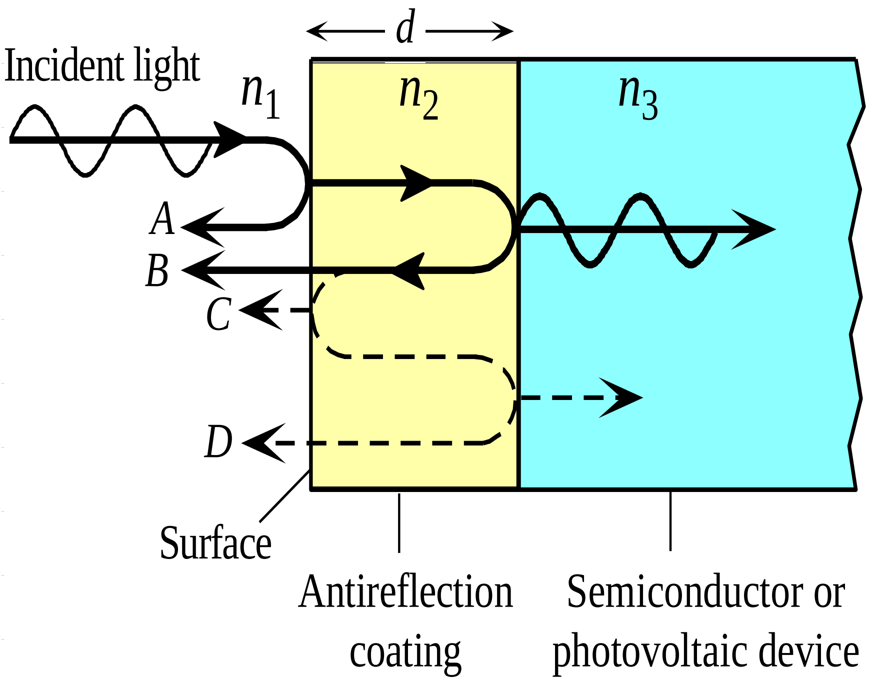
Illustration of how an antireflection coating reduces the reflected light intensity.

Light is first incident on the air/coating surface. Some of it becomes reflected as *A* in the figure. Wave *A* has experienced a 180° phase change on reflection because this is an external reflection. The wave that enters and travels in the coating then becomes reflected at the coating/semiconductor surface.

AR Coatings on Solar Cells

This reflected wave *B*, also suffers a 180° phase change since $n_3 > n_2$.

When *B* reaches *A*, it has suffered a total delay of traversing the thickness *d* of the coating twice. The **phase difference** is equivalent to $k_c(2d)$ where $k_c = 2\pi/\lambda_c$ is the propagation constant in the coating, *i.e.* $k_c = 2\pi/\lambda_c$ where λ_c is the wavelength in the coating.



Since $\lambda_c = \lambda/n_2$, where λ is the free-space wavelength, the phase difference $\Delta\phi$ between *A* and *B* is $(2\pi n_2/\lambda)(2d)$. To reduce the reflected light, *A* and *B* **must interfere destructively.** This requires the **phase difference to be π or odd-multiples of π, $m\pi$ where m = 1,3,5,**… is an odd-integer. Thus

AR Coatings on Solar Cells

Destructive interference requires

$$\text{Phase change} = (n_2 k)(2d) = m(\pi)$$

***m* = 1,3,5…odd integer**

$$\left(\frac{2\pi n_2}{\lambda}\right)2d = m\pi \quad \text{or} \quad d = m\left(\frac{\lambda}{4n_2}\right)$$

Thus, the thickness of the coating must be **odd multiples** of the quarter wavelength in the coating and depends on the wavelength

To obtain a good degree of destructive interference between waves *A* and *B*, the two amplitudes must be comparable. We need $n_2 = \sqrt{n_1 n_3}$

AR Coatings on Solar Cells

For a Si solar cell, √(3.5) or 1.87. Thus, Si_3N_4 is a good choice as an **antireflection coating material** on Si solar cells.

Taking the wavelength to be 700 nm,

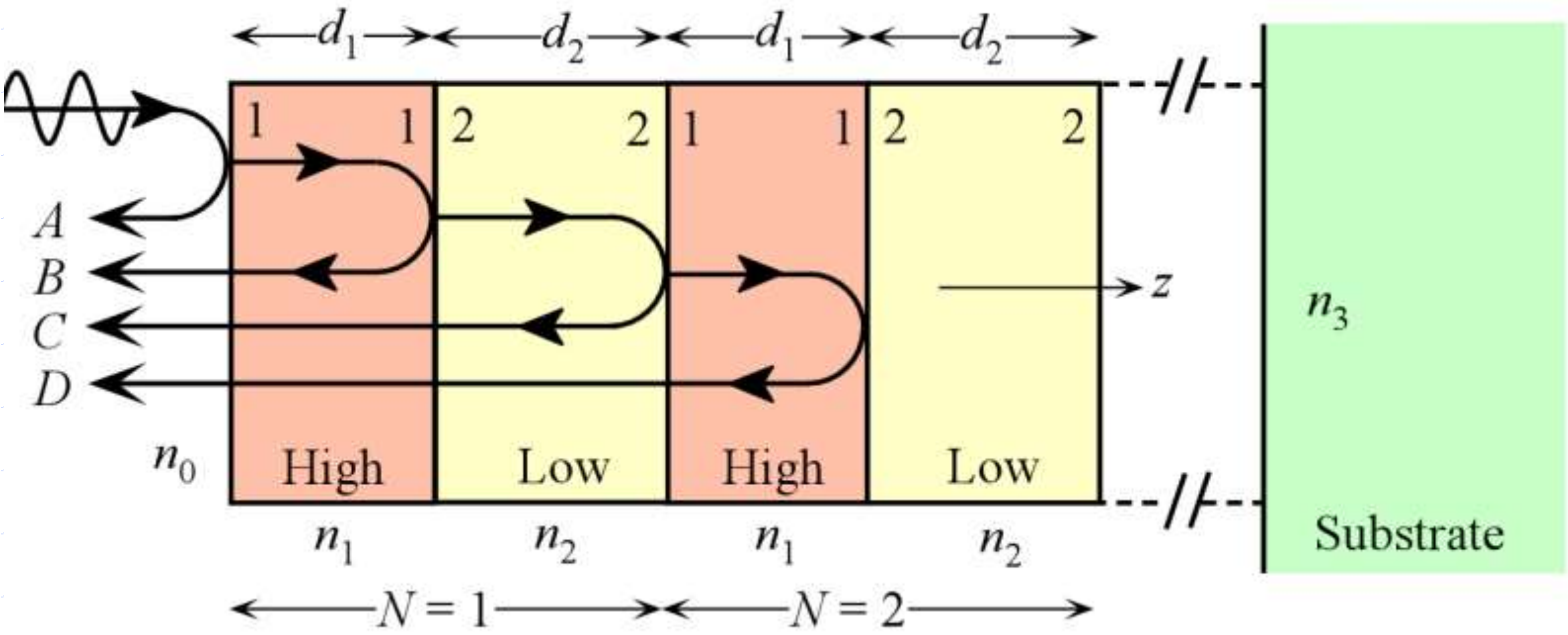
d = (700 nm)/[4 (1.9)] = **92.1 nm** or odd-multiples of d.

$$R_{\min} = \left(\frac{n_2^2 - n_1 n_3}{n_2^2 + n_1 n_3} \right)^2$$

$$R_{\min} = \left(\frac{1.9^2 - (1)(3.5)}{1.9^2 + (1)(3.5)} \right)^2 = 0.00024 \, or \; 0.24\%$$

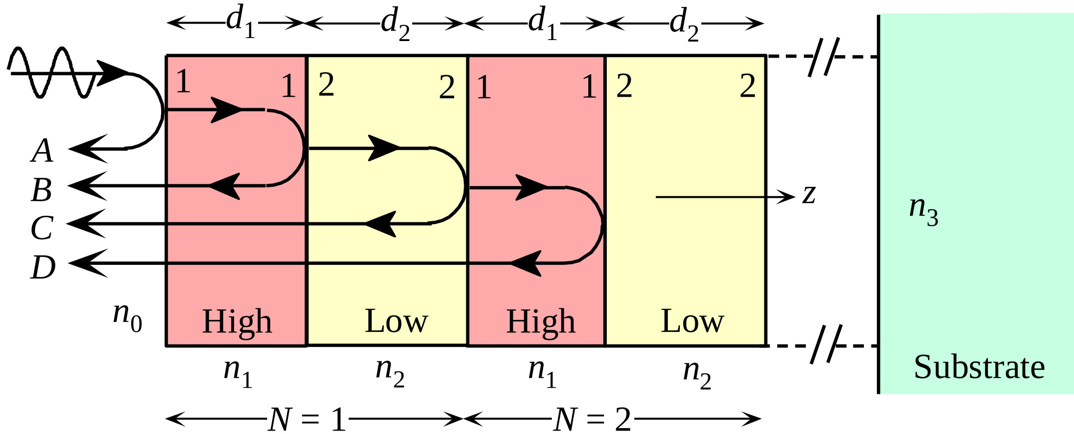
Reflection is almost entirely extinguished
However, only at 700 nm.

Dielectric Mirror or Bragg Reflector



Schematic illustration of the principle of the dielectric mirror with many low and high refractive index layers

Dielectric Mirror or Bragg Reflector



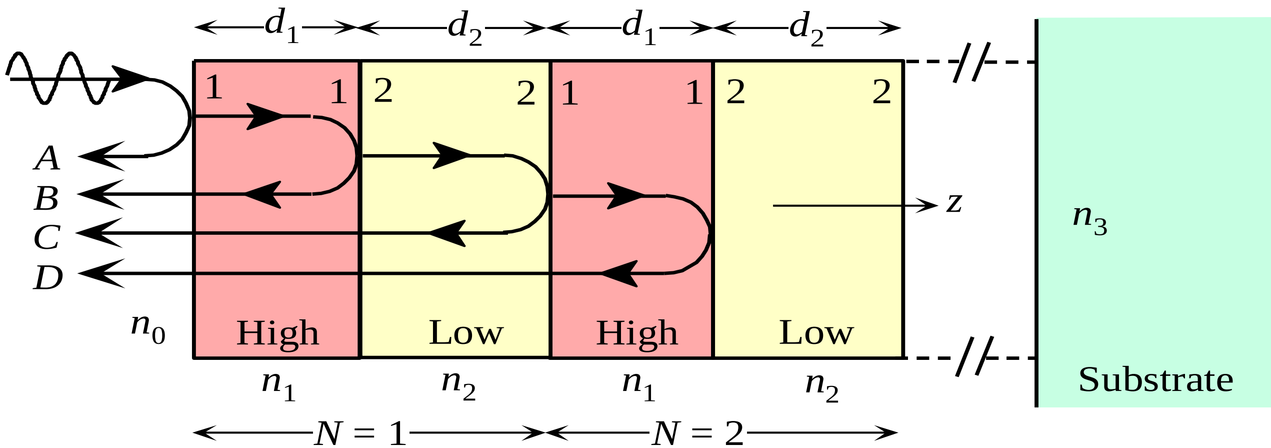
A **dielectric mirror** has a stack of dielectric layers of alternating refractive indices. Let $n_1(=n_H) > n_2(N_L)n_1\ (= n_H) > n_2\ (= n_L)$

Layer thickness d = **Quarter of wavelength or $\lambda_{layer}/4$**

$\boldsymbol{\lambda_{layer} = \lambda_o/n}$; λ_o is the free space wavelength at which the mirror is required to reflect the incident light, n = refractive index of layer.

Reflected waves from the interfaces **interfere constructively** and give rise to a substantial reflected light. If there are sufficient number of layers, the reflectance can approach unity at λ_o.

Dielectric Mirror or Bragg Reflector



r_{12} for light in layer 1 being reflected at the 1-2 boundary is $r_{12} = (n_1 - n_2)/(n_1 + n_2)$ and is a **positive number** indicating **no phase change**.

r_{21} for light in layer 2 being reflected at the 2-1 boundary is $r_{21} = (n_2 - n_1)/(n_2 + n_1)$ which is $-r_{12}$ (negative) indicating **a π phase change**.

The reflection coefficient alternates in sign through the mirror

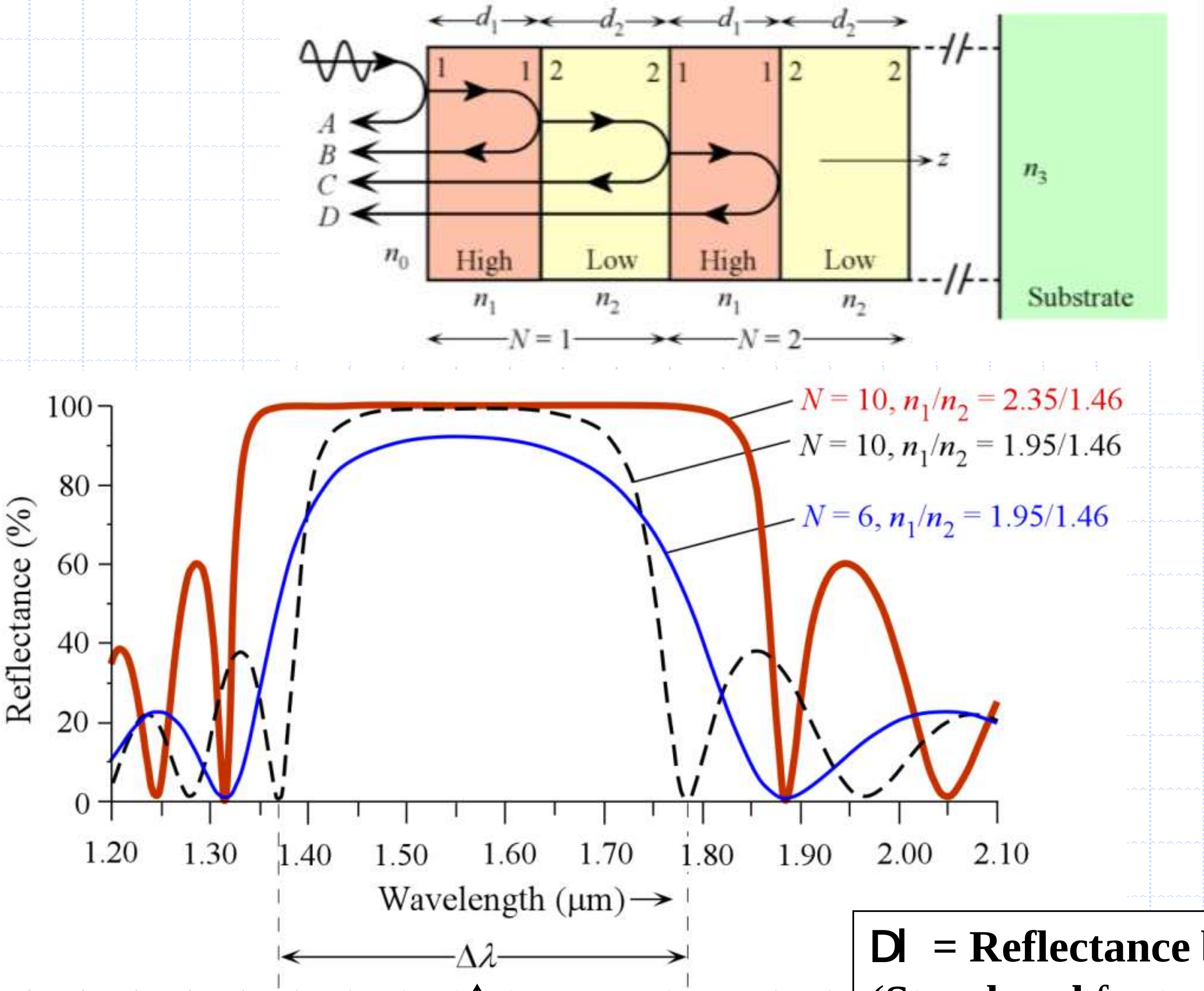
The **phase difference between *A* and *B*** is

$$0 + \pi + 2k_1d_1 = 0 + \pi + 2(2\pi n_1/\lambda_o)(\lambda_o/4n_1) = 2\pi.$$

Thus, waves *A* and *B* are in phase and **interfere constructively**.

Dielectric mirrors are widely used in modern vertical cavity surface emitting semiconductor lasers.

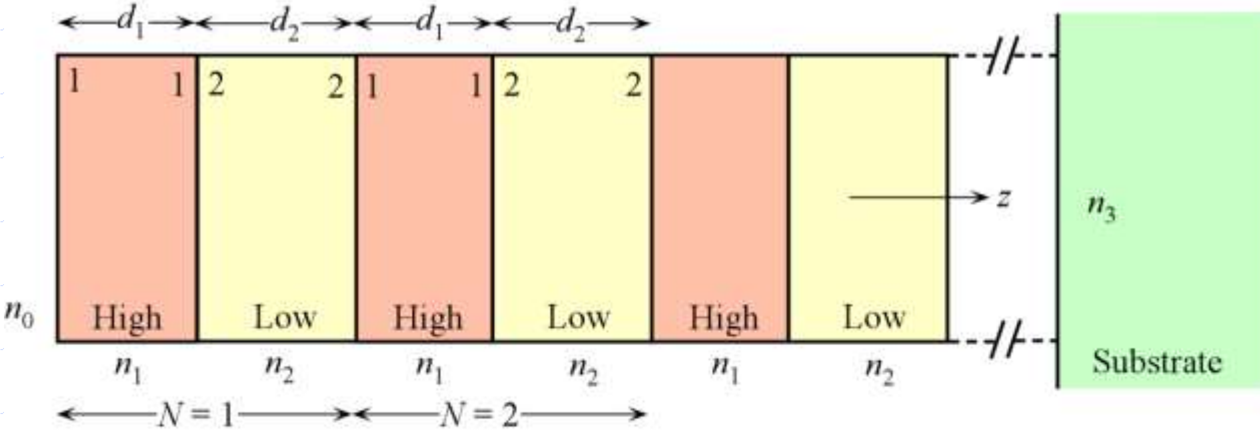
Dielectric Mirror or Bragg Reflector

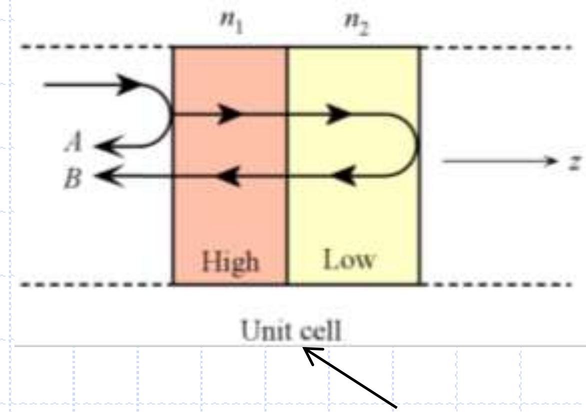


Dl = Reflectance bandwidth
(**Stop-band** for transmittance)

Dielectric Mirror or Bragg Reflector

Consider an "infinite stack"





This is a "unit cell"

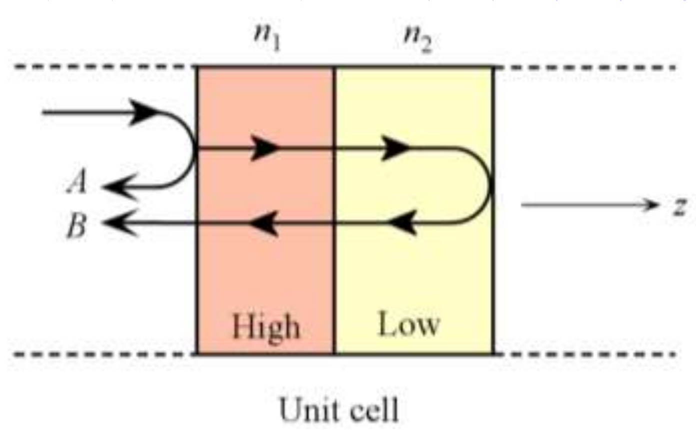
For reflection, the **phase difference between *A* and *B*** must be

$$2k_1d_1 + 2k_2d_2 = m(2\pi)$$

$$2(2\pi n_1/\lambda)d_1 + 2(2\pi n_2/\lambda)d_2 = m(2\pi)$$

$$\boxed{n_1d_1 + n_2d_2 = m\lambda/2}$$

Dielectric Mirror or Bragg Reflector



$$n_1 d_1 + n_2 d_2 = \lambda/2$$

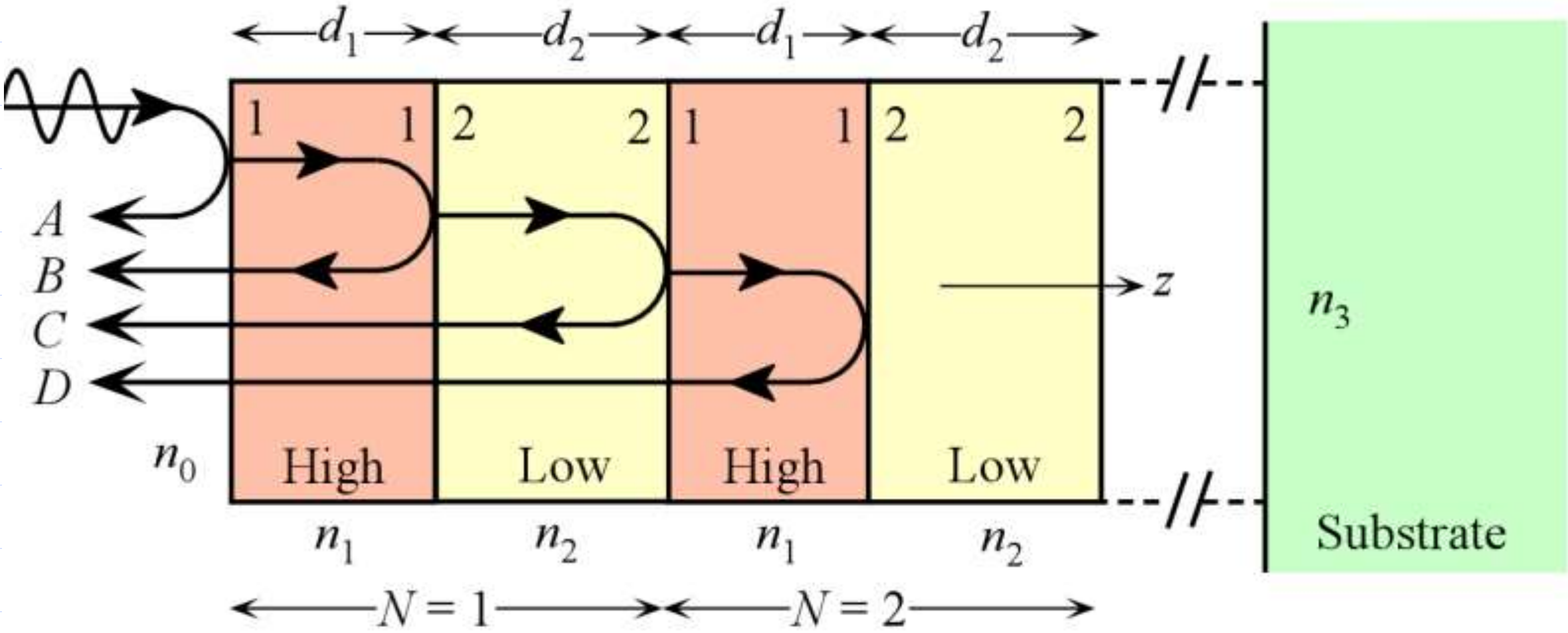
$$d_1 = \lambda/4n_1$$

$$d_2 = \lambda/4n_2$$

Quarter-Wave Stack

$d_1 = \lambda/4n_1$ and $d_2 = \lambda/4n_2$

Dielectric Mirror or Bragg Reflector



$$R_N = \left[\frac{n_1^{2N} - (n_0/n_3)n_2^{2N}}{n_1^{2N} + (n_0/n_3)n_2^{2N}} \right]^2$$

$$\frac{\Delta\lambda}{\lambda_o} \approx (4/\pi)\arcsin\left(\frac{n_1 - n_2}{n_1 + n_2}\right)$$

Example: Dielectric Mirror

A dielectric mirror has quarter wave layers consisting of Ta_2O_5 with n_H = 1.78 and SiO_2 with n_L = 1.55 both at 850 nm, the central wavelength at which the mirror reflects light. The substrate is Pyrex glass with an index n_s = 1.47 and the outside medium is air with n_0 = 1. Calculate the maximum reflectance of the mirror when the number N of double layers is 4 and 12. What would happen if you use TiO_2 with n_H = 2.49, instead of Ta_2O_5? Consider the N = 12 mirror. What is the bandwidth and what happens to the reflectance if you interchange the high and low index layers? Suppose we use a Si wafer as the substrate, what happens to the maximum reflectance?

Solution

n_0 = 1 for air, $n_1 = n_H$ = 1.78, $n_2 = n_L$ = 1.55, $n_3 = n_s$ = 1.47, N = 4. For 4 pairs of layers, the maximum reflectance R_4 is

$$R_4 = \left[\frac{(1.78)^{2(4)} - (1/1.47)(1.55)^{2(4)}}{(1.78)^{2(4)} + (1/1.47)(1.55)^{2(4)}}\right]^2 = 0.4 \text{ or } 40\%$$

Solution

N = 12. For 12 pairs of layers, the maximum reflectance R_{12} is

$$R_{12} = \left[\frac{(1.78)^{2(12)} - (1/1.47)(1.55)^{2(12)}}{(1.78)^{2(12)} + (1/1.47)(1.55)^{2(12)}} \right]^2 = 0.906 \text{ or } 90.6\%$$

Now use TiO_2 for the high-n layer with $n_1 = n_H = 2.49$,

R_4 = 94.0% and **R_{12} = 100%** (to two decimal places).

The refractive index contrast is important. For the TiO_2-SiO_2 stack we only need 4 double layers to get roughly the same reflectance as from 12 pairs of layers of Ta_2O_5-SiO_2. If we interchange n_H and n_L in the 12-pair stack, *i.e.* $n_1 = n_L$ and $n_2 = n_H$, the Ta_2O_5-SiO_2 reflectance falls to 80.8% but the TiO_2-SiO_2 stack is unaffected since it is already reflecting nearly all the light.

Solution

We can only compare bandwidths D/ for "infinite" stacks (those with $R \approx 100\%$)

For the TiO_2-SiO_2 stack

$$\Delta\lambda \approx \lambda_o (4/\pi)\arcsin\left(\frac{n_2 - n_1}{n_2 + n_1}\right)$$

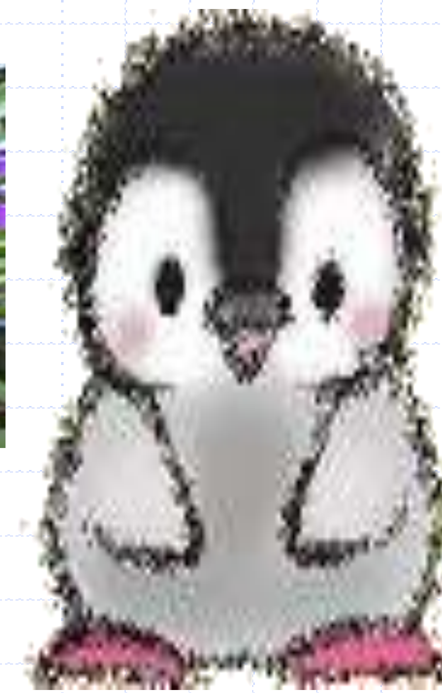
$$\Delta\lambda \approx (850\text{ nm})(4/\pi)\arcsin\left(\frac{2.49 - 1.55}{2.49 + 1.55}\right) = 254\text{nm}$$

For the Ta_2O_5-SiO_2 infinite stack, we get D/ =74.8 nm

As expected D/ is narrower for the smaller contrast stack

Thank you



Have a nice day.