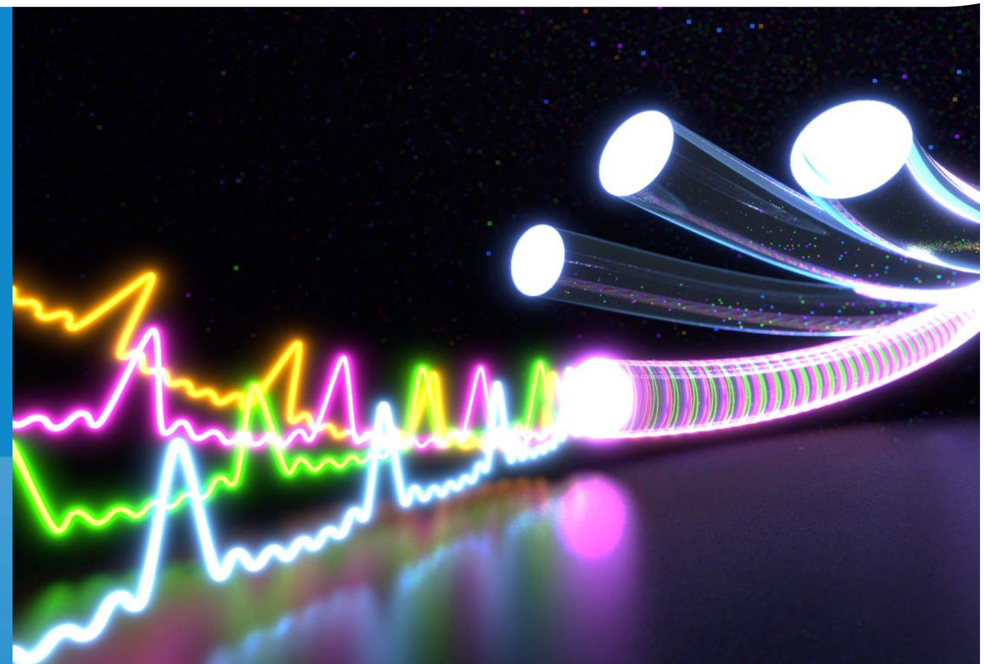
Lecture 4
Dielectric Waveguides and Optical Fibers I

ECE 325
OPTOELECTRONICS

Menoufia University



Kasap–2.1, 2.3, 2.4, and 2.5

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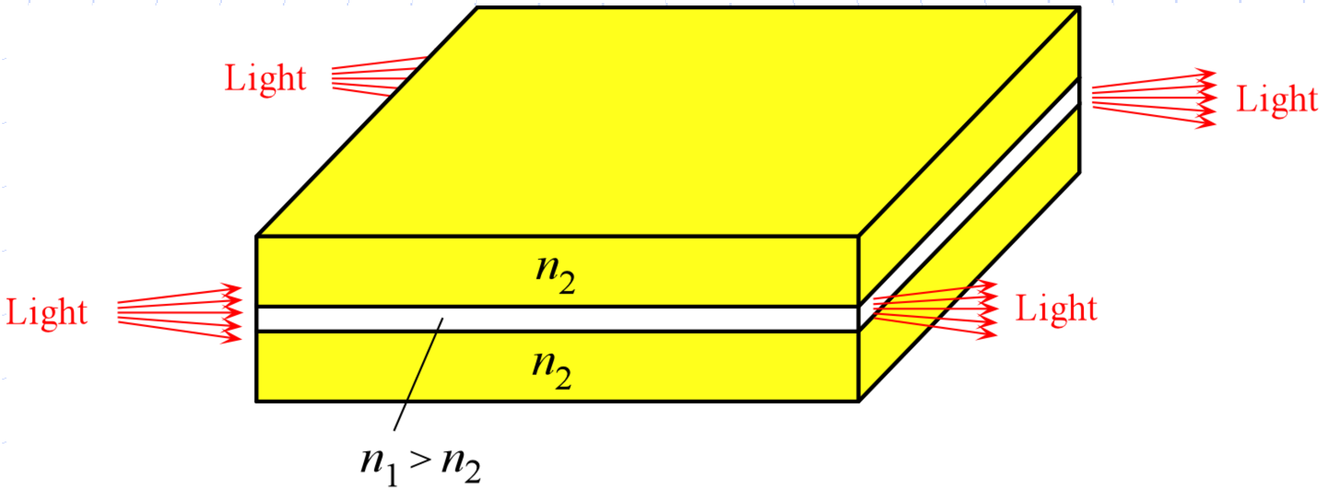
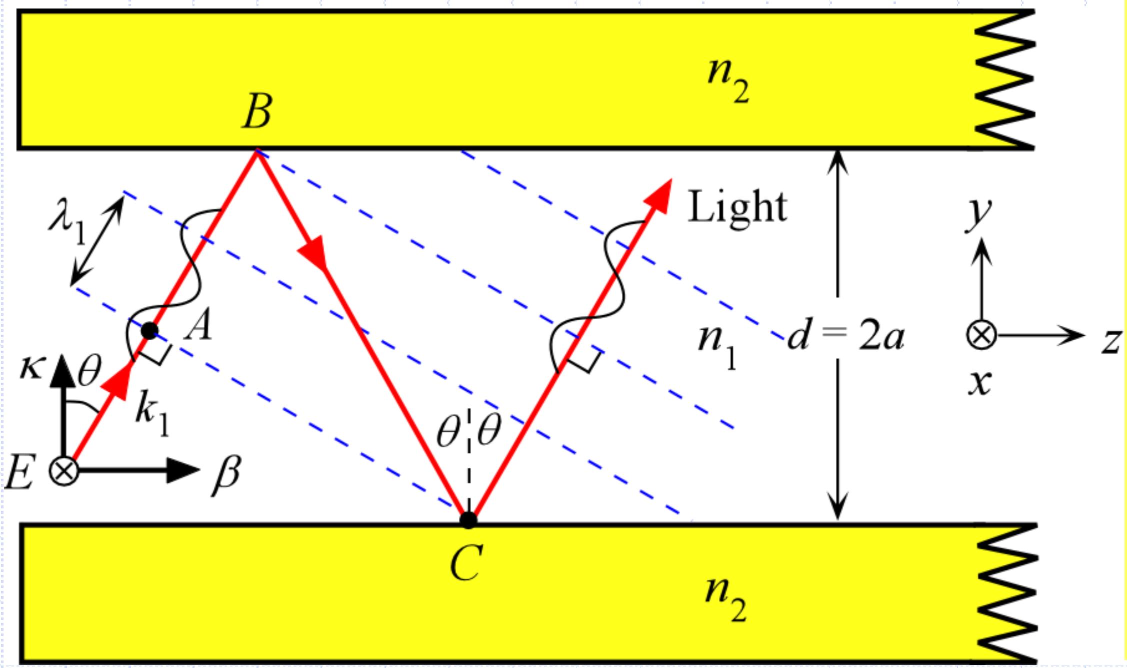
Planar Optical Waveguide



- A slab of dielectric of thickness $2a$ and refractive index n_1 is sandwiched between two semi-infinite regions both of refractive index n_2. ($\boldsymbol{n_1 > n_2}$)
- n_1 is called the **core** and n_2 is called the **cladding**
- Light ray can readily propagate along such a waveguide, in a **zigzag fashion**, provided it can undergo **TIR** at the boundaries between n_1 and n_2
- Any light wave having an angle $\theta > \theta_c$ will propagate.
- The light beam should be **very thin**, much lower that the thickness $2a$ of the core.

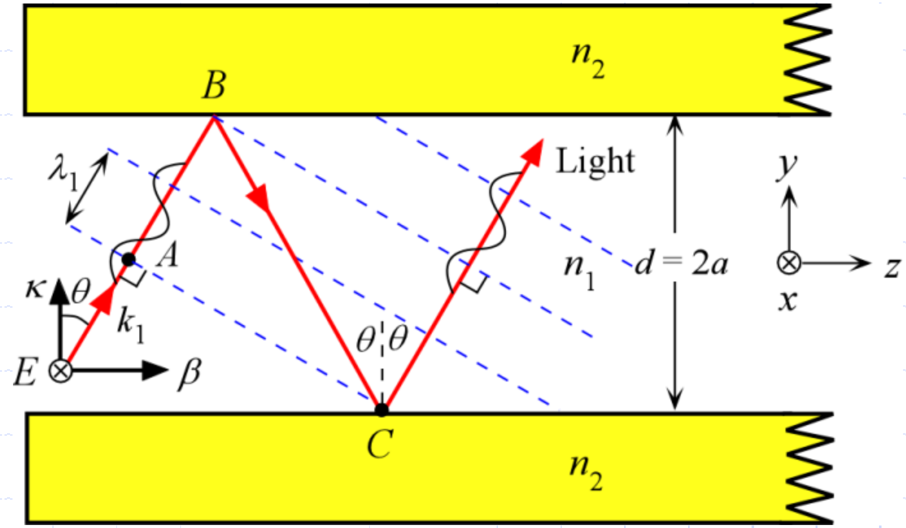
Waves Inside the Core



A light ray traveling in the guide must interfere constructively with itself to propagate successfully. Otherwise destructive interference will destroy the wave. *E* is parallel to *x*. (λ_1 and k_1 are the wavelength and the propagation constant inside the core medium n_1 *i.e.* $\lambda_1 = \lambda/n_1$.)

- Wave coming from A observes TIR at points B and C
- Wave coming from C interferes with its original wave coming from A in the wavefront (AC).
- This interference could be **constructive** or **destructive** depending on the phase shift between the two waves at the interference location (wavefront AC).
- To be constructive this phase shift should be **multiple of 2π**.

Waveguide Condition and Modes



ϕ TIR phase shift at the boundaries (at B and C)

$$k_1 = kn_1 = 2\pi n_1/\lambda,$$

$$\Delta\phi(AC) = k_1(AB + BC) - 2\phi = m(2\pi),\quad m = 0,1,2,3,\ldots$$

$$BC = d/\cos\theta \quad \text{and} \quad AB = BC\cos(2\theta)$$

$$AB + BC = BC\cos(2\theta) + BC = BC[(2\cos^2\theta - 1) + 1] = 2d\cos\theta$$

For each value of m, we find a possible pair of angles (θ_m, ϕ_m)

$$\mathbf{k_1[2d\cos\theta] - 2\phi = m(2\pi)}$$

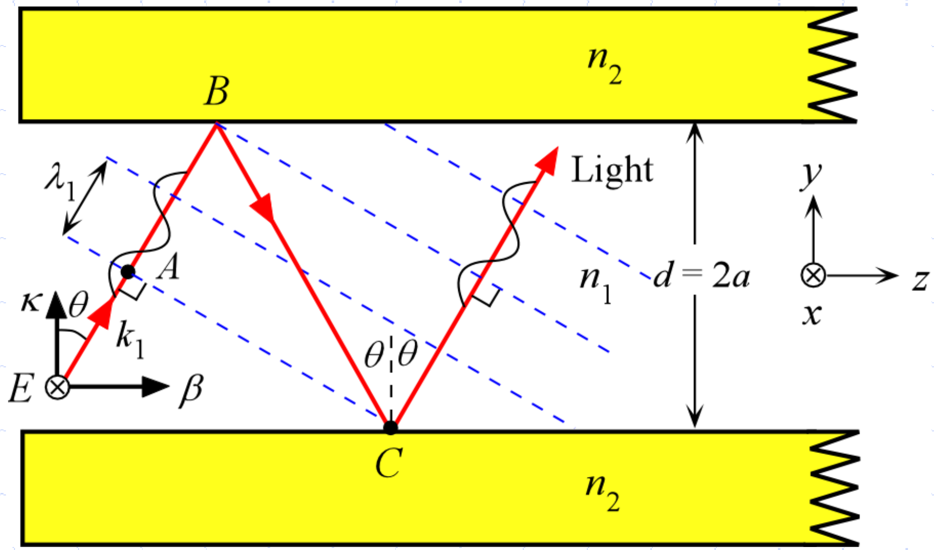
$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

m* = 0, 1, 2, 3 *etc
Integer

"Mode number"

Waveguide condition

Propagation Constant and Vector



- Propagation vector $\mathbf{k_1}$ has an angle θ_m with z-axis. We decompose this in two components:

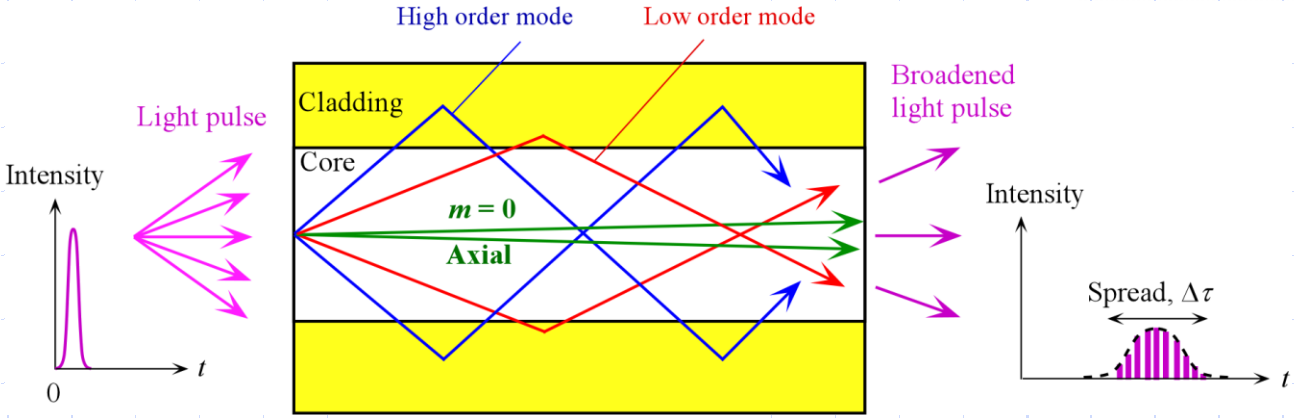
$$\beta_m = k_1 \sin\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\sin\theta_m$$

Propagation constant along the guide (along *z)*

$$\kappa_m = k_1 \cos\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\cos\theta_m$$

Transverse Propagation constant (along *y)*

Waveguide Condition and Waveguide Modes



To get a propagating wave along a guide you must have constructive interference. All these rays interfere with each other. Only certain angles are allowed . Each allowed angle represents a **mode** of propagation.

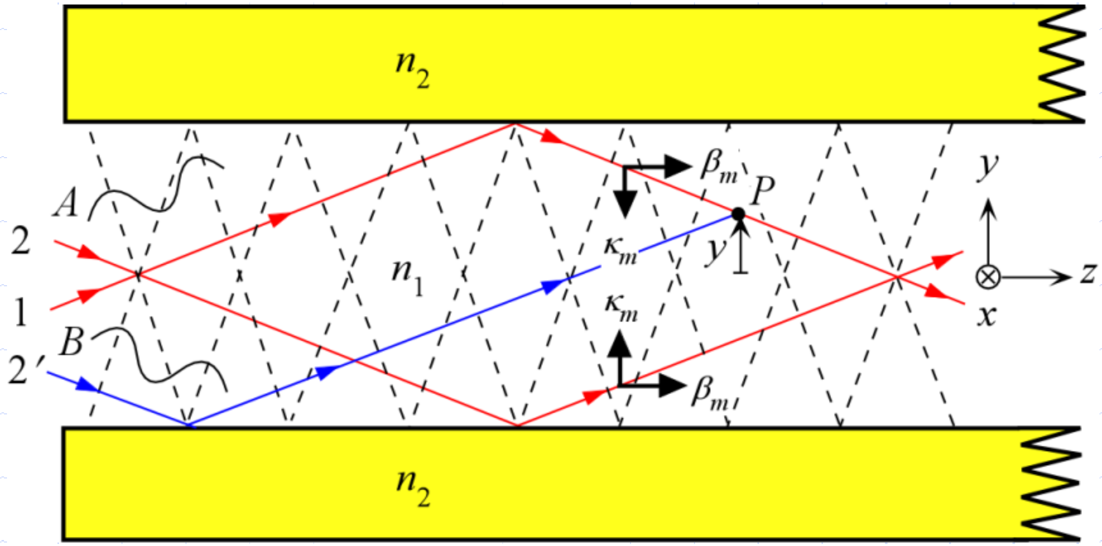
$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

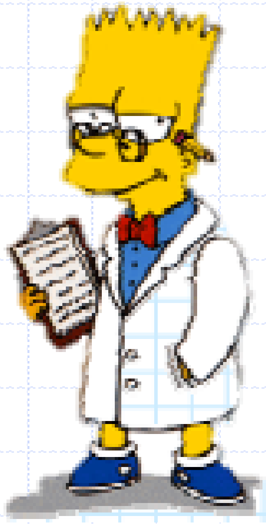
m **= integer,** n_1 **= core refractive index,** θ_m **is the incidence angle, 2*a* is the core thickness.**

Minimum θ_m **and maximum** ***m*** **must still satisfy TIR.**

There are only a finite number of modes.

Modes in a Planar Waveguide

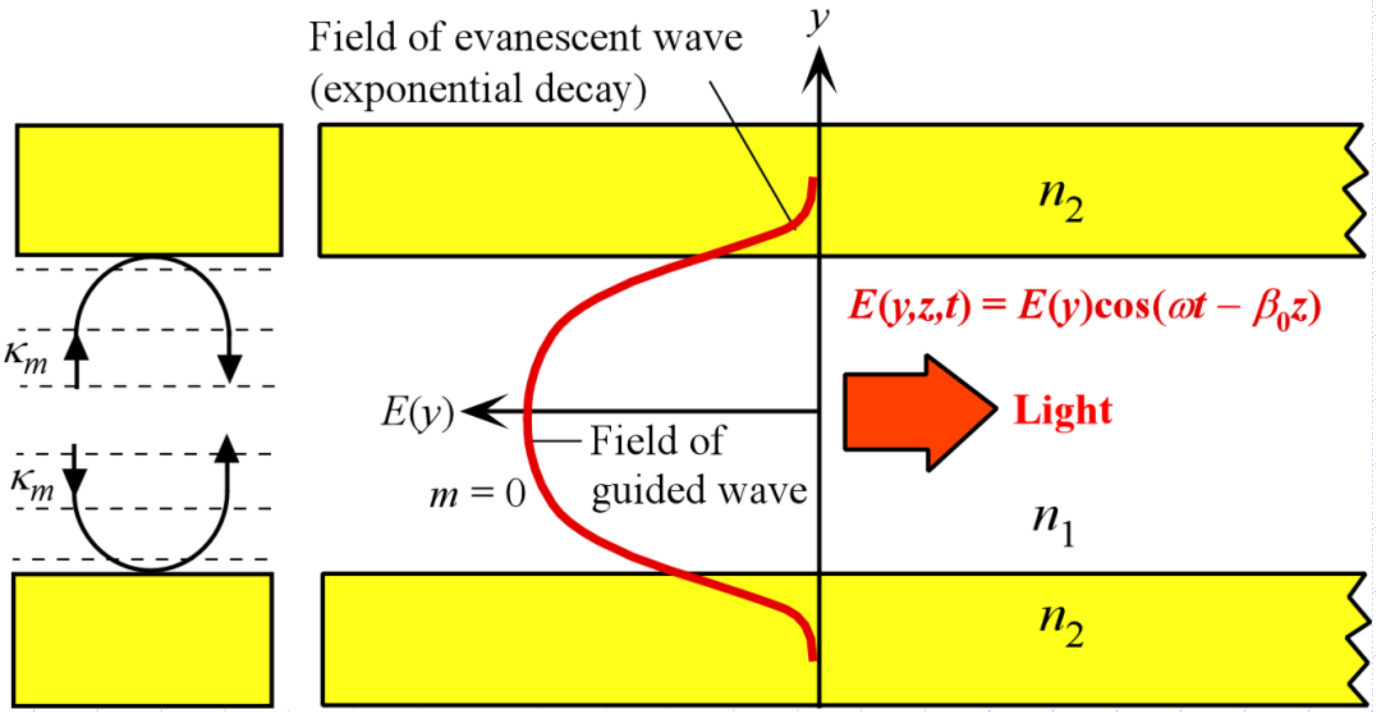




We can identify upward (A) and downward (B) traveling waves in the guide which interfere to set up a **standing wave** along y and a wave that is **propagating** along z. Rays 2 and 2′ belong to the same wave front but 2′ becomes reflected before 2. The interference of 1 and 2′ determines the field at a height y from the guide center. The field $E(y, z, t)$ at P can be written as

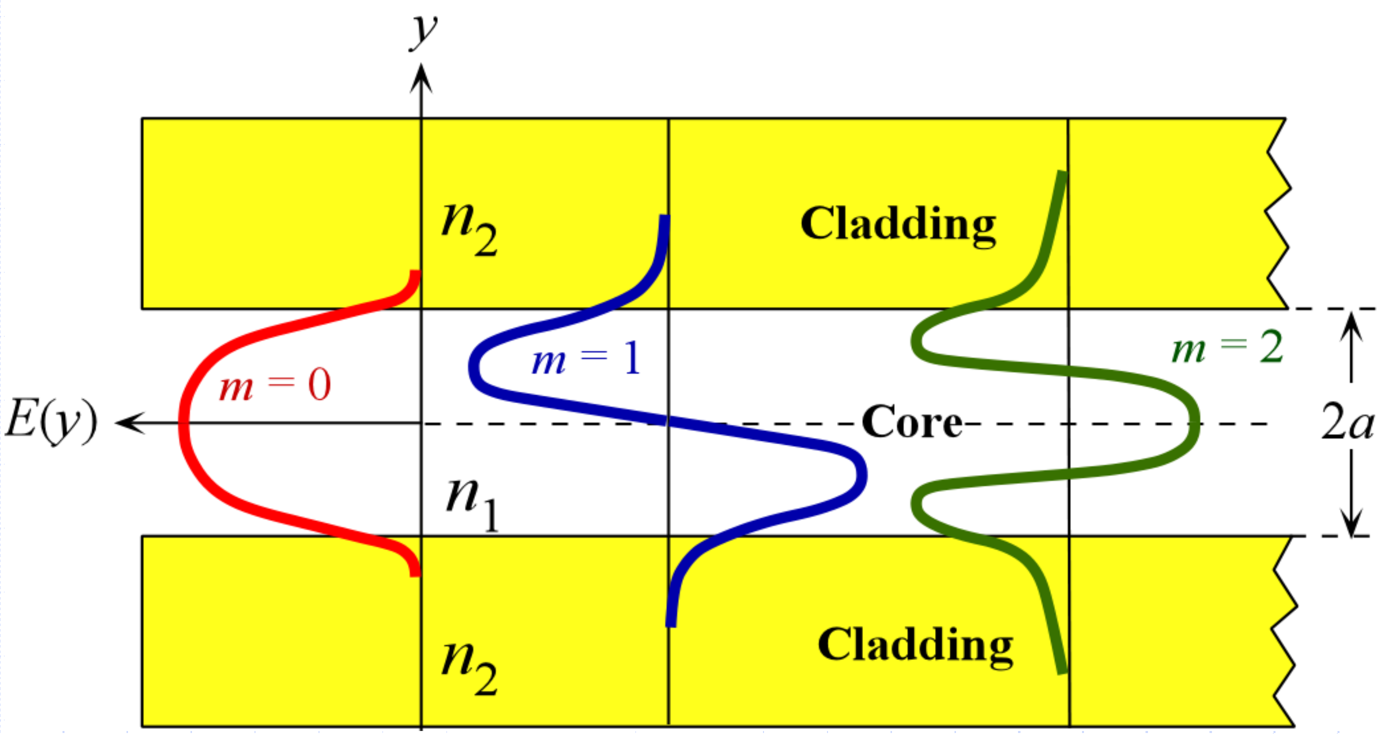
$$E(y,z,t) = \underbrace{E_m(y)}_{\text{Field pattern along } y}\underbrace{\cos(\omega t - \beta_m z)}_{\text{Traveling wave along } z}$$

Mode Field Pattern



Left: The upward and downward traveling waves have equal but opposite wavevectors k_m and interfere to set up a standing electric field pattern across the guide. **Right:** The E pattern of the lowest mode traveling wave along the guide. This mode has $\boldsymbol{m = 0}$ and the **lowest $\boldsymbol{\theta}$**. It is often referred to as the glazing incidence ray. It has the **highest phase velocit**y along the guide

Modes in a Planar Waveguide



The electric field patterns of the first three modes ($m = 0, 1, 2$) traveling wave along the guide.

Notice **different extents** of field penetration into the cladding.

Each of these traveling wave constitutes a **mode of propagation**.

Normalized Frequency or *V*-Number

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}$$

For a specific wavelength λ, V-number depends on:

- The **waveguide geometry** ($2a$) and
- Material characteristics (n_1 and n_2)

The V-number is a characteristic parameter of the waveguide

- The mode number, m, must satisfy

$$m \le \frac{(2V - \phi)}{\pi}$$

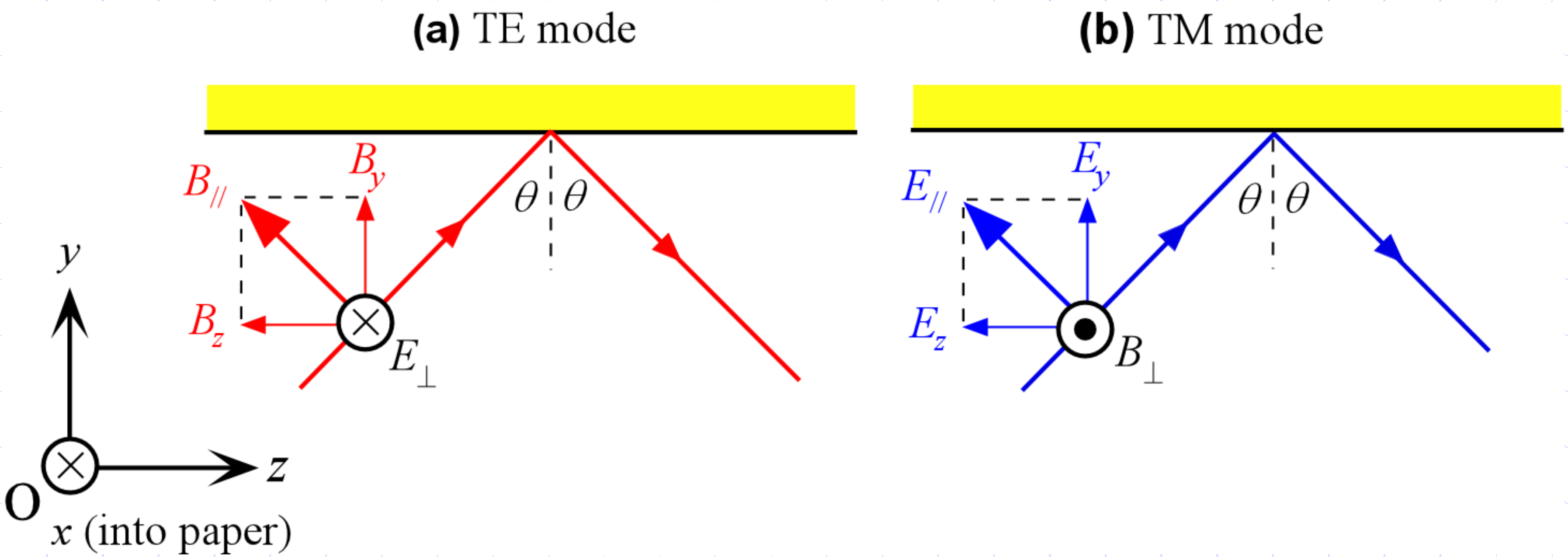
$V < \pi/2$, $m = 0$ is the only possibility and only the **fundamental mode** ($m = 0$) propagates along the dielectric slab waveguide: a **single mode** planar waveguide.

$\lambda = \lambda_c$ for $V = \pi/2$ is the **cut-off wavelength**, and above this wavelength, only one-mode, the fundamental mode will propagate.

Multimode when $V > \pi/2$

$$M = \mathrm{Int}\left(\frac{2V}{\pi}\right) + 1$$

TE and TM Modes



$E_\perp$ is along x, so that $E_\perp = E_x$

$B_\perp$ is along $-x$, so that $B_\perp = -B_x$

Possible modes can be classified in terms of (a) transverse electric field (TE) and (b) transverse magnetic field (TM). Plane of incidence is the paper.

Example on Waveguide Modes

Consider a planar dielectric guide with a core thickness 20 μm, n_1 = 1.455, n_2 = 1.440, light wavelength of 900 nm. Find angles θ_m for all modes.

TIR phase change ϕ_m for TE mode

$$\tan\left(\tfrac{1}{2}\phi_m\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m}$$

TE mode

Waveguide condition

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

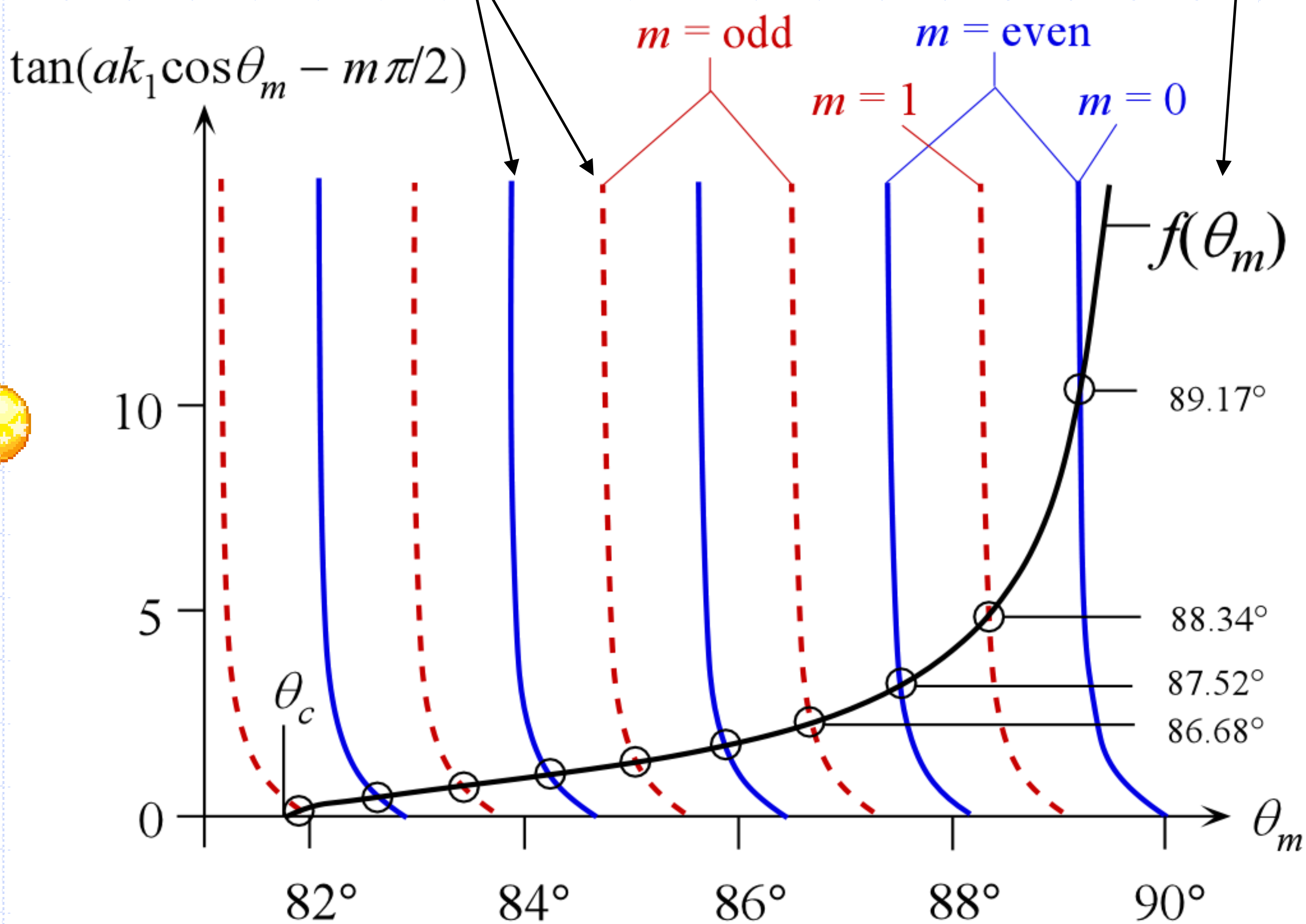
Waveguide condition

$$\phi_m = 2ak_1\cos\theta_m - m\pi$$

Example on Waveguide Modes

$$\tan\left(ak_1\cos\theta_m - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m} = f(\theta_m)$$

TE mode



Example on Waveguide Modes

$$\frac{1}{\delta_m} = \alpha_m = \frac{2\pi n_2 \left[\left(\frac{n_1}{n_2} \right)^2 \sin^2 \theta_m - 1 \right]^{1/2}}{\lambda}$$

Mode m, incidence angle θ_m and penetration δ_m for a planar dielectric waveguide with $d = 2a = 20$ μm, $n_1 = 1.455$, $n_2 = 1.440$ ($\lambda = 900$ nm)

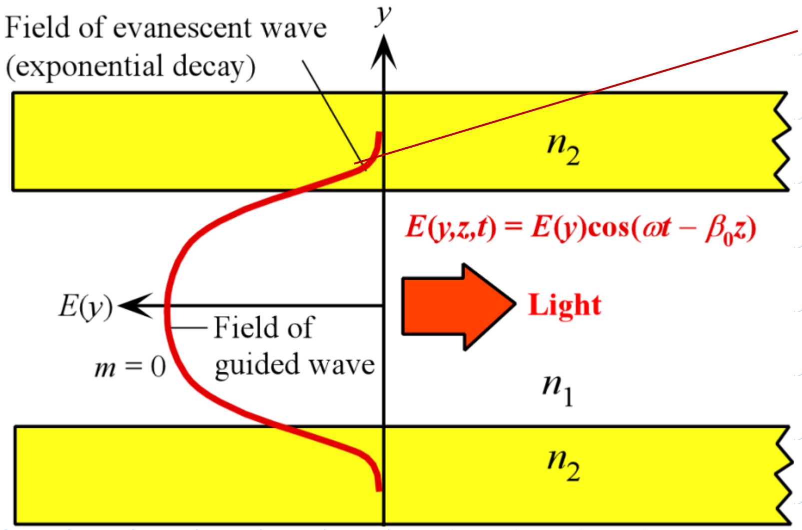
m	0	1	2	3	4	5	6	7	8	9
θ_m	89.2°	88.3°	87.5°	86.7°	85.9°	85.0°	84.2°	83.4°	82.6°	81.9°
δ_m (μm)	0.691	0.702	0.722	0.751	0.793	0.866	0.970	1.15	1.57	3.83

$m = 0$
Fundamental mode

Highest mode

Critical angle $\theta_c = \arcsin(n_2/n_1) = 81.77°$

Mode Field Width $2w_o$



$E_{\text{cladding}}(y') = E_{\text{cladding}}(0)\exp(-\alpha_{\text{cladding}}\, y')$

For the axial mode: θ_i close to 90°

$$\alpha_{\text{cladding}} = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

$$\approx \frac{2\pi}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{V}{a}$$

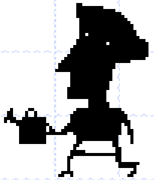
$$\therefore \quad 2w_o = 2a + 2\delta \approx 2a\frac{(V+1)}{V}$$

Mode Field Width $2w_o$

When V increases:

$$V \uparrow \Rightarrow MFW \quad \Rightarrow \quad 2a$$

OPTICAL FIBERS



Why Fiber ?

• Enormous capacity: 1.3 mm-1.55 mm allocates bandwidth of 37 THz!!

• Cables and equipment have small size and weight

- A large number of fibers fit easily into an optical cable
- Applications in special environments as in aircrafts, satellites, ships

• Longer Distances (SMF)

- Less attenuation per distance: Optical fiber loss can be as low as 0.2dB/km Compared to loss of coaxial cables: 10-300dB/km)
- Almost zero frequency dependent loss
- Dispersion Limited (Chromatic ~5ps/nm/km)

• Lower Power

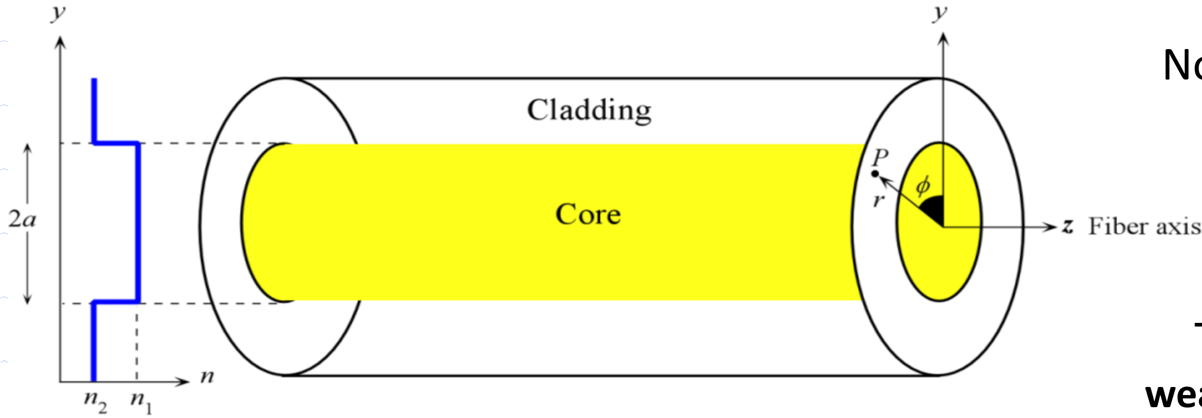
- Less attenuation

Why Fiber ?

- Less Noise
  - No crosstalk between fibers
  - No reflections
- Immunity to interference
  - Nuclear power plants, hospitals, EMP resistive systems (installations for defense)
- Electrical isolation
  - Electrical hazardous environments
  - Negligible crosstalk
- Signal security
  - Banking, computer networks, military systems
- Silica fibers have abundant raw material

Optical Fibers

- Typically made of silica
- Commonly used fibers have core and cladding layers.



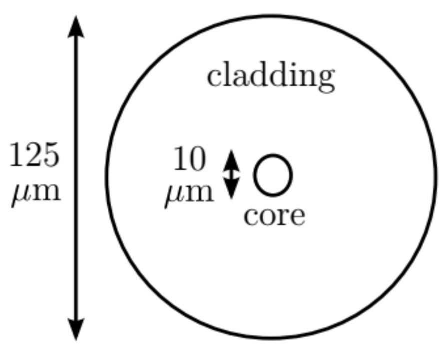
Normalized index difference Δ

$$\Delta = \frac{(n_1 - n_2)}{n_1}$$

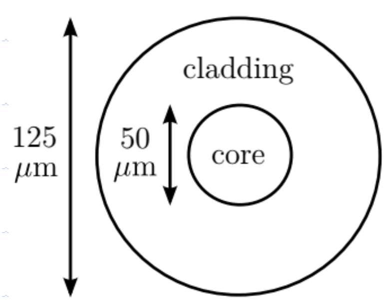
Typical value $\Delta \ll 1$

weakly guiding fibers

The step index optical fiber. The central region, the core, has greater refractive index than the outer region, the cladding. The fiber has cylindrical symmetry. The coordinates *r*, ϕ, z are used to represent any point *P* in the fiber. Cladding is normally much thicker than shown.



SMF cross section



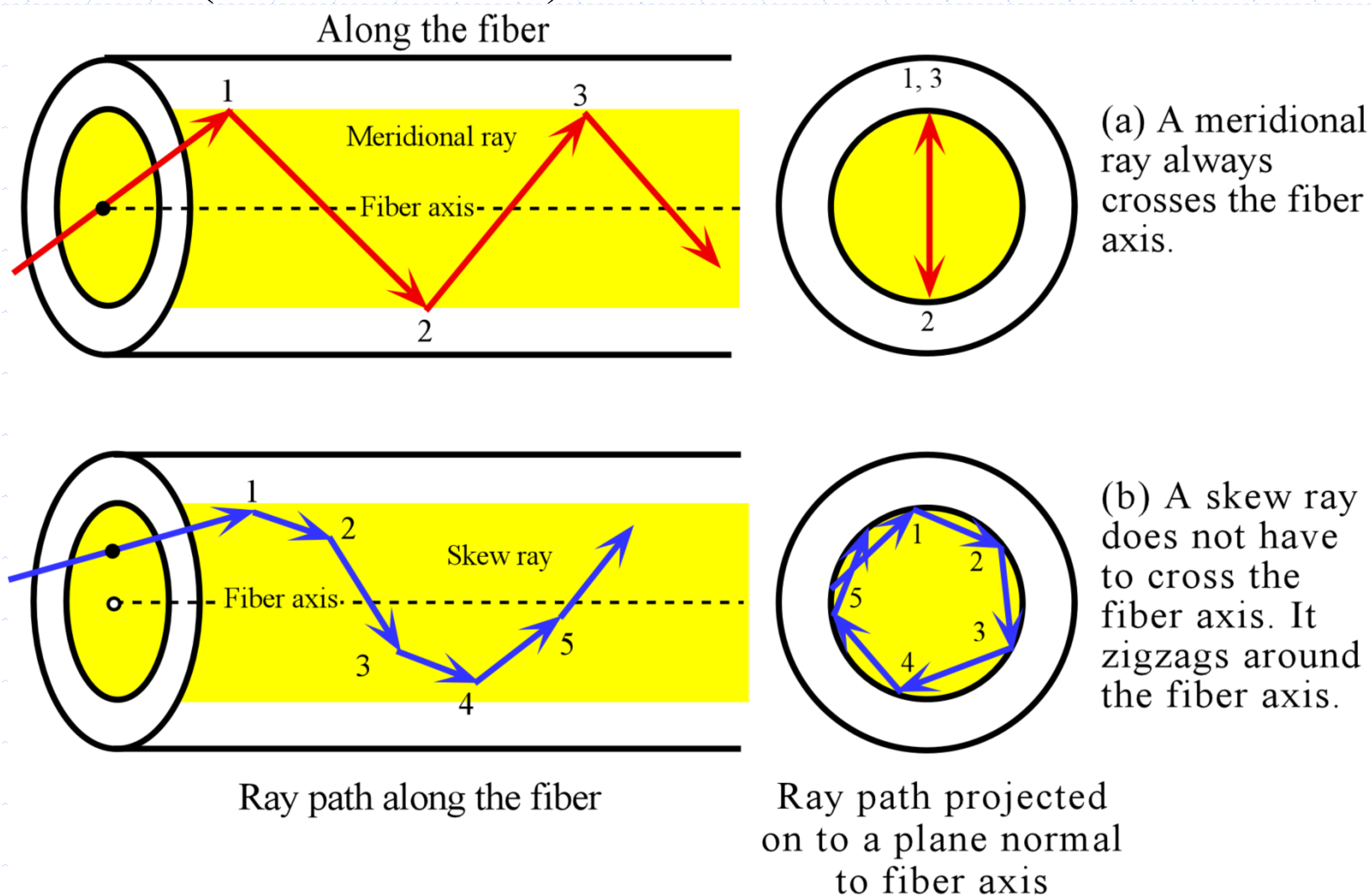
MMF cross section

- 50/125 µm,
- 62.5/125 µm,
- 100/140 µm

Ray Representation of the propagation

Meridional ray enters the fiber through the fiber axis and hence also crosses the fiber axis on each reflection as it zigzags down the fiber. It travels in a plane that contains the fiber axis. (TE or TM wave)

Skew ray enters the fiber off the fiber axis and zigzags down the fiber without crossing the axis. (HE or EH mode)



Modes LP_{lm}

$\Delta << 1$ weakly guiding fibers

Guided modes in a step index fiber with $\Delta << 1$ are generally visualized by LP wave.

LP (linearly polarized) mode

- Both have TE and TM wave
- Can be represented by the propagation of an electric field distribution $E(r,\varphi)$

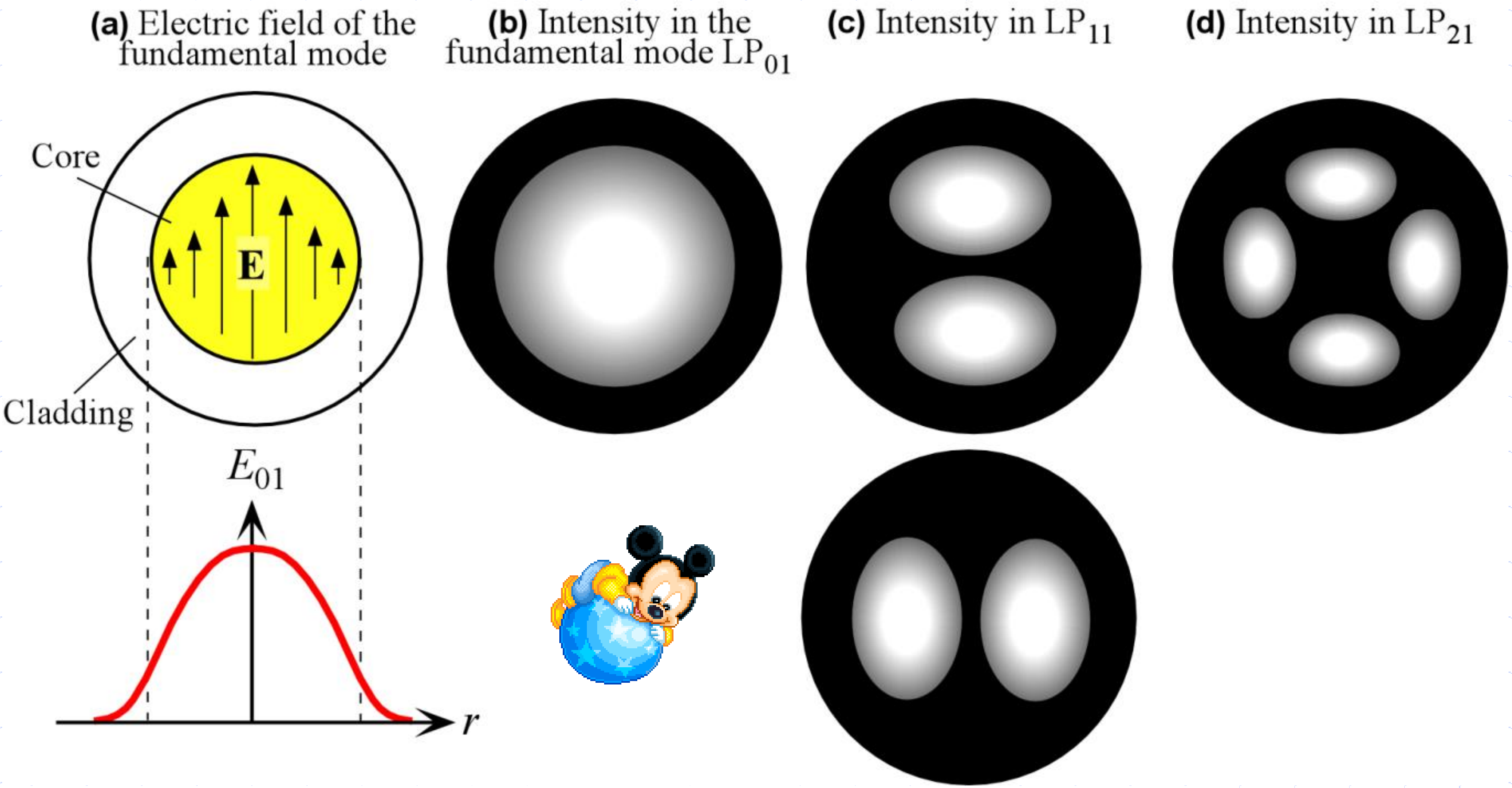
$$E_{\mathrm{LP}} = \underbrace{E_{lm}(r,\phi)}_{\text{Field Pattern}} \; \underbrace{\exp j(\omega t - \beta_{lm} z)}_{\text{Traveling wave}}$$

E and **B** are 90^{o} to each other and *z*

m = number of maxima along r starting from the core center.
Determines the reflection angle θ

$2l$ = number of maxima around a circumference

Modes LP_{lm}



The electric field distribution of the fundamental mode in the transverse plane to the fiber axis *z*. The light intensity is greatest at the center of the fiber. Intensity patterns in LP_{01}, LP_{11} and LP_{21} modes. (a) The field in the fundamental mode. (b)-(d) Indicative light intensity distributions in three modes, LP_{01}, LP_{11} and LP_{21}.

Modes in an Optical Fiber

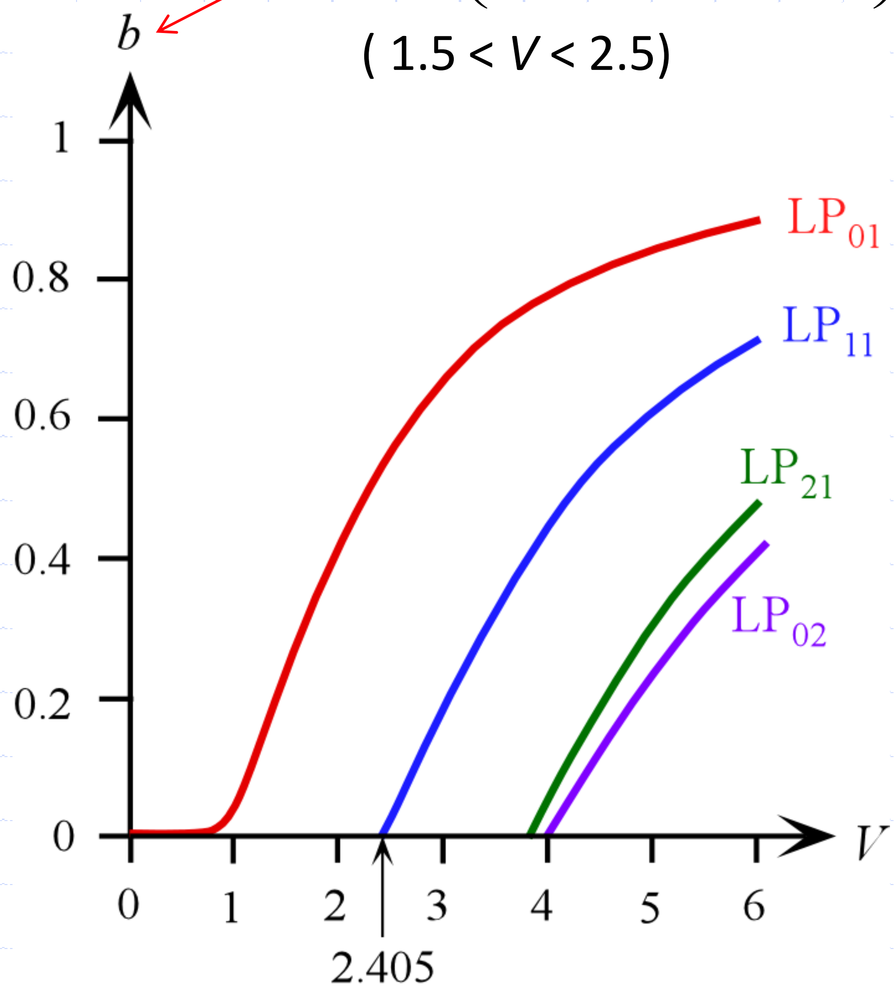
Normalized propagation constant

$$\boxed{b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}}$$

$$k = 2\pi/\lambda$$

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2$$

($1.5 < V < 2.5$)



Normalized propagation constant *b* vs. *V*-number for a step-index fiber for various LP modes

Optical Fiber Parameters

$n = (n_1 + n_2)/2 =$ **average refractive index**

$\Delta =$ **normalized index difference**

$\Delta = (n_1 - n_2)/n_1 \approx (n_1{}^2 - n_2{}^2)/2n_1{}^2$

***V*-number** $$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = \frac{2\pi a}{\lambda}(2n_1 n\Delta)^{1/2}$$

$V < 2.405$ only 1 mode exists. Fundamental mode

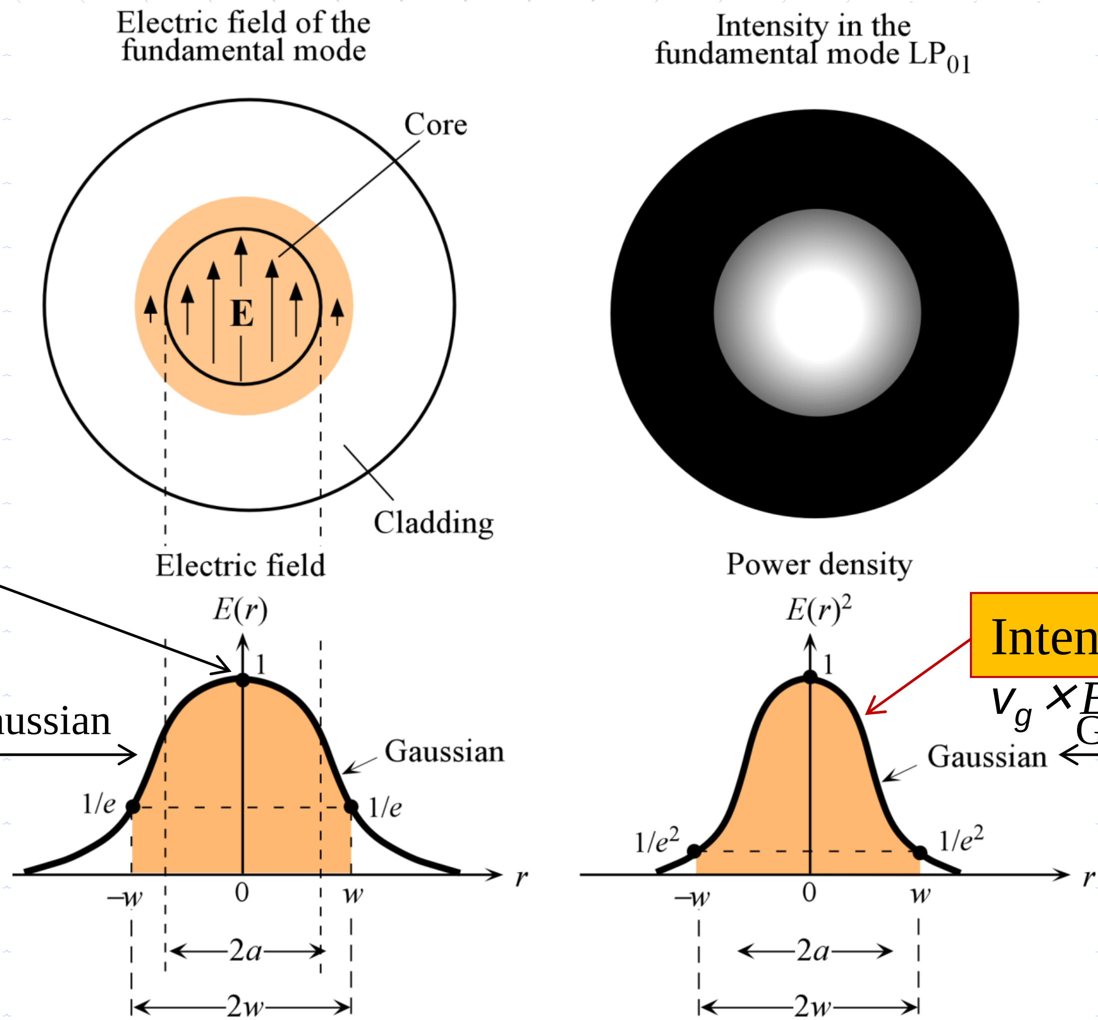
$V < 2.405$ or $\lambda > \lambda_c$ Single mode fiber

$V > 2.405$ Multimode fiber

Exists a good approximation to the number of modes M

Number of modes $$M \approx \frac{V^2}{2}$$

Mode Field Diameter (2w)



Note: Maximum set arbitrarily to 1

Gaussian

Intensity $\propto v_g \times E(r)^2$

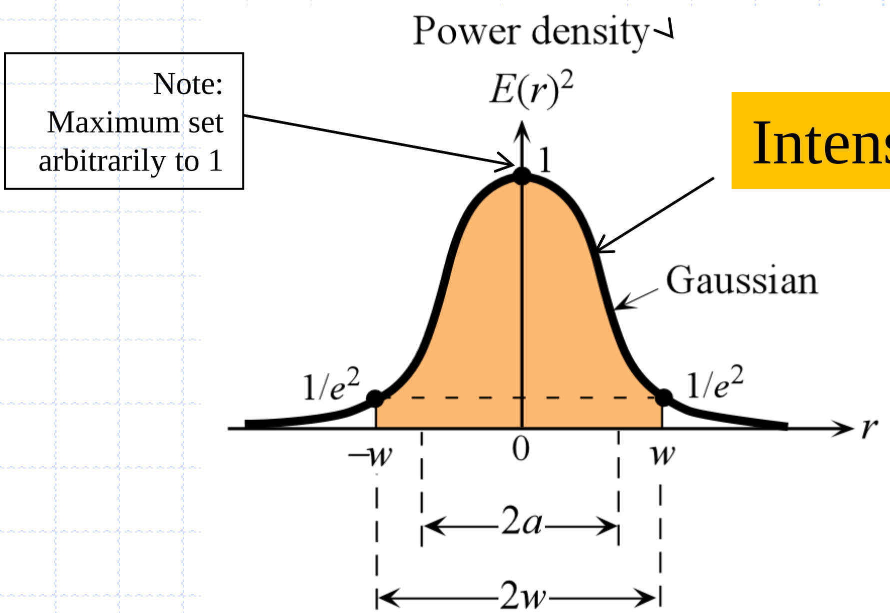
Gaussian

$$E(r) = E(0)\exp[-(r/w)^2]$$

$$E(r)^2 = E(0)^2\exp[-2(r/w)^2]$$

Mode Field Diameter

$$E(r)^2 = E(0)^2 \exp[-2(r/w)^2]$$



Intensity $\propto v_g \times E(r)^2$

$2w$ = Mode Field Diameter (MFD)

Marcuse MFD Equation

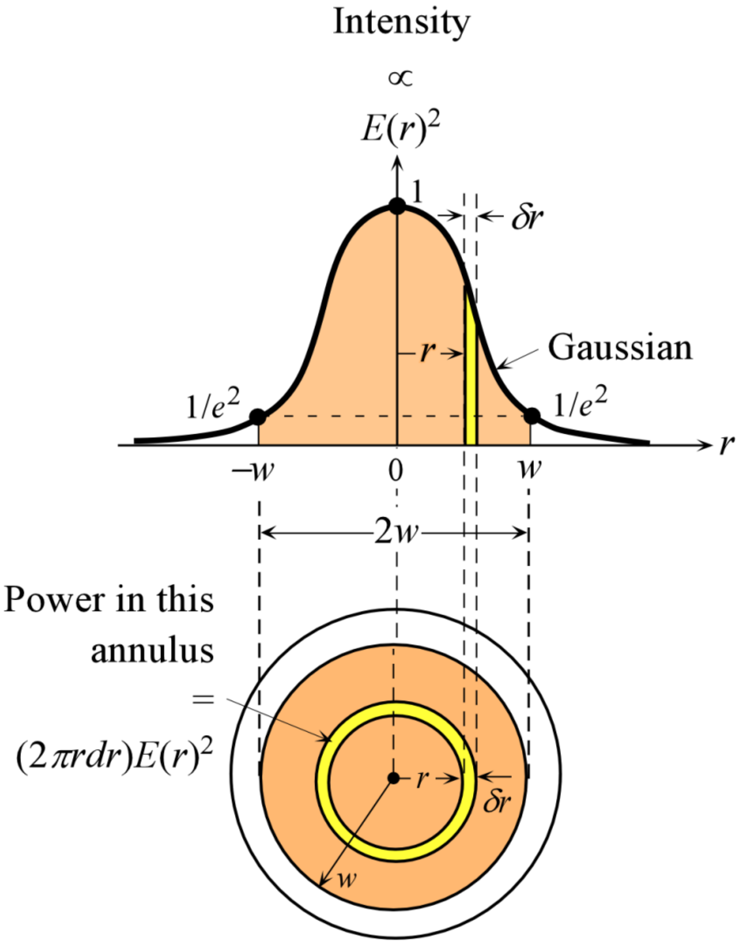
$$2w = 2a(0.65 + 1.619V^{-3/2} + 2.879V^{-6})$$

$0.8 < V < 2.5$

$$2w \approx (2a)(2.6/V)$$

$1.6 < V < 2.4$

Mode Field Diameter (2w)



$$E(r)^2 = E(0)^2 \exp[-2(r/w)^2]$$

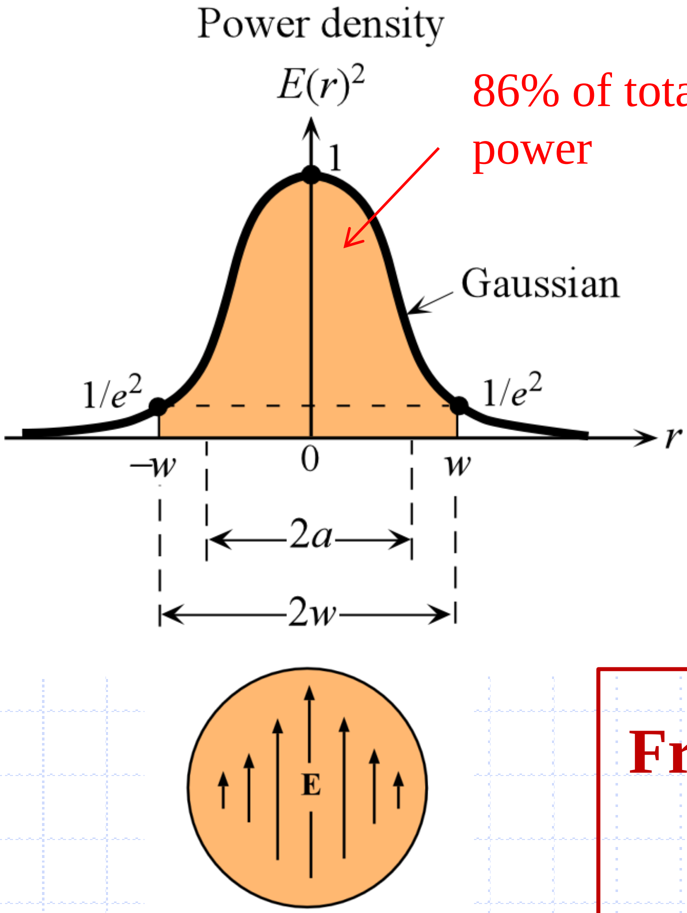
and

$$\text{Intensity} \propto v_g \times E(r)^2$$

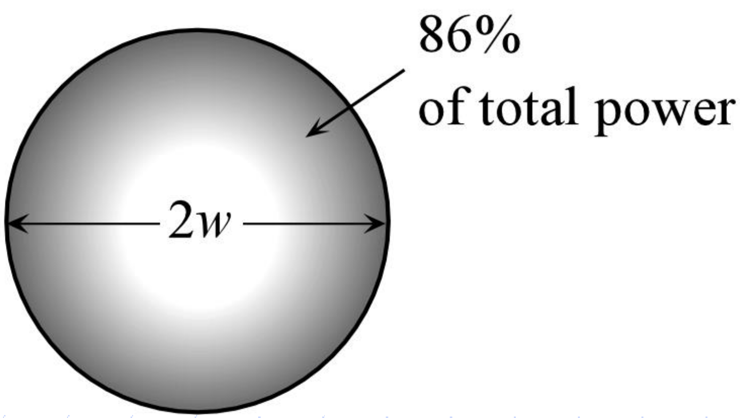
Area of a circular thin strip (annulus) with radius r is $2\pi r dr$. Power passing through this strip is proportional to $E(r)^2(2\pi r)dr$

$$\text{Fraction of optical power within MFD} = \frac{\int_0^w E(r)^2 \boxed{2\pi r dr}}{\int_0^\infty E(r)^2 2\pi r dr} = 0.865$$

$$E(r)^2 = E(0)^2 \exp[-2(r/w)^2]$$



Mode Field Diameter (2*w*)



Fraction of optical power within MFD = 86 %

This is the same as the fraction of optical power within a radius *w* from the axis of a Gaussian beam (See Chapter 1)

Example: A multimode fiber

Calculate the number of allowed modes in a multimode step index fiber which has a core of refractive index of 1.468 and diameter 100 μm, and a cladding of refractive index of 1.447 if the source wavelength is 850 nm.

Solution

Substitute, $a = 50$ μm, $\lambda = 0.850$ μm, $n_1 = 1.468$, $n_2 = 1.447$ into the expression for the V-number,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} = (2\pi 50/0.850)(1.468^2 - 1.447^2)^{1/2}$$
$$= 91.44.$$

Since $V >> 2.405$, the number of modes is

$$M \approx V^2/2 = (91.44)^2/2 = 4181$$

which is large.

Example: A single mode fiber

What should be the core radius of a single mode fiber which has a core of $n_1 = 1.4680$, cladding of $n_2 = 1.447$ and it is to be used with a source wavelength of 1.3 μm?

Solution

For single mode propagation, $V \leq 2.405$. We need,

$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$

or

$[2\pi a/(1.3\ \mu\text{m})](1.468^2 - 1.447^2)^{1/2} \leq 2.405$

which gives $\boldsymbol{a \leq 2.01\ \mu\text{m}}$.

Rather thin for easy coupling of the fiber to a light source or to another fiber; *a* is comparable to *l* which means that the geometric ray picture, strictly, cannot be used to describe light propagation.

Example: Single mode cut-off wavelength

Calculate the cut-off wavelength for single mode operation for a fiber that has a core with diameter of 8.2 μm, a refractive index of 1.4532, and a cladding of refractive index of 1.4485. What is the *V*-number and the mode field diameter (MFD) for operation at λ = 1.31 μm?

Solution

For single mode operation,

$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$

Substituting for a, n_1 and n_2 and rearranging we get,

$\lambda > [2\pi(4.1\ \mu m)(1.4532^2 - 1.4485^2)^{1/2}]/2.405 = 1.251\ \mu m$

Wavelengths shorter than 1.251 μm give multimode propagation.

At λ = 1.31 μm,

$V = 2\pi[(4.1\ \mu m)/(1.31\ \mu m)](1.4532^2 - 1.4485^2)^{1/2} = 2.30$

Mode field diameter MFD

Example: Single mode cut-off wavelength

Solution (continued)

Mode field diameter MFD from the Marcuse Equation is

$$\boxed{\begin{aligned}2w &= 2a(0.65+1.619V^{-3/2}+2.879V^{-6})\\ &= 2(4.1)[0.65+1.62(2.30)^{-3/2}+2.88(2.30)^{-6}]\end{aligned}}$$

$2w$ = 9.30 μm

86% of total power is within this diameter

$2w = (2a)(2.6/V) = 2(4.1)(2.6/2.30) = 9.28$ μm

$2w = 2a[(V+1)/V] = 11.8$ μm

This is for a planar waveguide, and the definition is different than that for an optical fiber

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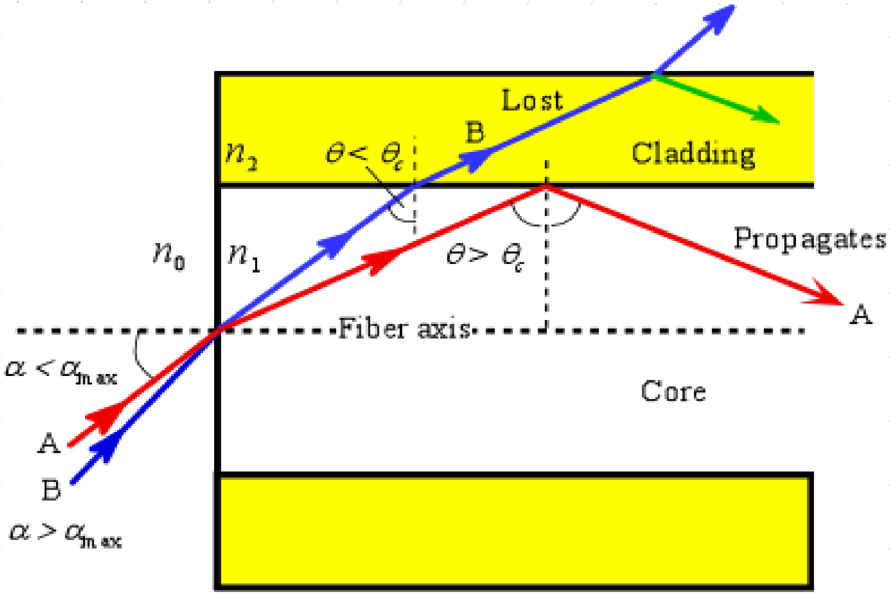
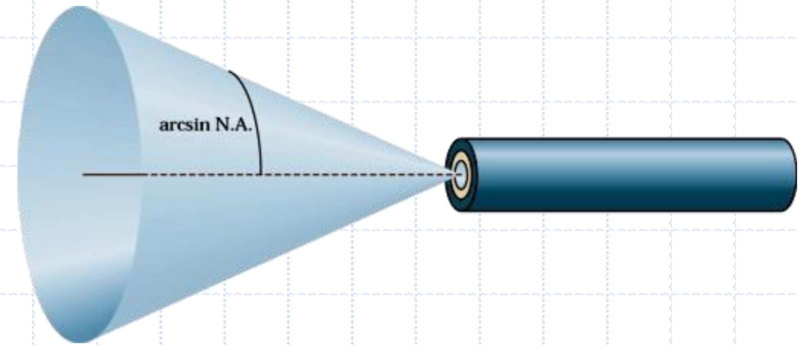
Numerical Aperture (NA)





- At $n_0 - n_1$ interface, Snell's law gives
$$\frac{\sin \alpha_{max}}{\sin(\pi/2 - \theta_c)} = \frac{n_1}{n_0}$$
- θ_c is determined by the onset of TIR, that is, $\sin \theta_c = n_2/n_1$.
- We can now eliminate θ_c to obtain
$$\sin \alpha_{max} = \frac{(n_1^2 - n_2^2)^{1/2}}{n_o} = \frac{\text{NA}}{n_o}$$
- The **numerical aperture (NA)** is a characteristic parameter of an optical fiber defined by
$$\text{NA} = (n_1^2 - n_2^2)^{1/2}$$
$$V = \frac{2\pi a}{\lambda}\text{NA}$$

$2\alpha_{max}$ = **total acceptance angle**

NA is an important factor in light launching designs into the optical fiber.

Example: A multimode fiber and total acceptance angle

A step index fiber has a core diameter of 100 μm and a refractive index of 1.480. The cladding has a refractive index of 1.460. Calculate the numerical aperture of the fiber, acceptance angle from air, and the number of modes sustained when the source wavelength is 850 nm.

Solution

Normalized refractive index
$\Delta = (n_1 - n_2)/n_1 = \mathbf{0.0135 \text{ or } 1.35\%}$

The numerical aperture is

$$\mathrm{NA} = (n_1^2 - n_2^2)^{1/2} = (1.480^2 - 1.460^2)^{1/2} = \mathbf{0.2425 \text{ or } 25.3\%}$$

From, $\sin\alpha_{max} = \mathrm{NA}/n_o = 0.2425/1$

Acceptance angle $\alpha_{max} = \mathbf{14°}$

Total acceptance angle $2\alpha_{max} = \mathbf{28°}$

V-number in terms of the numerical aperture can be written as,

$$V = (2\pi a/\lambda)\mathrm{NA} = [(2\pi 50\ \mu\mathrm{m})/(0.85\ \mu\mathrm{m})](0.2425) = 89.62$$

The number of modes, $M \approx V^2/2 = \mathbf{4016}$

Example: A single mode fiber

A typical single mode optical fiber has a core of diameter 8 μm and a refractive index of 1.460. The normalized index difference is 0.3%. The cladding diameter is 125 μm. Calculate the numerical aperture and the total acceptance angle of the fiber. What is the single mode cut-off frequency λ_c of the fiber?

Solution

The numerical aperture

$$\mathrm{NA} = (n_1^2 - n_2^2)^{1/2} = [(n_1 + n_2)(n_1 - n_2)]^{1/2}$$

Substituting $(n_1 - n_2) = n_1\Delta$ and $(n_1 + n_2) \approx 2n_1$, we get

$$\mathrm{NA} \approx [(2n_1)(n_1\Delta)]^{1/2} = n_1(2\Delta)^{1/2} = 1.46(2\times0.003)^{1/2} = \mathbf{0.113\ or\ 11.3\ \%}$$

The acceptance angle is given by

$$\sin\alpha_{max} = \mathrm{NA}/n_o = 0.113/1 \text{ or } \alpha_{max} = \mathbf{6.5^\circ}\ \text{, and } 2\alpha_{max} = \mathbf{13^\circ}$$

The condition for single mode propagation is $V \le 2.405$ which corresponds to a minimum wavelength λ_c is given by

$$\lambda_c = [2\pi a\mathrm{NA}]/2.405 = [(2\pi)(4\ \mu\mathrm{m})(0.113)]/2.405 = \mathbf{1.18\ \mu m}$$

Wavelengths shorter than 1.18 μm will result in multimode operation.

Dispersion = Spread of Information

- **Intermode (Intermodal) Dispersion: Multimode fibers only**
- **Material Dispersion**

  Group velocity depends on N_g and hence on λ

- **Waveguide Dispersion**

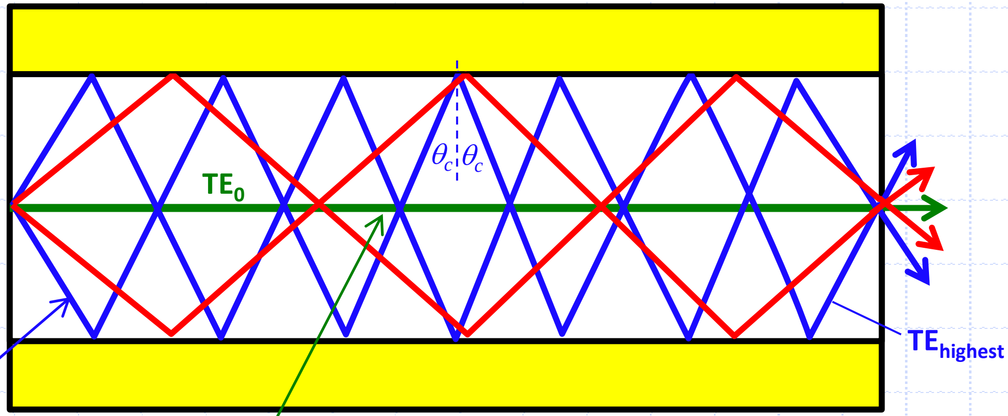
  Group velocity depends on waveguide structure
- **Chromatic Dispersion**

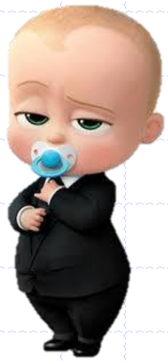
  Material dispersion + Waveguide Dispersion

- **Profile Dispersion**

  Like material and waveguide dispersion.
- **Add all 3 Material + Waveguide + Profile**
- **Polarization Dispersion**

Intermode Dispersion (MMF)





$$v_{g\min} \approx \frac{c}{n_1}\sin\theta_c = \frac{c}{n_1}\left(\frac{n_2}{n_1}\right)$$

$$v_{g\max} \approx \frac{c}{n_1}$$

$$\Delta\tau = \frac{L}{v_{g\min}} - \frac{L}{v_{g\max}}$$

$$\frac{\Delta\tau}{L} = \frac{(n_1 - n_2)}{c}\left(\frac{n_1}{n_2}\right)$$

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

(Since $n_1/n_2 \approx 1$)

$\Delta\tau/L \approx 10 - 50$ ns / km

Depends on length!

Group Velocity & Group Delay

- The group velocity is given by:

$$V_g = \frac{d\omega}{d\beta}$$

- The group delay is given by:

$$\tau_g = \frac{l}{V_g} = l\,\frac{d\beta}{d\omega}$$

- It is important to note that all above quantities depend both on **frequency** & the **propagation mode**. In order to see the effect of these parameters on group velocity and delay, the following analysis would be helpful.

Group Velocity and Group Delay

Consider a single mode fiber with core and cladding indices of 1.4480 and 1.4400, core radius of 3 μm, operating at 1.5 μm. ***What are the group velocity and group delay at this wavelength*****?**

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2 \qquad 1.5 < V < 2.5$$

$$b = \frac{(\beta/k) - n_2}{n_1 - n_2} \Rightarrow \beta = n_2 k[1 + b\Delta]$$

$k = 2\pi/\lambda = 4{,}188{,}790\ \text{m}^{-1}$ and $\omega = 2\pi c/\lambda = 1.255757 \times 10^{15}\ \text{rad s}^{-1}$

$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} = 1.910088$

$b = 0.3860859$, and $\beta = 6.044796 \times 10^6\ \text{m}^{-1}$.

Increase wavelength by 0.1% and recalculate. Values in the table

Group Velocity and Group Delay

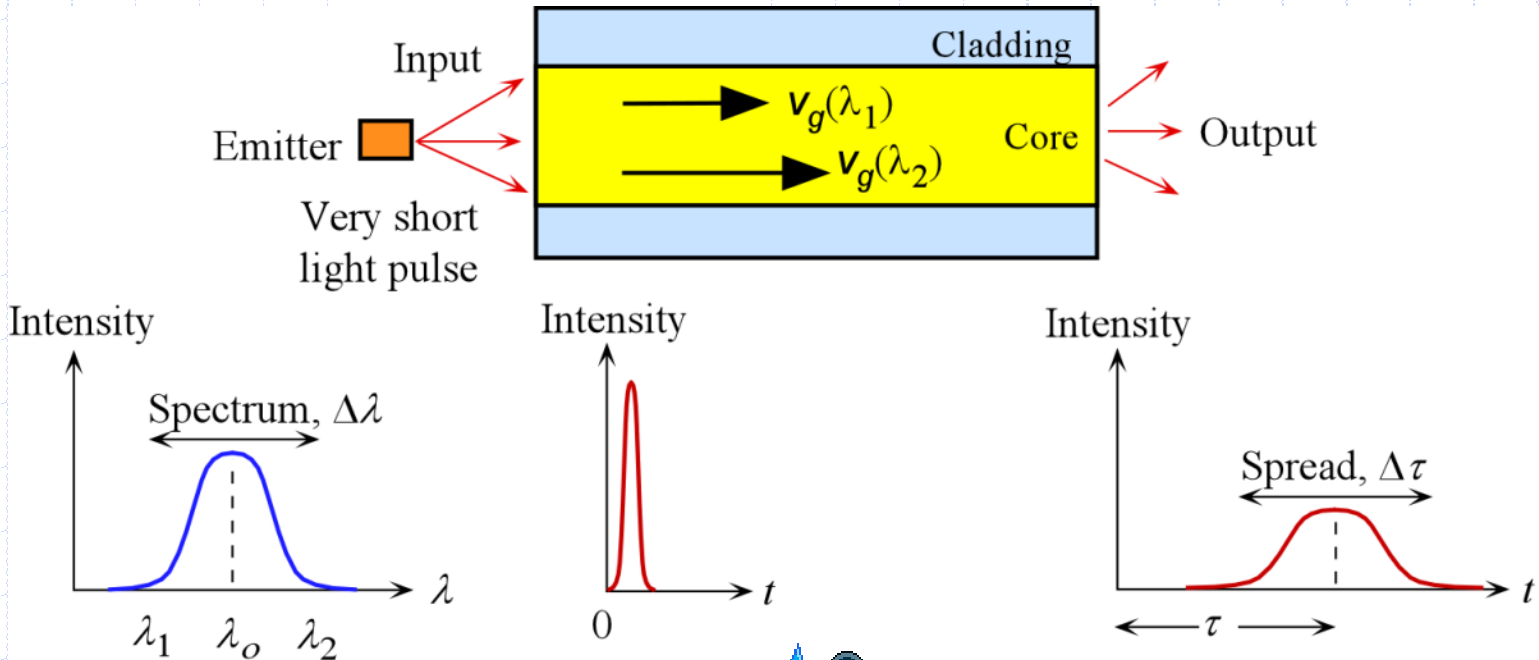
Calculation →	V	k (m^{-1})	ω (rad s^{-1})	b	β (m^{-1})
λ = 1.500000 μm	1.910088	4188790	1.255757×10^{15}	0.3860859	6.044818×10^{6}
λ' = 1.50150 μm	1.908180	4184606	1.254503×10^{15}	0.3854382	6.038757×10^{6}

$$v_g = \frac{d\omega}{d\beta} = \frac{\omega' - \omega}{\beta' - \beta} = \frac{(1.254503 - 1.255757)\times10^{15}}{(6.038757 - 6.044818)\times10^{6}} \approx 2.07\times10^{8}\ \mathrm{m\,s^{-1}}$$

The group delay τ_g over 1 km is 4.83 μs

Intramode Dispersion (SMF)

Dispersion in the fundamental mode



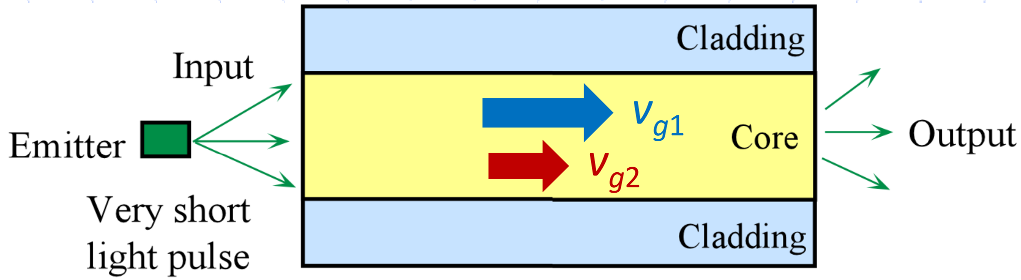
Group Delay $\tau = L / v_g$

Group velocity v_g depends on

Refractive index = $n(\lambda)$	**Material Dispersion**
V-number = $V(\lambda)$	**Waveguide Dispersion**
$\Delta = (n_1 - n_2)/n_1 = \Delta(\lambda)$	**Profile Dispersion**

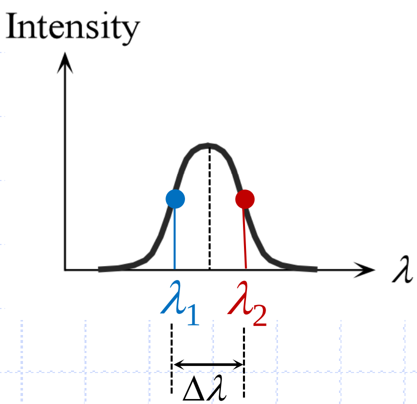
Intramode Dispersion (SMF)

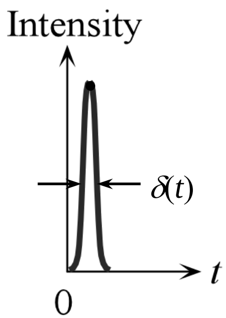
Chromatic dispersion in the fundamental mode

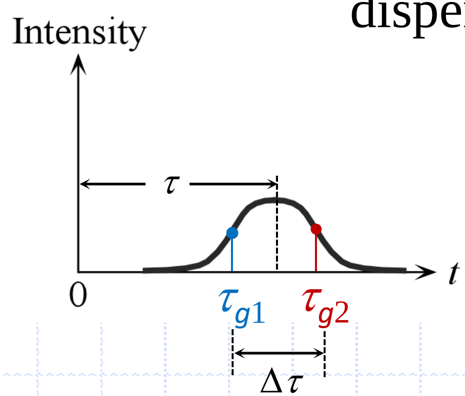


$$\Delta\tau = DL\Delta\lambda$$

Output pulse dispersed







$$\boxed{\Delta\lambda = \lambda_2 - \lambda_1}$$

Chromatic spread

$$D = \frac{\Delta\tau}{L\Delta\lambda}$$

OR

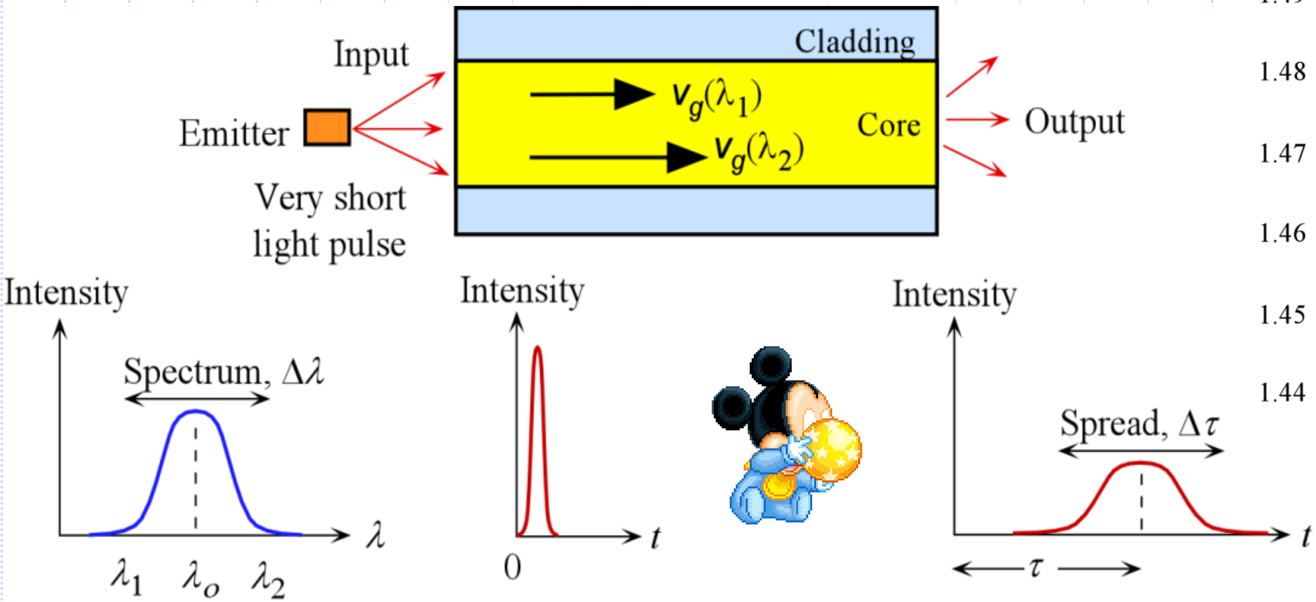
$$D = \frac{d\tau}{Ld\lambda}$$

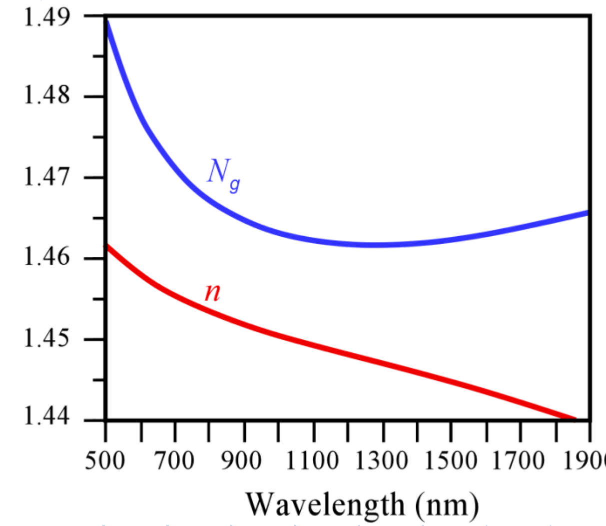
$$\boxed{\Delta\tau = \tau_{g1} - \tau_{g2}}$$

Dispersion

Definition of Dispersion Coefficient

Material Dispersion





Emitter emits a spectrum $\Delta\lambda$ of wavelengths.

Waves in the guide with different free space wavelengths travel at different group velocities due to the wavelength dependence of n_1. The waves arrive at the end of the fiber at different times and hence result in a broadened output pulse.

$$v_g = c / N_g$$

Group velocity

Depends on the wavelength

$$D_m \approx -\frac{\lambda}{c}\left(\frac{d^2 n}{d\lambda^2}\right)$$

$$\boxed{\frac{\Delta\tau}{L} = D_m \Delta\lambda}$$

D_m = Material dispersion coefficient, ps nm^{-1} km^{-1}

Waveguide Dispersion

b* hence *β* depend on *V* and hence on *λ

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}$$

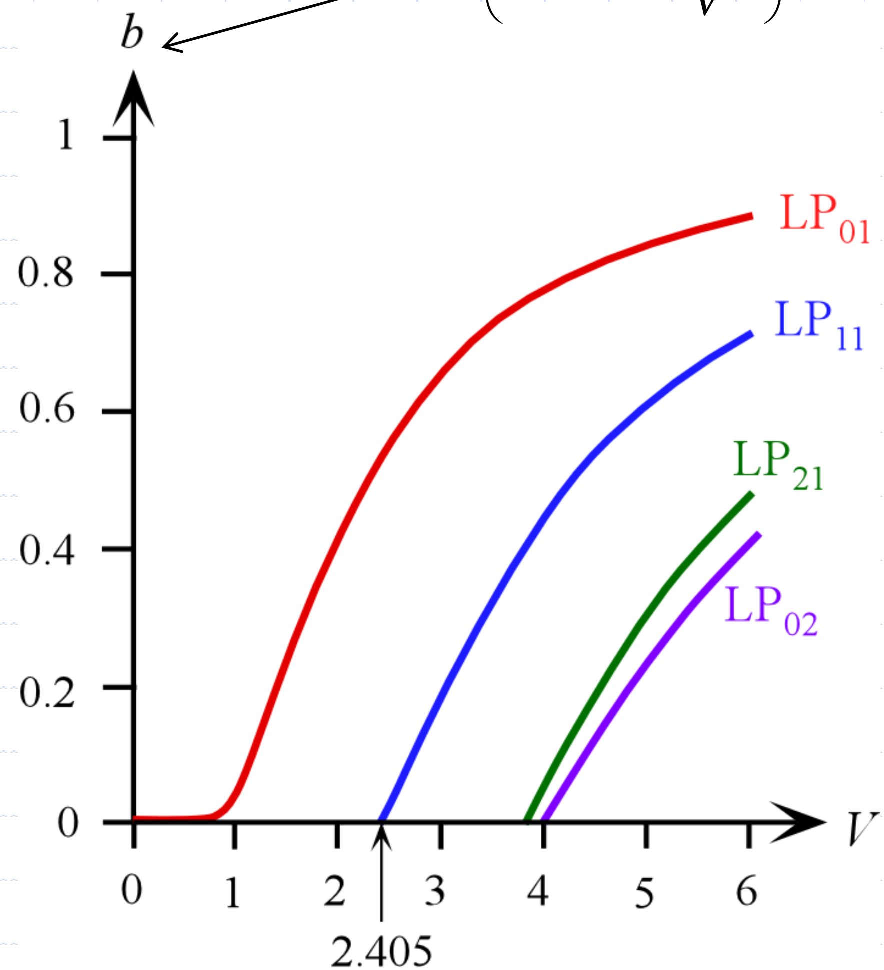
Normalized propagation constant

$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}$$

$$k = 2\pi/\lambda$$

$$\beta \approx n_2 k(1 + b\Delta)$$

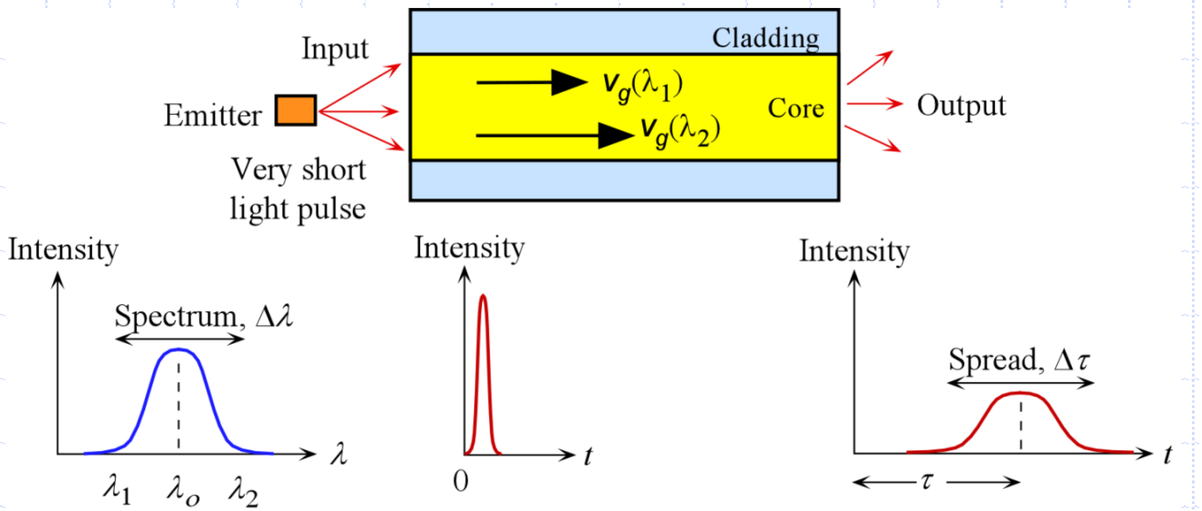
$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2$$



Using *V* number:

$$V = ka(n_1^2 - n_2^2)^{1/2} \approx kan_2\sqrt{2\Delta}$$

Waveguide Dispersion



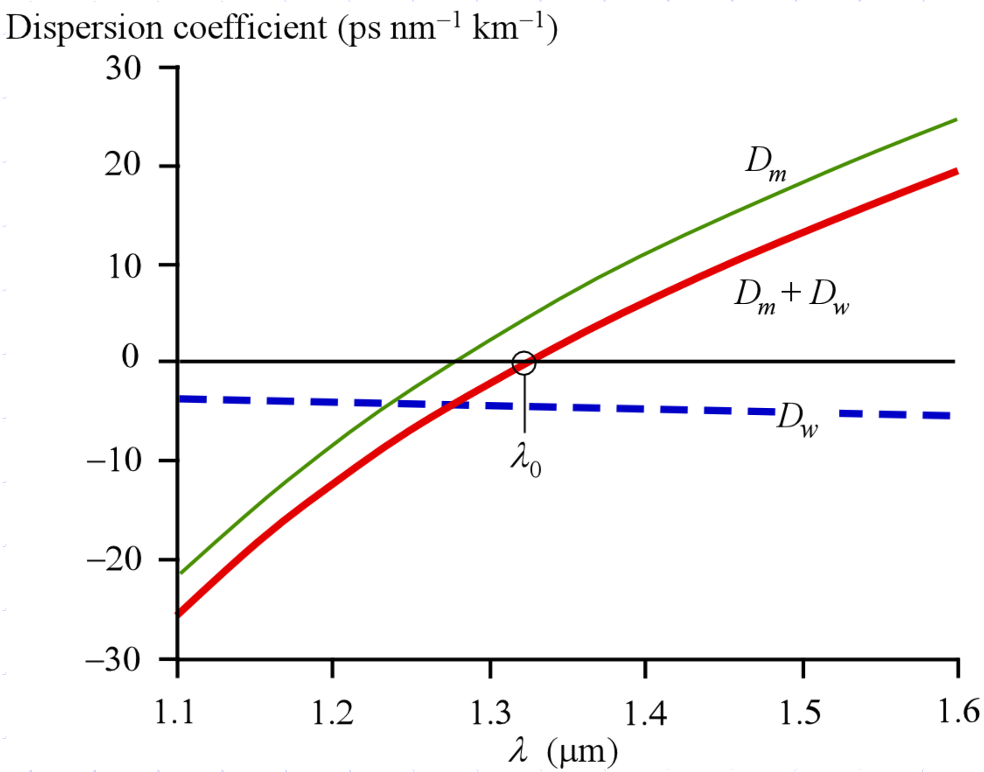
Waveguide dispersion The group velocity $v_g(01)$ of the fundamental mode depends on the V-number, which itself depends on the source wavelength λ, even if n_1 and n_2 were constant. Even if n_1 and n_2 were wavelength independent (no material dispersion), we will still have waveguide dispersion by virtue of $v_g(01)$ depending on V and V depending inversely on λ. Waveguide dispersion arises as a result of the guiding properties of the waveguide which imposes a nonlinear ω vs. β_{lm} relationship.

$$\frac{\Delta\tau}{L} = D_w \Delta\lambda$$

D_w = waveguide dispersion coefficient

D_w depends on the waveguide structure, ps nm^{-1} km^{-1}

Chromatic Dispersion



Material dispersion coefficient (D_m) for the core material (taken as SiO_2), waveguide dispersion coefficient (D_w) (a = 4.2 μm) and the total or chromatic dispersion coefficient D_{ch} (= $D_m + D_w$) as a function of free space wavelength, λ

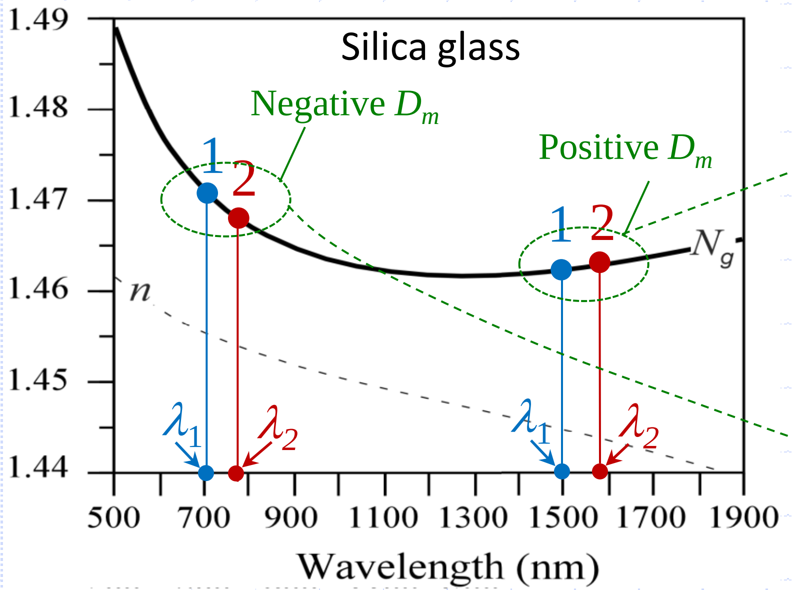
Chromatic = Material + Waveguide

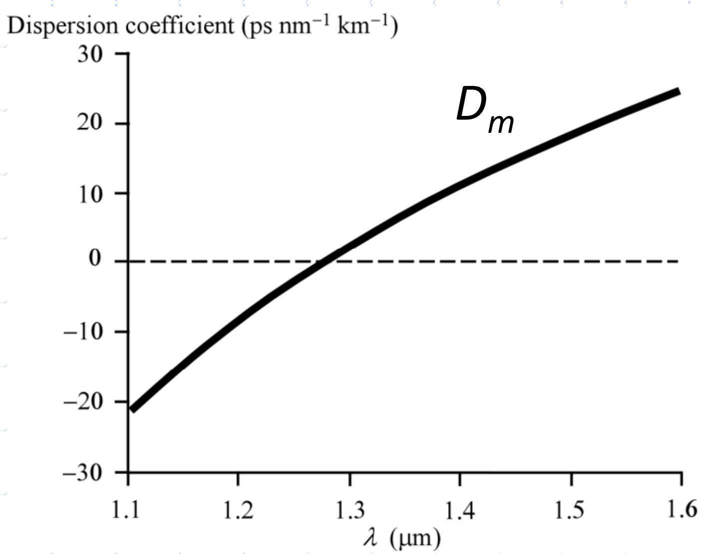
$$\frac{\Delta\tau}{L} = (D_m + D_w)\Delta\lambda$$

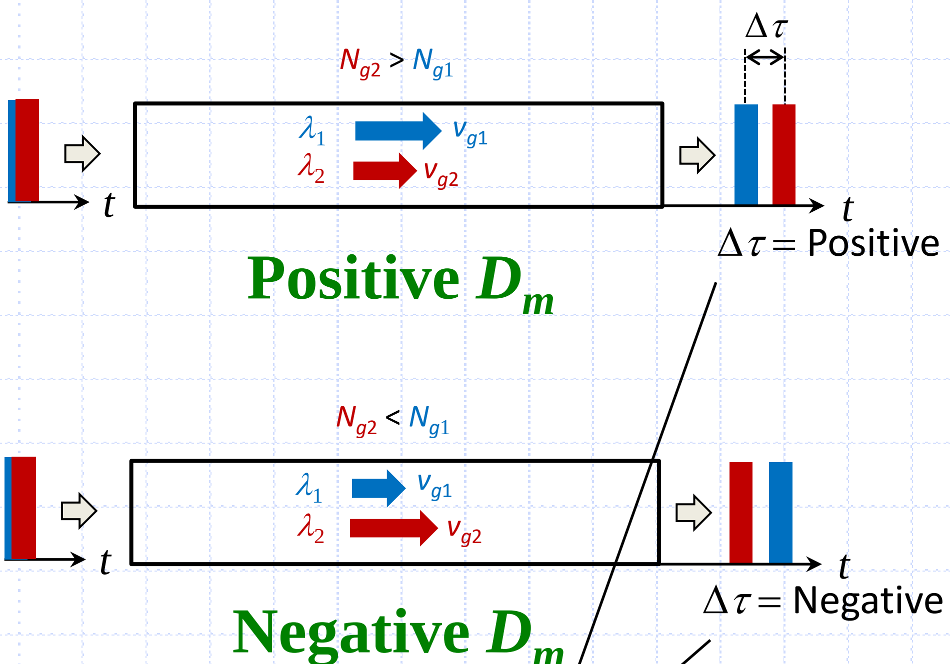
Chromatic dispersion coefficient

$$D_{ch} = D_m + D_w$$

What do Negative and Positive D_m mean?



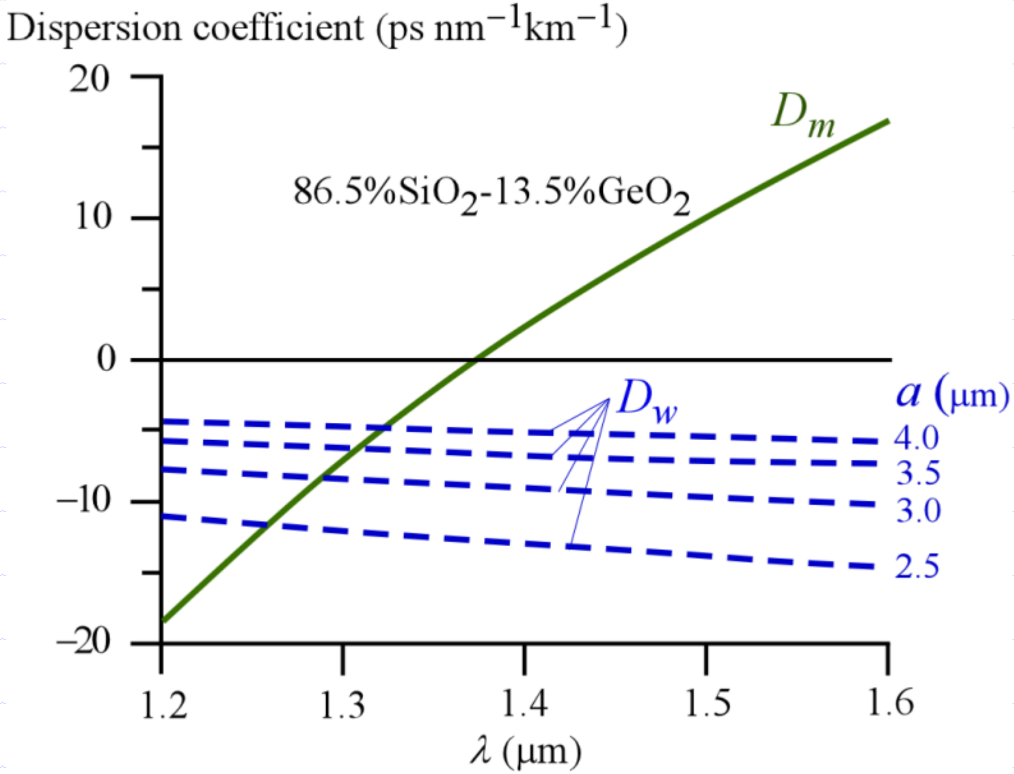




$$D_m = \frac{\Delta\tau}{L\Delta\lambda}$$

$$\boxed{\Delta\lambda = \lambda_2 - \lambda_1}$$

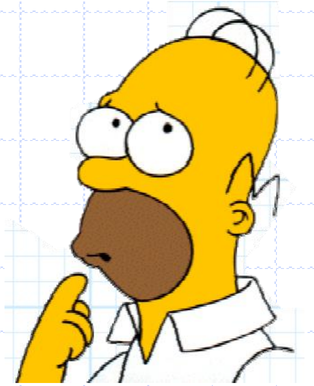
Waveguide Dimension and Chromatic Dispersion



$$D_w = \frac{n_2 \Delta}{c\lambda}\left[V \frac{d^2(bV)}{dV^2}\right]$$

$$D_w \approx -\frac{0.025\lambda}{a^2 c n_2}$$

$$D_w(\mathrm{ps\,nm^{-1}\,km^{-1}}) \approx -\frac{83.76\,\lambda(\mu\mathrm{m})}{[a(\mu\mathrm{m})]^2 n_2}$$



Waveguide dispersion depends on the guide properties

Profile Dispersion

Group velocity $v_g(01)$ of the fundamental mode depends on Δ, refractive index difference.

Δ may not be constant over a range of wavelengths: $\Delta = \Delta(\lambda)$

$$\frac{\Delta\tau}{L} = D_p \Delta\lambda$$

D_p = *Profile dispersion coefficient*

$D_p < 0.1$ ps nm^{-1} km^{-1}

Can generally be ignored

NOTE

Total intramode (chromatic) dispersion coefficient D_{ch}

$$D_{ch} = D_m + D_w + D_p$$

where D_m, D_w, D_p are material, waveguide and profile dispersion coefficients respectively

Chromatic Dispersion

$$D_{ch} = D_m + D_w + D_p$$

$$\frac{\Delta\tau}{L} = D_{ch}\Delta\lambda$$

S_0 = Chromatic dispersion slope at λ_0

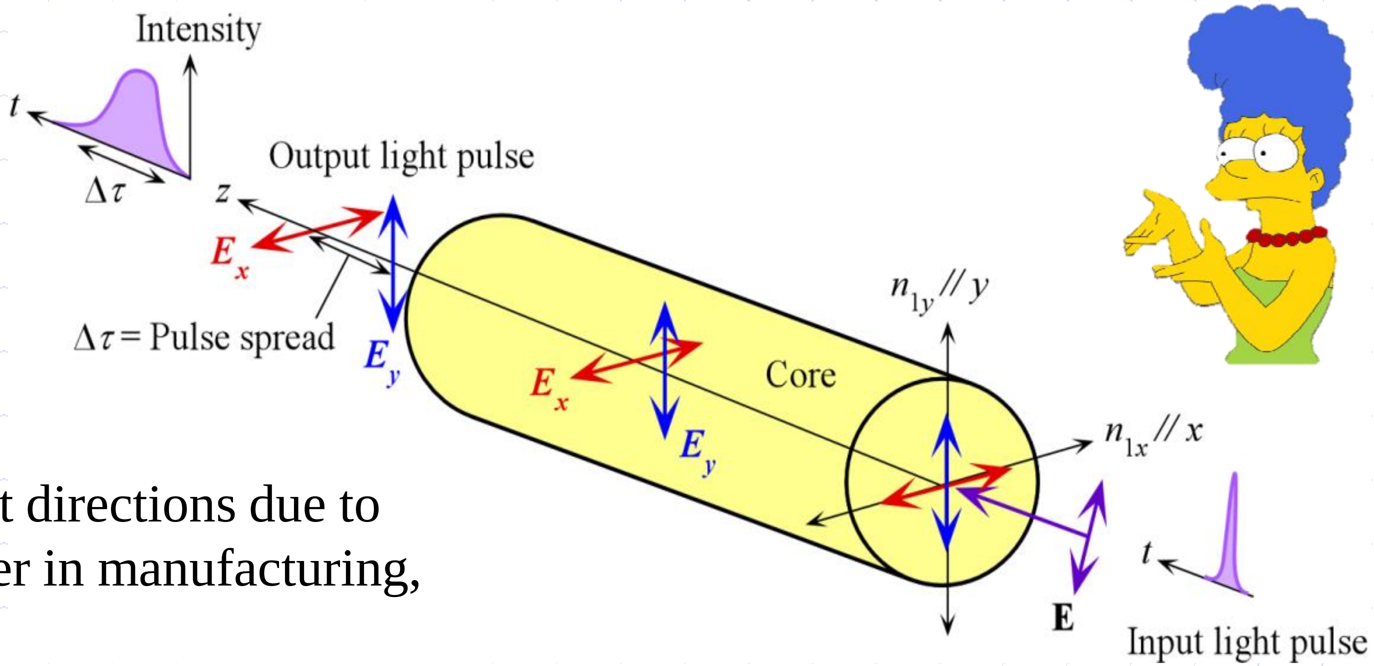
Chromatic dispersion is zero at $\lambda = \lambda_0$

$$\boxed{D_{ch} = \frac{S_0\lambda}{4}\left[1-\left(\frac{\lambda_0}{\lambda}\right)^4\right]}$$

Polarization Dispersion

Polarization mode dispersion (PMD) is due to slightly different velocity for each polarization mode because of the lack of perfectly symmetric & anisotropicity of the fiber. If the group velocities of two orthogonal polarization modes are v_{gx} and v_{gy}, then the differential time delay $\Delta\tau$ between these two polarization over a distance L is

$$\Delta\tau = \left|\frac{L}{v_{gx}} - \frac{L}{v_{gy}}\right|$$



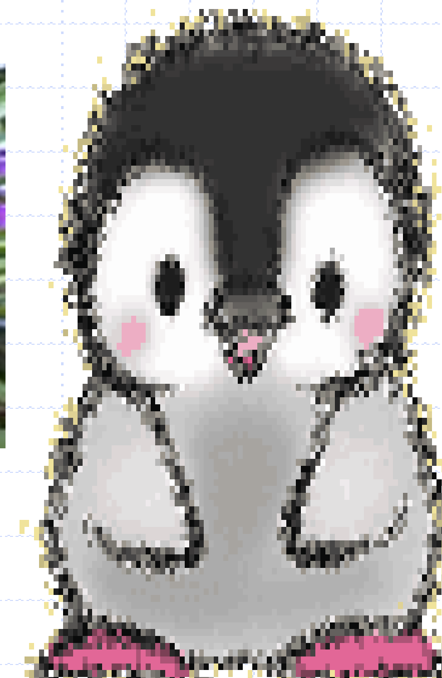
n different in different directions due to induced strains in fiber in manufacturing, handling and cabling.

$$\Delta\tau = D_{\text{PMD}}\sqrt{L}$$

D_{PMD} = Polarization dispersion coefficient

Typically $D_{\text{PMD}} = 0.1 – 0.5$ ps nm^{-1} km$^{-1/2}$

Thank you



Have a nice day.