Lecture 6
Polarization and Modulation of light I

ECE 325

OPTOELECTRONICS

Menoufia University

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Kasap–6.1, 6.2, 6.3 and 6.4

Polarization

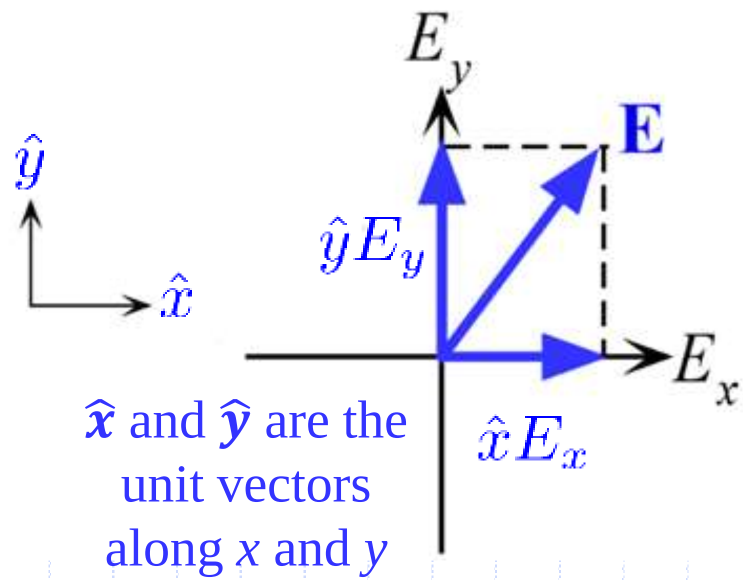
- **Polarization**: the **E**-field vector behavior in the EM wave as it propagates through a medium.
- **Linearly polarized EM wave:** the field oscillations of **E**-field at a given point at all times are confined to a well defined line.
- **Unpolarized light beam:** light beam has waves with **E**-field propagating with random orientation **⊥ to *z***.
- The field vector for a plane wave propagating in *z* with oscillations along a line in the ***xy* plane** is

$$E_x = E_{xo}\cos(wt - kz)$$

$$E_y = E_{yo}\cos(wt - kz + \phi)$$

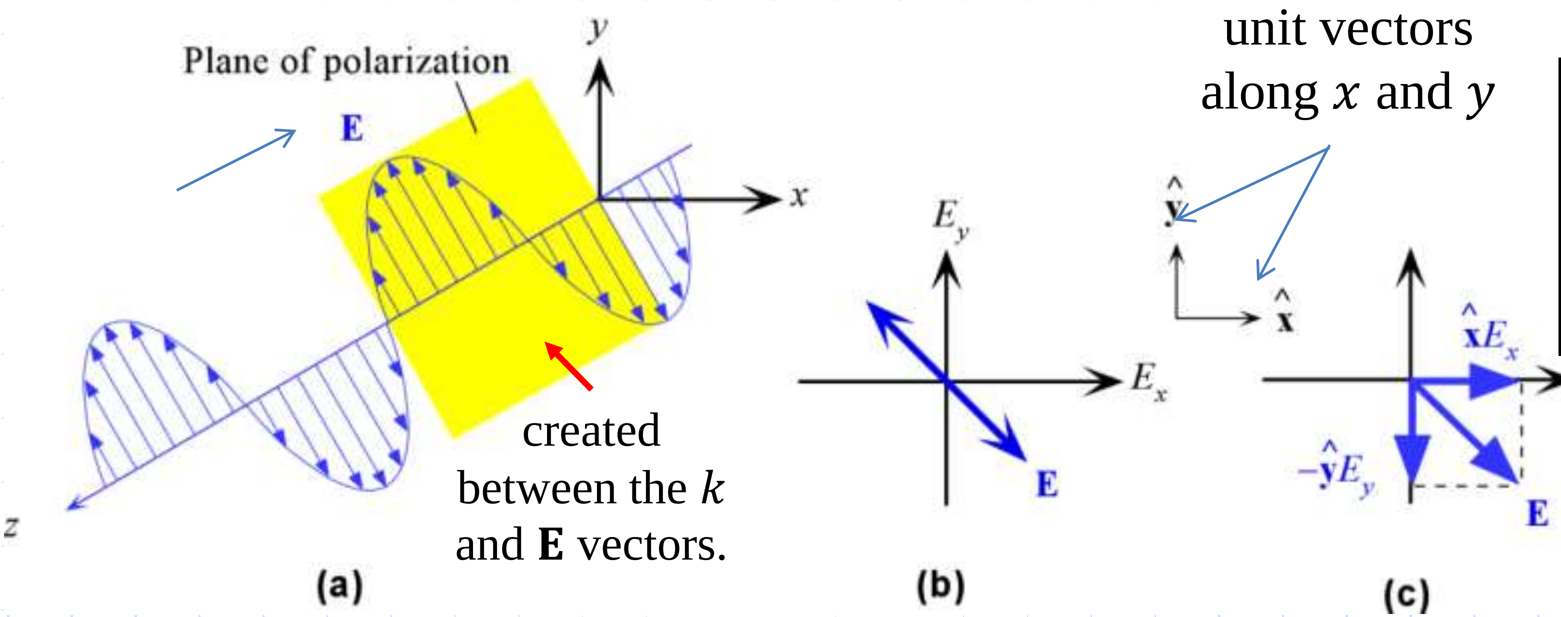
ϕ = phase difference between E_y and E_x; ϕ can arise if one of the components is delayed (retarded)

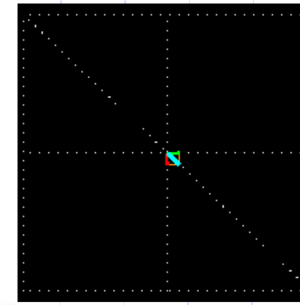
The electric field **E** in the wave at any space and time location can be found by adding $\mathbf{E}_x$ and $\mathbf{E}_y$ *vectorially*.



$\hat{x}$ and $\hat{y}$ are the unit vectors along *x* and *y*

Linearly Polarized Light





- Linearly polarized light beam with **E**-field oscillations at $-45°$ to x-axis is obtained by choose $E_{xo} = E_{yo}$ and $\phi = \pm 180°$.
- Use $\phi = \pi$

$$\mathbf{E} = E_x\hat{x} + E_y\hat{y} \;= \hat{x}E_{xo}\cos(wt - kz) - \hat{y}E_{yo}\cos(wt - kz)$$

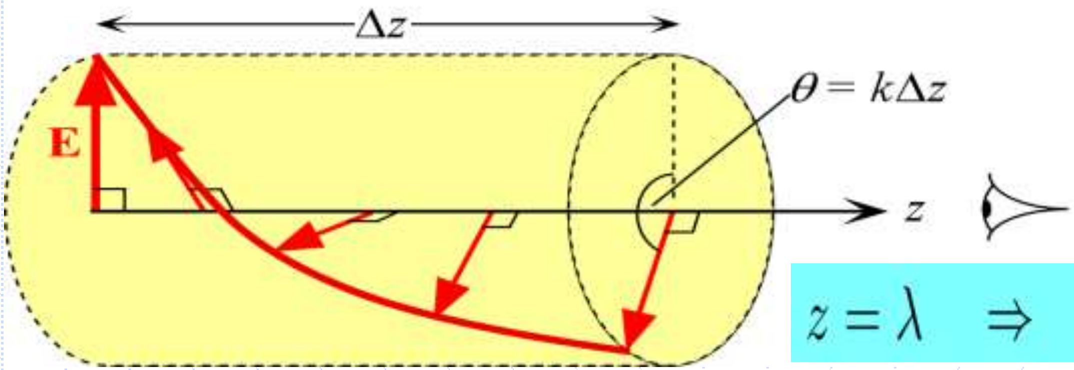
$$= E_o\cos(wt - kz)$$

describes the propagation of E_o at $-45°$ to x-axis along the z-direction.

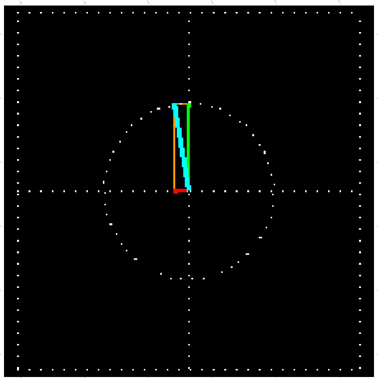
$$E_o = \hat{x}E_{xo} - \hat{y}E_{yo}$$

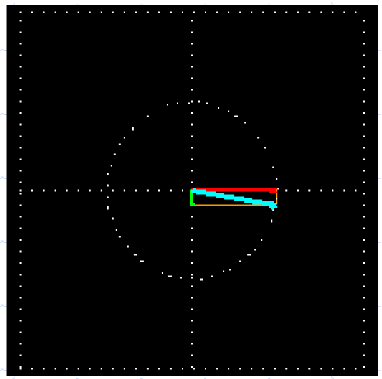
Vector amplitude which is at $-45°$ to the x-axis

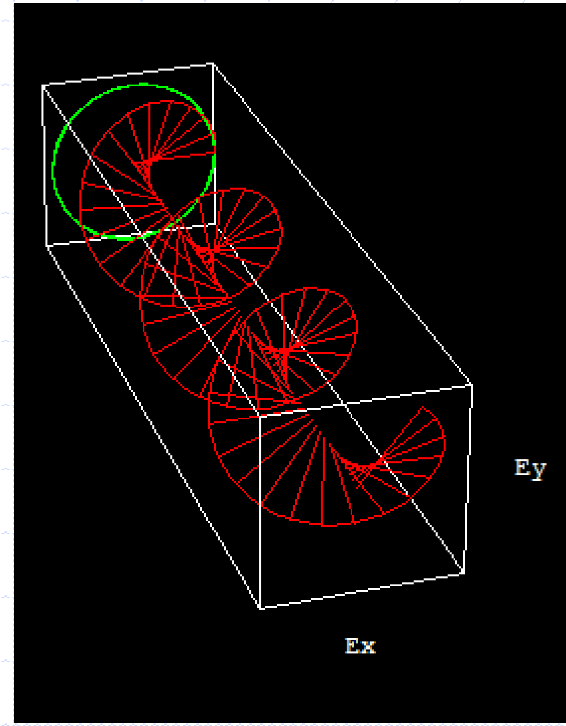
Circularly Polarized Light



$$z = \lambda \quad \Rightarrow \quad \theta = 2\pi$$



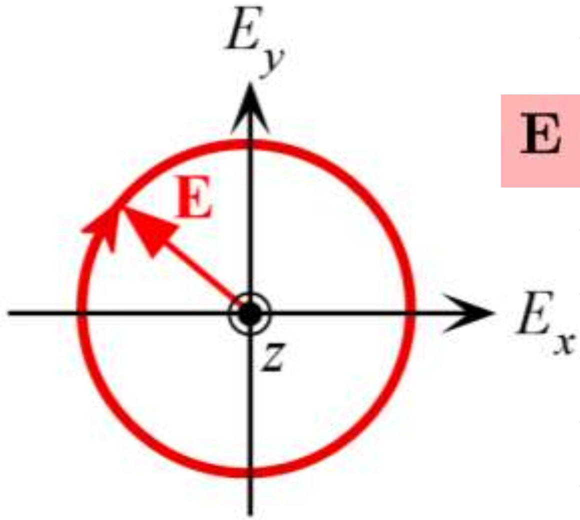




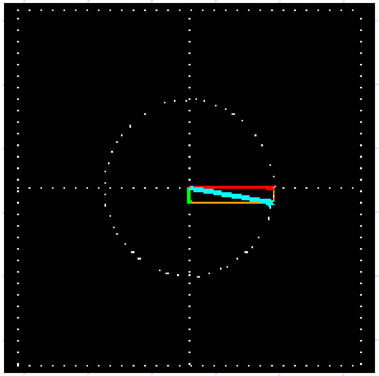
If the magnitude of the field vector **E** remains **constant** but its end point at a given location on *z* traces out **a circle** by rotating in a **clockwise** sense with time, as observed by the receiver of the wave, then the wave is said to be **right circularly polarized**.

If the rotation of the tip of **E** is **counterclockwise**, the wave is said to be **left circularly polarized**.

Circularly Polarized Light



$$\mathbf{E} = E_x\hat{x} + E_y\hat{y}$$



A **right circularly polarized** light that is traveling along *z* (**out of paper**). The field vector **E** is always at right angles to *z*, rotates clockwise around *z* with time, and traces out a full circle over one wavelength of distance propagated.

Assume that $\phi = 90°$ and $E_{xo} = E_{yo} = A$. Then

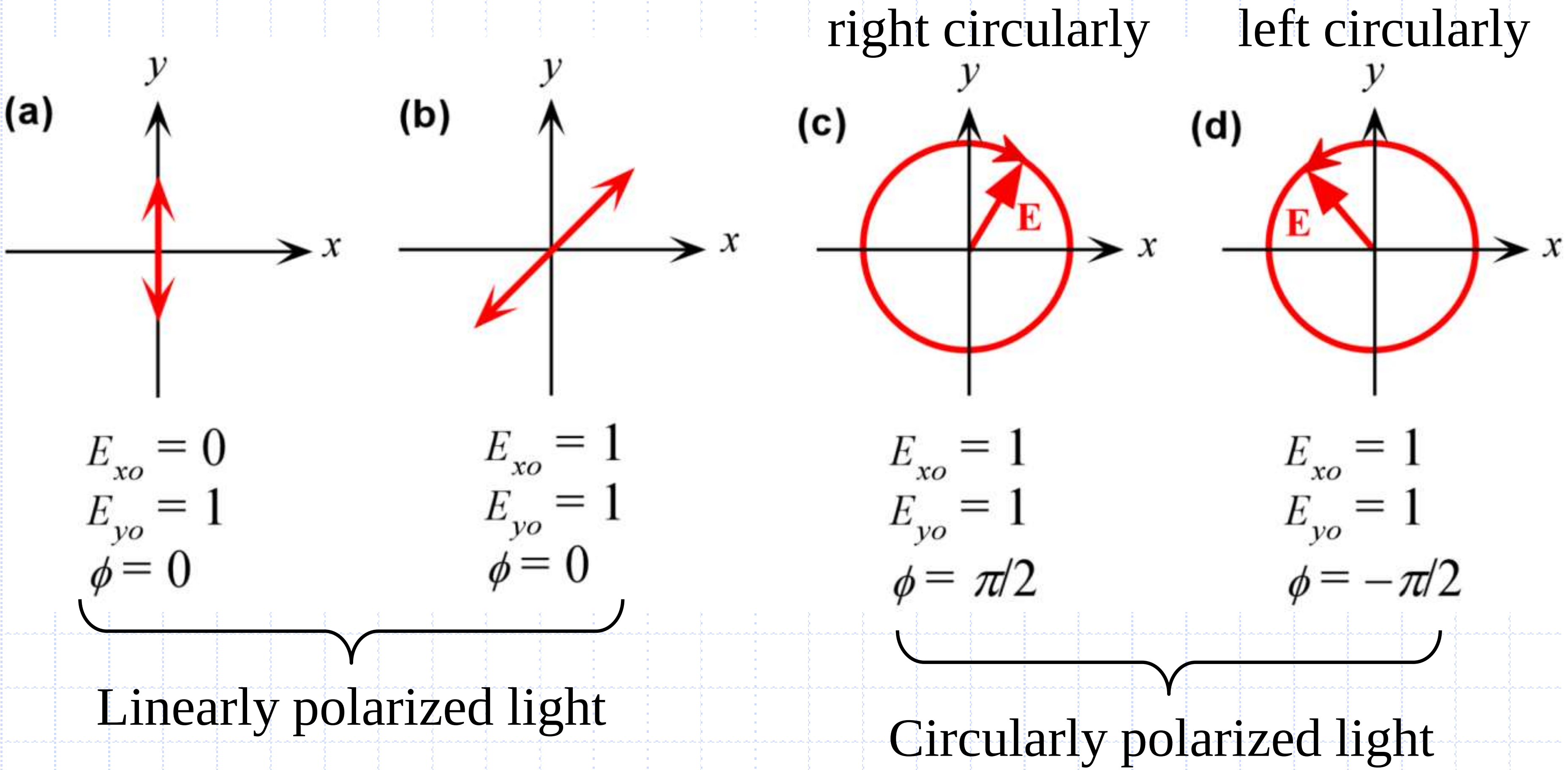
$$E_x = A\cos(wt - kz)$$

$$E_y = E_{yo}\cos\left(wt - kz + \frac{\pi}{2}\right) = -A\sin(wt - kz)$$

$$E_x^2 + E_y^2 = A^2$$

$\Rightarrow$ represent a circle

Linear and Circular Polarization

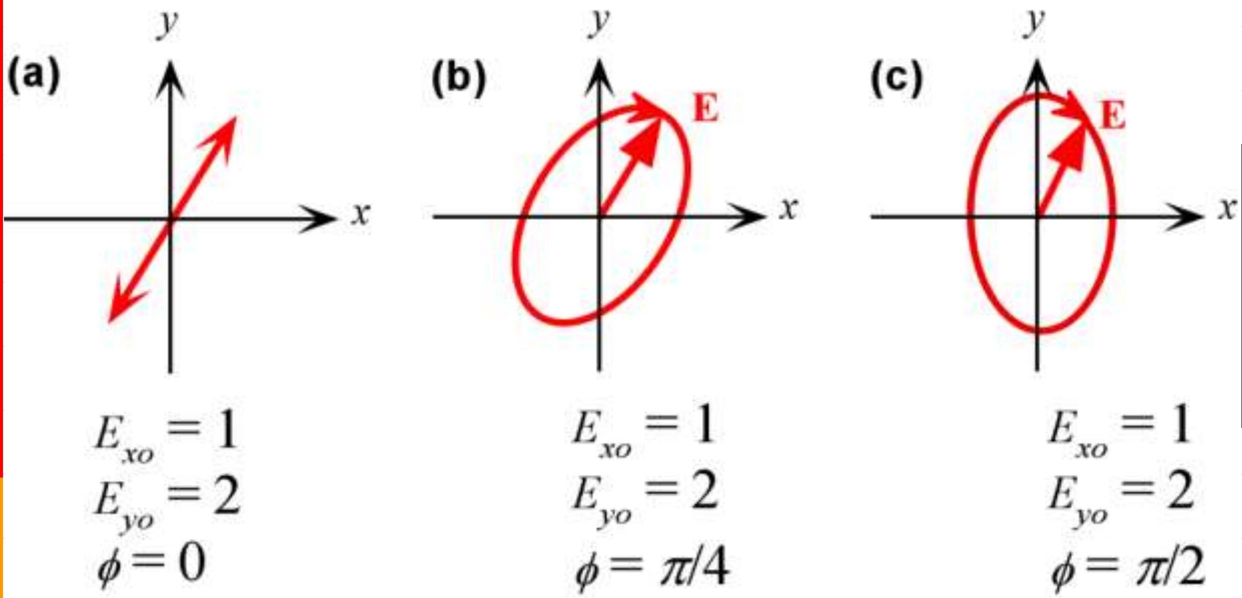


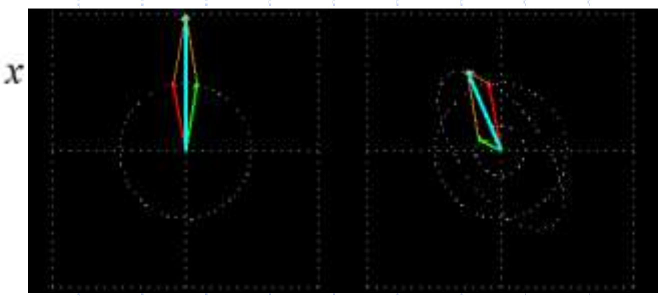
$$E_x = E_{xo}\cos(\omega t - kz)$$
$$E_y = E_{yo}\cos(\omega t - kz + \phi)$$

Linear and Elliptical Polarization

- Assume that the magnitude of one vector component in **E** is larger then the other, i.e., $\boldsymbol{E_{xo} \neq E_{yo}}$.

Instead of a **circle**, the wave generates an **ellipse** as it propagates along k in the z direction.

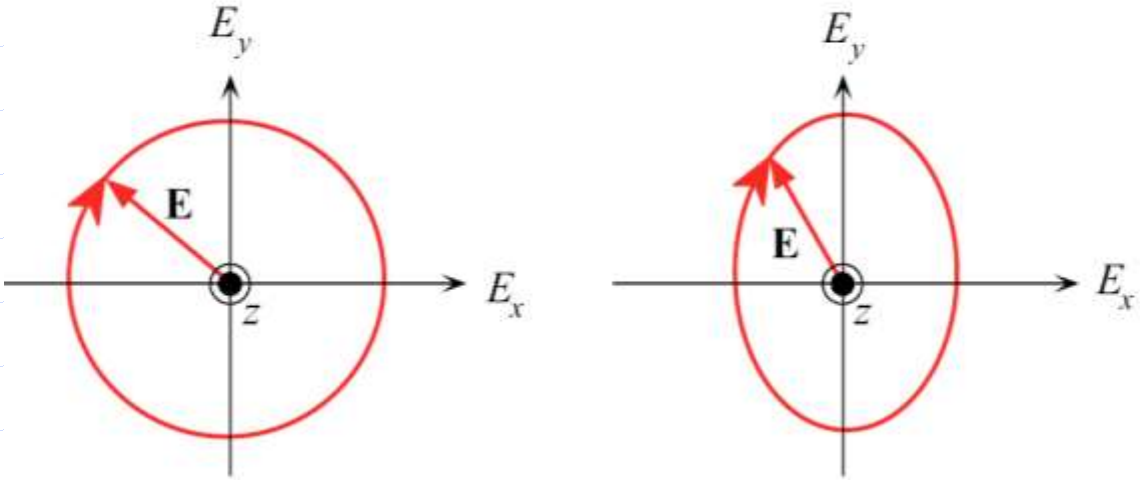




(a) Linearly polarized light with $E_{yo} = 2E_{xo}$ and $\phi = 0$. (b) When $\phi = \pi/4$, the light is right elliptically polarized with a tilted major axis. (c) When $\phi = \pi/2$, the light is right elliptically polarized.

If E_{xo} and E_{yo} were **equal**, this would be right circularly polarized light.

Circular and Elliptical Polarization



A right circularly polarized wave has $E_{xo} = E_{yo} = A$ (an amplitude), and $\phi = \pi/2$. This means that,

$$E_x = A\cos(\omega t - kz)$$

and $$E_y = -A\sin(\omega t - kz)$$

When $E_{xo} = E_{yo} = A$ and the phase difference ϕ is other than 0, $\pm\pi$ or $\pm\pi/2$, i.e. $\pm\pi/4$ or $\pm 3\pi/4$, the resultant wave is **elliptically polarized** and the tip of the vector in the figure traces out an *ellipse*.

S.O. Kasap, *Optoelectronics and Photonics: Principles and Practices*, Second Edition, © 2013 Pearson Education

Example: Elliptical and Circular Polarization

- **Show that if the magnitudes, E_{xo} and E_{yo}, are different and the phases difference is 90°, that the wave is elliptically polarized**

$$\mathbf{E} = E_x \hat{x} + E_y \hat{y}$$

$$E_x \neq E_y \qquad \phi = \frac{\pi}{2}$$

$$\frac{E_x}{E_{xo}} = \cos(wt - kz) \qquad \frac{E_y}{E_{yo}} = \cos\left(wt - kz + \frac{\pi}{2}\right) = -\sin(wt - kz)$$

$$1 = (\cos(wt - kz))^2 + (-\sin(wt - kz))^2 = \left(\frac{E_x}{E_{xo}}\right)^2 + \left(\frac{E_y}{E_{yo}}\right)^2$$

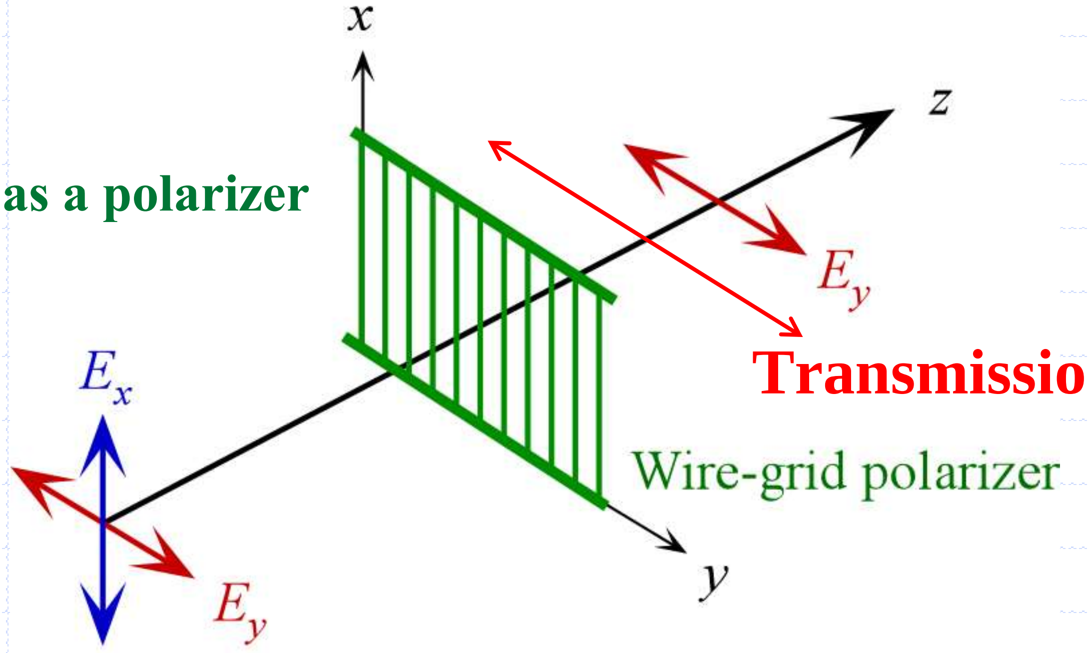
Equation for an ellipse if the denominators do not equal, $E_{xo} \neq E_{yo}$. Wheares $E_{xo} = E_{yo} \Rightarrow$ equation of circle.

- Further, at $z = 0$ and at $\omega t = 0, E = E_x = E_{xo}$. Moreover, at $\omega t = \pi/2, E = E_y = -E_{yo}$
  - Thus the field rotates in a **clockwise position**: Right Elliptically Polarized

Polarizers

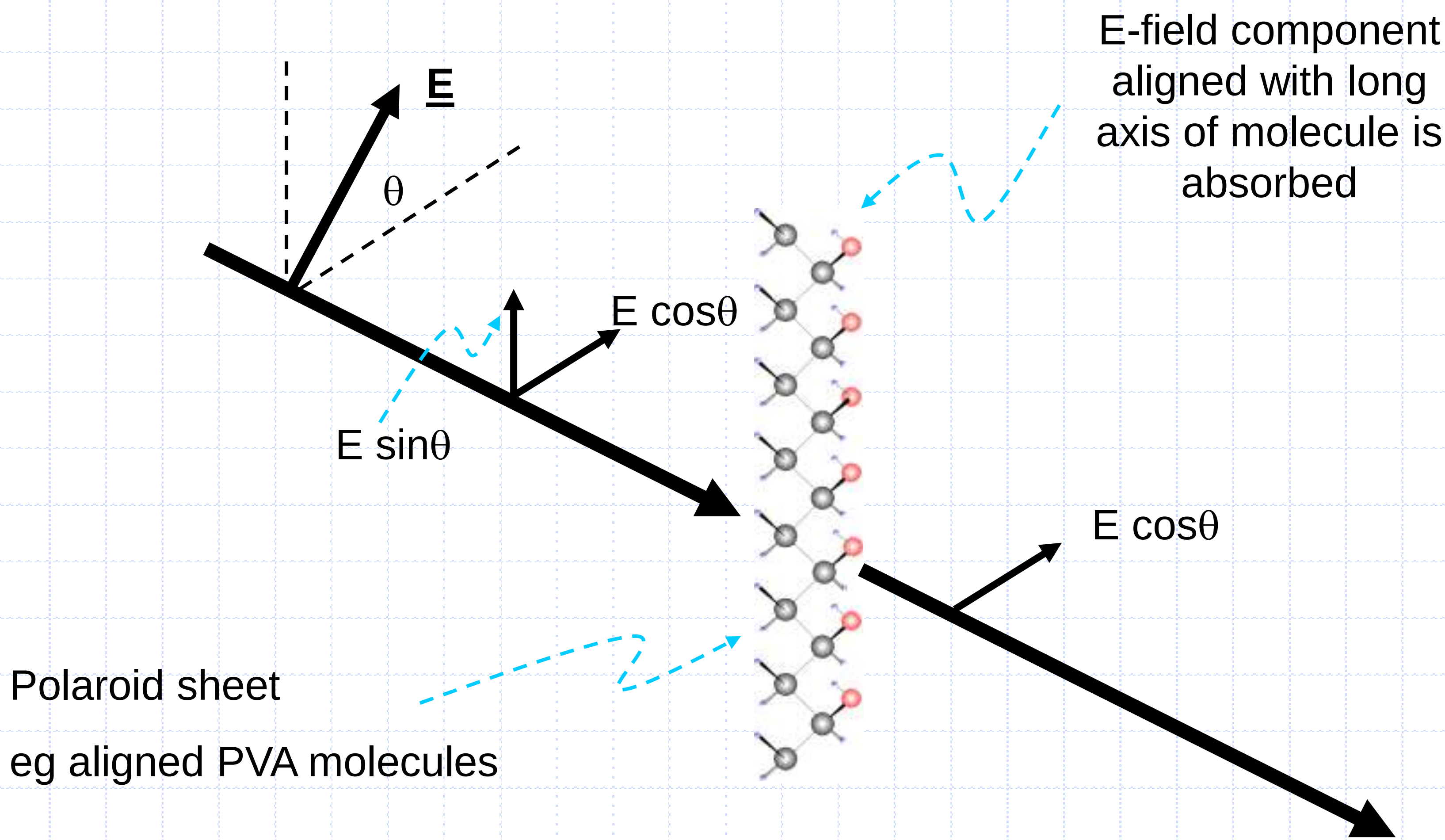
A **linear polarizer** allows only field oscillations along a particular/preferred direction, called the **transmission axis (TA)** to pass through.

The wire grid-acts as a polarizer



- The transmitted light is **polarized** based on the orientation of the polarizing medium.
- **Polaroid** sheets are common examples of linear polarizers.

Polaroid Sheets

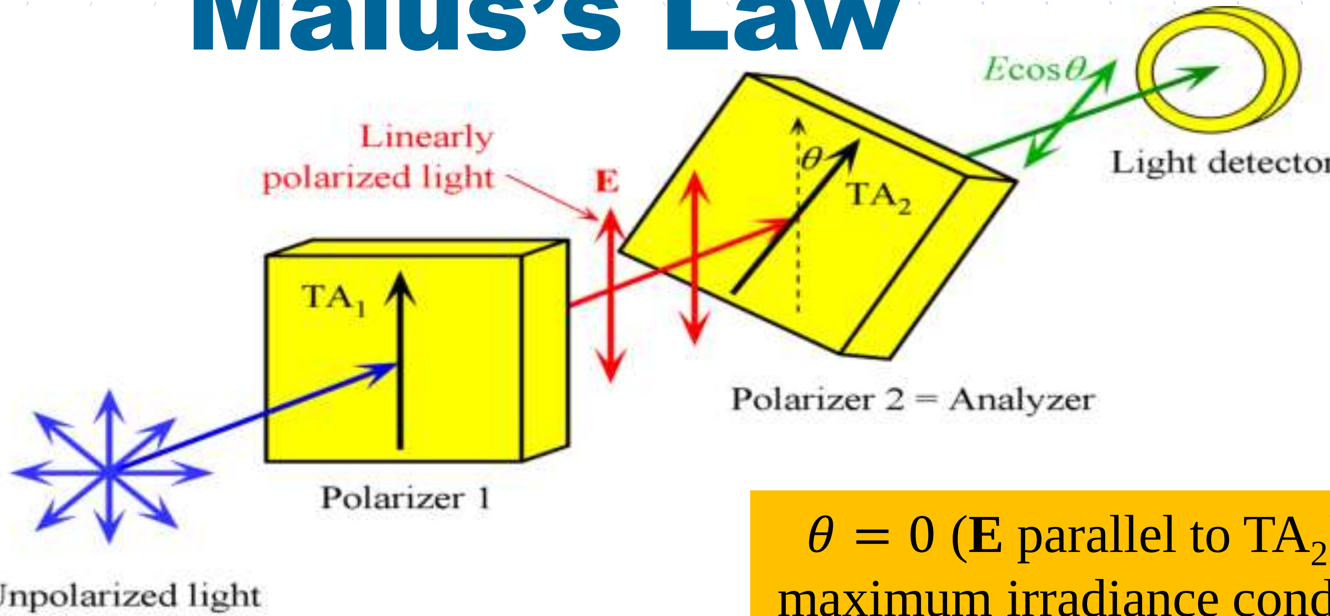


Made by heating and stretching a sheet of PVA laminated to a supporting sheet of cellulose acetate treated with iodine solution (H-type polaroid). Invented in 1928.

Malus's Law

Etienne-Louis Malus

1775-1812



$\theta = 0$ (**E** parallel to TA$_2$) $\Rightarrow$ maximum irradiance condition

- Combining two polarizers
  - 1st polarizer generates **initial** polarization.
  - 2nd polarizer, the **analyzer** is used to measure the degree of polarization of the 1st polarizer by reducing field intensity as a function of off axis polarization by the analyzer.
- **Malus's law** relates the intensity of a linearly polarized light passing through a polarizer to the angle between the TA and the **E** vector.

$$I(\theta) = I(0)\cos^2\theta$$

Principal Refractive Indices

We can describe light propagation in terms of *three* refractive indices, called **principal refractive indices** n_1, n_2 and n_3, along three mutually orthogonal directions in the crystal, say x, y and z called **principal axes**.

Optically Isotropic Materials

- **Optically isotropic** materials have **isotropic** crystal structures:
  - Generate **uniform polarization** along all three principle axis.
  - Have uniform refractive indices for all incident angles.

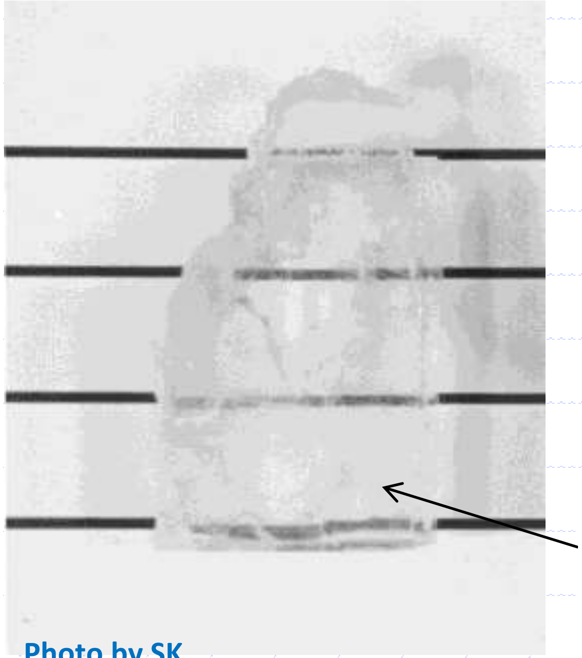


Photo by SK

Most noncrystalline materials such as liquids and glasses, and all cubic crystals are optically isotropic

The refractive index is the same in all directions for all polarizations of the field.

Sodium chloride (halite) crystal

Optical Anisotropy

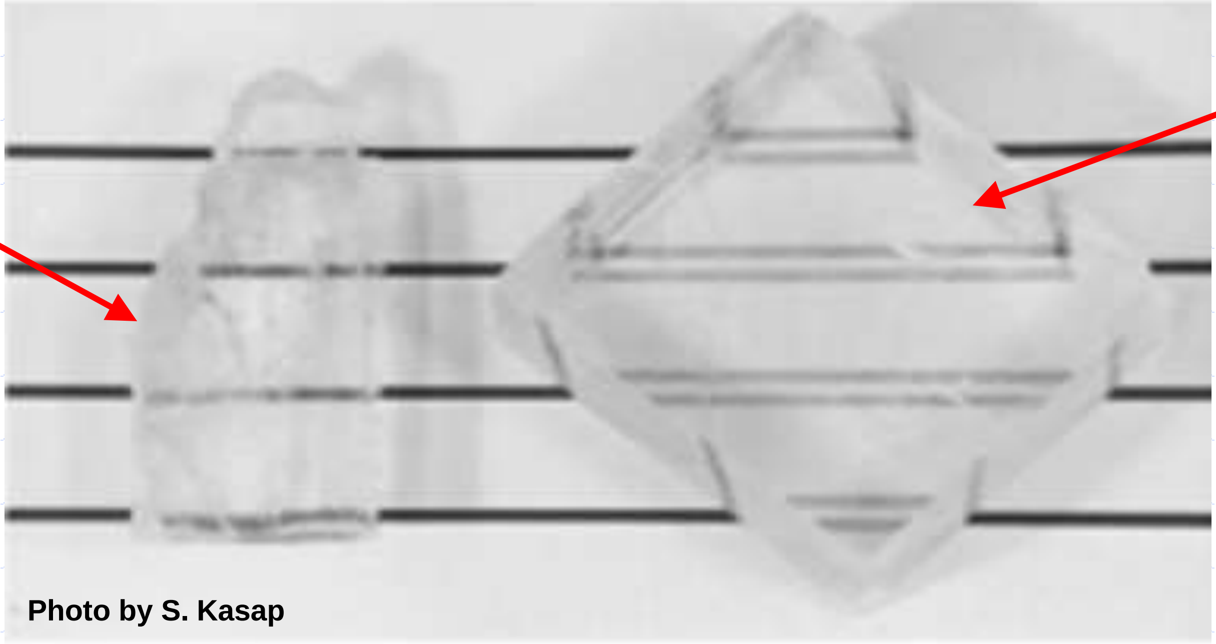
- **Optically anisotropic** materials have **anisotropic** crystal structure:
  - Generate **different degrees of polarization** in different directions
  - The refractive index n of a crystal depends on the direction of the E field in the propagating light beam.

- For all classes of crystals **excluding cubic structures**, the refractive index depends on the propagation direction and the state of polarization, i.e. the direction of E.
- Except along certain special directions (**optic axes**), any unpolarized light ray entering such a crystal breaks into two different rays (**different directions**) with different polarizations and phase velocities.
  - referred to as **birefringent** because incident light beams may be ***doubly refracted***.

Birefringence

When we view an image through a **calcite** crystal, an optically **anisotropic crystal**, we see two images, each constituted by light of different polarization passing through the crystal, whereas there is only one image through an optically **isotropic crystal** as depicted in the figure. **Optically anisotropic** crystals are called **birefringent** because an incident light beam may be doubly refracted.

sodium chloride (halite) crystal (optically isotropic)



calcite crystal (optically anisotropic)

A line viewed through a cubic sodium chloride (halite) crystal (**optically isotropic**) and a calcite crystal (**optically anisotropic**).

Optic Axis

An **optic axis** is a special direction in the crystal along which the velocity of propagation does ***not*** depend on the state of polarization, i.e., the propagation velocity along the optic axis is the same whatever the polarization of the EM wave.

Anisotropic crystals may posses one or two optic axes.

Biaxial crystals : have three distinct principal indices also have *two* optic axes.

Uniaxial crystals: have two of their principal indices the same ($n_1 = n_2$) and have only *one* optic axis.

- **Positive** uniaxial crystals, such as **quartz**, have $n_3 > n_1$
- **Negative** uniaxial crystals, such as **calcite** , have $n_3 < n_1$

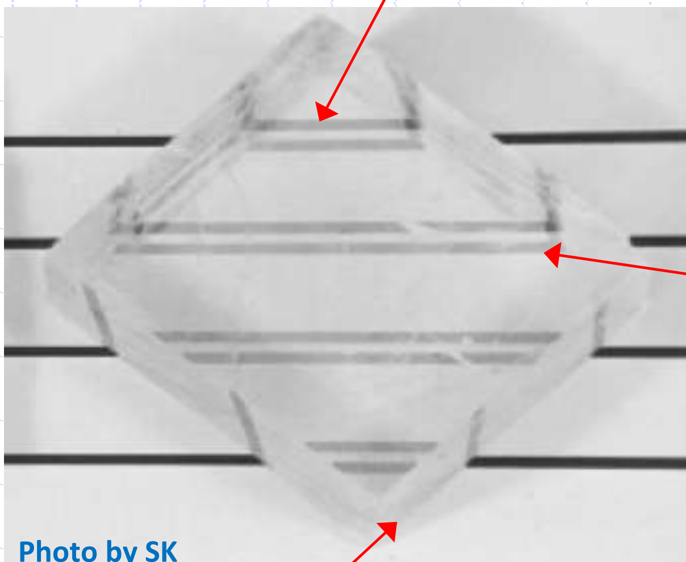
Uniaxial Crystals

- EM waves entering an **uniaxial crystals** will **split** into **two orthogonal** linearly polarized waves (ordinary (***o***) and extraordinary (***e***) waves) with different phase velocities based on the different refractive indices.
  - ***o*-wave** has the same phase velocity in all directions and behaves like an ordinary wave in which the field ⊥ to k.
  - ***e*-wave** has a phase velocity that depends on the direction of propagation and its state of polarization and further, the E field in the ***e*-wave** is not necessarily ⊥ to k.

- Both wave propagate with the **same velocity** only along an **optic axis**.
  - ***o*-wave** is always perpendicularly polarized to the optic axis and obeys **Snell's law**.
  - ***e*-waves** are polarized parallel to the optic axis.
- ***o* and *e* waves** are refracted differently inside the crystal and are split upon emerging from it.

Many crystals are optically anisotropic They exhibit birefringence

This line is due to the "extraordinary wave"

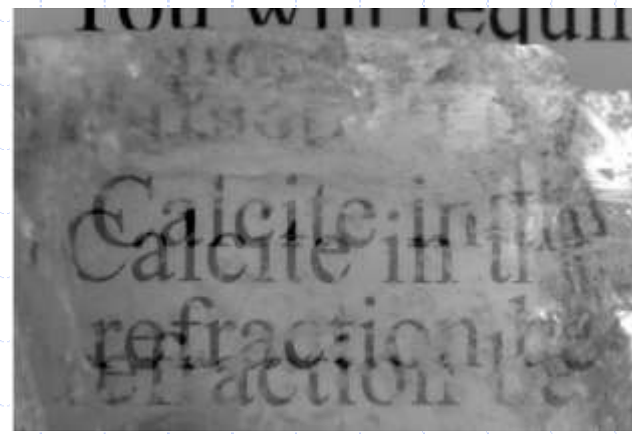


A calcite crystal (optically anisotropic)

The calcite crystal has two refractive indices

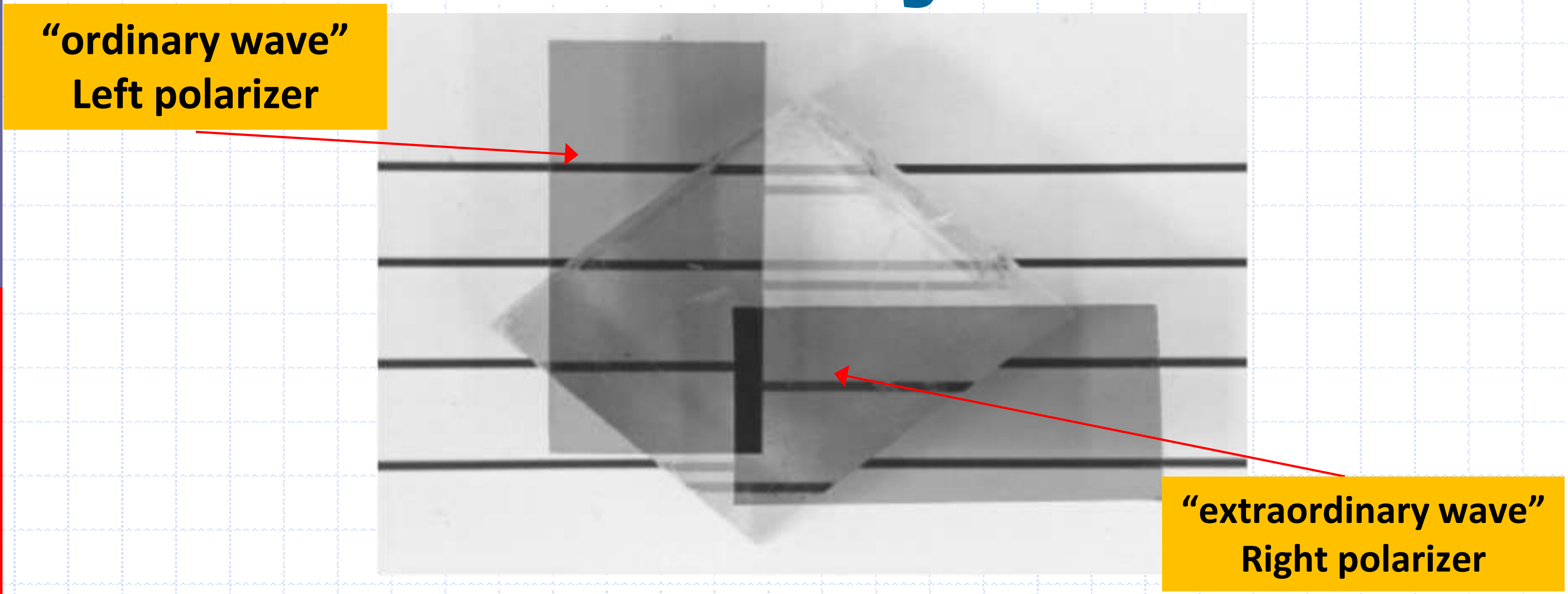
The crystal exhibits double refraction

This line is due to the "ordinary wave"



Calcite have $n_3 < n_1 \Rightarrow$ called **negative** uniaxial crystals

Uniaxial Crystals



Images viewed through a calcite crystal have orthogonal polarizations.

Two polaroid analyzers are placed with their **transmission axes**, along the long edges, at **right angles** to each other.

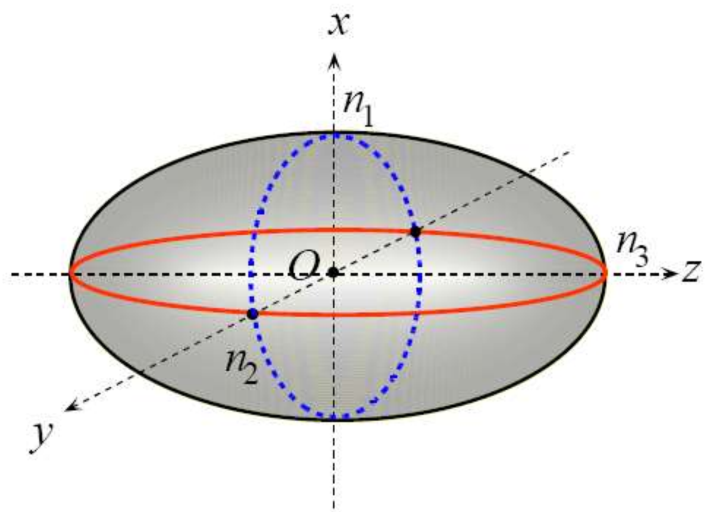
The ***o*-wave**, undeflected, goes through the left polarizer whereas the ***e*-wave**, deflected, goes through the right polarizer. The two waves therefore have **orthogonal polarizations**.

Principal refractive indices of some optically isotropic and anisotropic crystals (near 589 nm, yellow Na-D line)

Optically isotropic		$n = n_o$		
	Glass (crown)	1.510		
	Diamond	2.417		
	Fluorite (CaF_2)	1.434		
Uniaxial - Positive		$n_1(n_o)$	$n_3(n_e)$	
	Ice	1.309	1.3105	$n_3(n_e) > n_1(n_o)$
	Quartz	1.5442	1.5533	
	Rutile (TiO_2)	2.616	2.903	
Uniaxial - Negative		$n_1(n_o)$	$n_3(n_e)$	
	Calcite ($CaCO_3$)	1.658	1.486	$n_3(n_e) < n_1(n_o)$
	Tourmaline	1.669	1.638	
	Lithium niobate ($LiNbO_3$)	2.29	2.20	
Biaxial		n_1	n_2	n_3
	Mica (muscovite)	1.5601	1.5936	1.5977

Fresnel's Optical Indicatrix

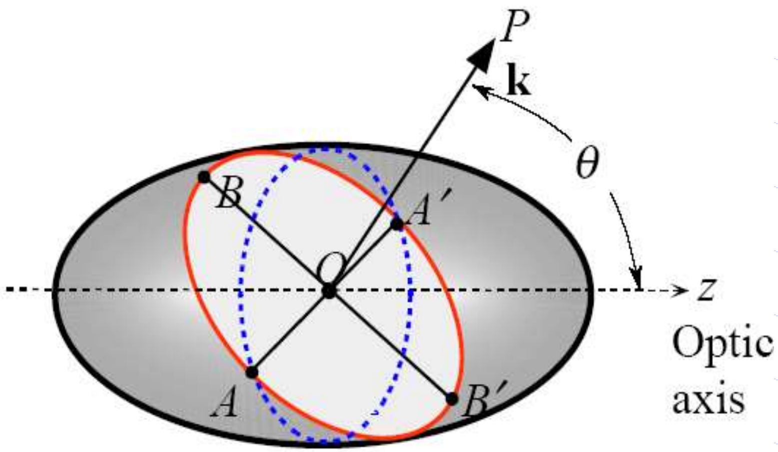
Fresnel's ellipsoid
(for $n_1 = n_2 < n_3$; quartz)
Positive uniaxial crystals



- The optical properties of a crystal can be represented in terms of the *three* refractive indices (n_1, n_2, n_3) along each principle axis (x, y, z).
- **Optical indicatrix** is a refractive index surface placed in the center of the principal axes, as shown in the figure, where the x-, y-, and z-axes have intercepts n_1, n_2, and n_3, respectively.
- Optical indicatrix for an **isotropic** medium
  - take the shape of a **sphere**, and
  - the refractive index would be the *same* in all directions ($n_1 = n_2 = n_3 = n$).

Ordinary Wave and Extraordinary Wave

An EM wave propagating along *OP* at an angle θ to the optic axis *z*.



The line *AOA′*, the *minor axis*, corresponds to the polarization of the ***o*-wave** and its semiaxis *AA′* is the refractive index $n_o = n_2$ of this *o*-wave. The electric displacement and the electric field are in the same direction and parallel to *AOA′*.

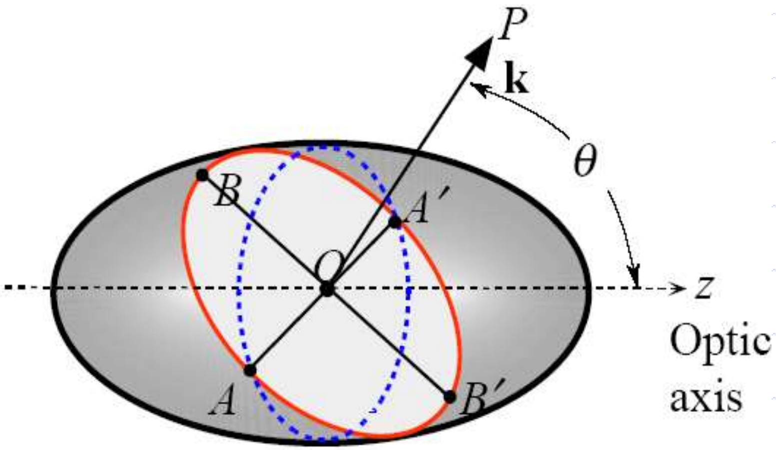
The line *BOB′* the *major axis*, corresponds to the electric displacement field (**D**) oscillations in the ***e*-wave** and its semiaxis *OB* is the refractive **index $n_e(\theta)$** of this *e*-wave. This refractive index is smaller than n_3 but greater than n_2 ($= n_o$).

Ordinary Wave and Extraordinary Wave

When the ***e*-wave** is traveling along the *y*-axis, or along the *x*-axis, $n_e(\theta) = n_3 = n_e$ and the *e*-wave has its slowest phase velocity.

Along any ***OB* direction** that is at an angle θ to the optic axis, the *e*-wave has a refractive index $n_e(\theta)$ given by

$$\frac{1}{n_e(\theta)^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$



along the optic axis

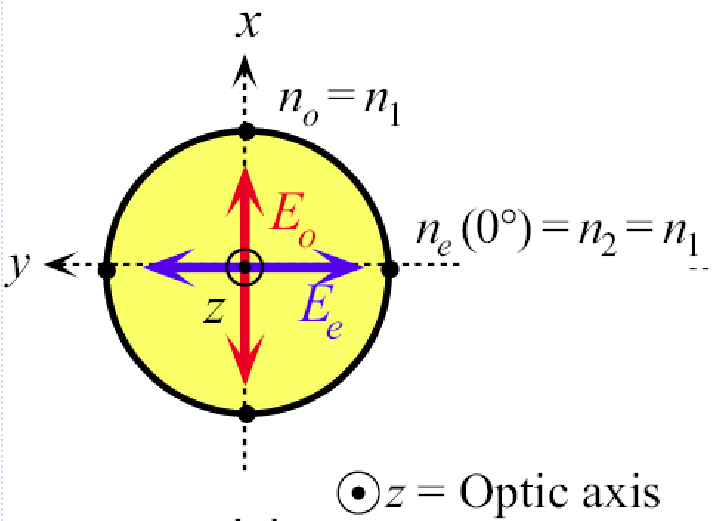
normal to optic axis

Clearly, for $\theta = 0°$, $n_e(0°) = n_o$ and for $\theta = 90°$, $n_e(90°) = n_e$.

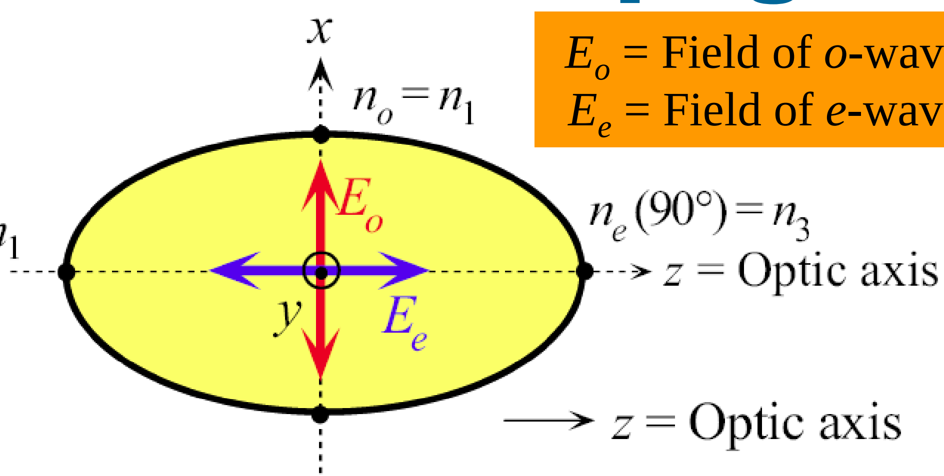
The electric field $\mathbf{E}_{e\text{-wave}}$ of the *e*-wave is orthogonal to that of the *o*-wave, and it is in the **plane determined by k and the optic axis**.

O-Wave and E-Wave Propagation

E_o = Field of *o*-wave = $E_{o\text{-wave}}$
E_e = Field of *e*-wave = $E_{e\text{-wave}}$



(a) Wave propagation along the optic axis.



(b) Wave propagation normal to optic axis

- When ***e*-wave** is traveling along the z-axis (**optic axis**), $\theta = 0°$, as in Fig. (a) ⇒ $n_e = n_o$.

All waves traveling along the **optic axis** have the <u>same phase velocity</u> whatever their polarization.

- When the ***e*-wave** is traveling along the y-axis (or along the x-axis) ⇒ $n_e(\theta = 90°) = n_3 = n_e$ and the e-wave has **its <u>slowest phase velocity</u>** as shown in Fig. (b).

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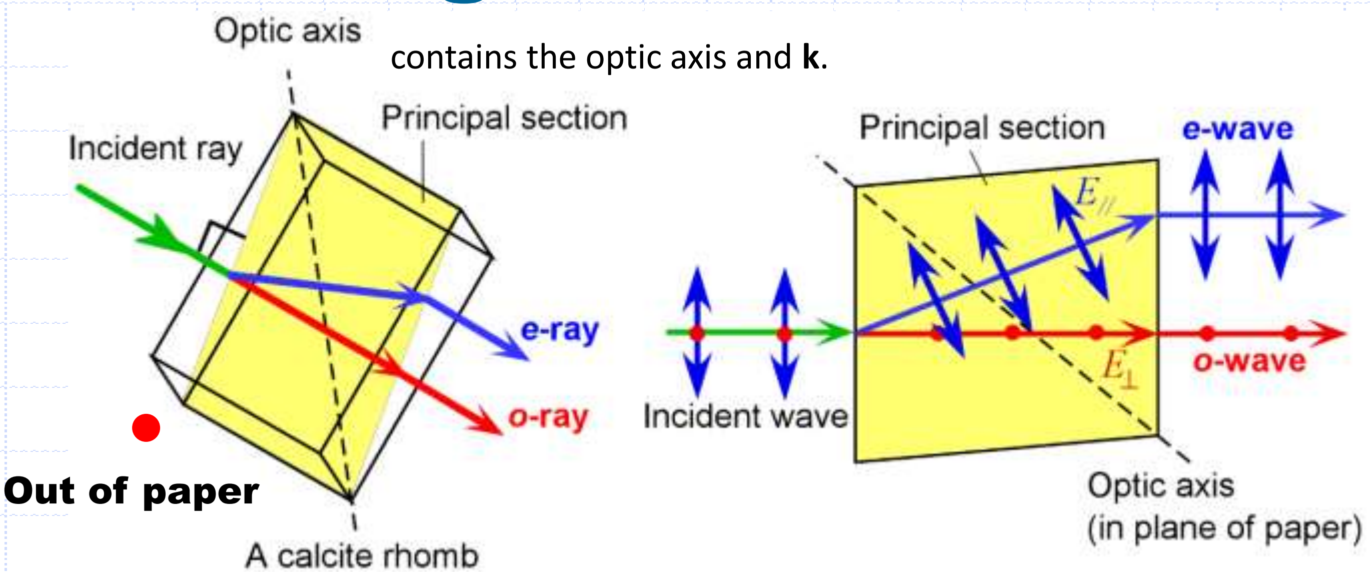
Birefringence of Calcite ($CaCO_3$)

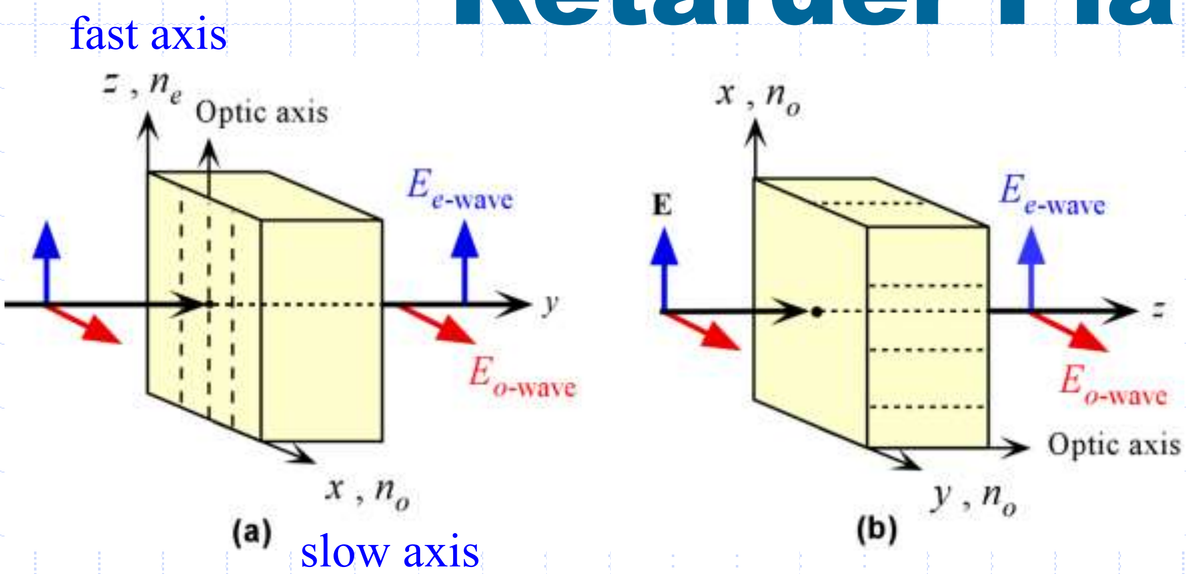


Negative uniaxial ($n_3 < n_1$)

crystal orientation is a rhombohedron (parallelogram with 78.08° and 101.92° at principle axis)

- **Unpollarized light** that is off the optic axis entering the structure is broken into an ***o***- and ***e*-wave** propagating through at different angles and mutually orthogonal polarizations.
  - **o-wave** has its field oscillations ⊥ the **optic axis (**out of the paper $E_{\perp}$**)**. It obeys Snell's law, i.e., it enters the crystal **undeflected**.
  - ***e*-wave** polarization is in the plane of the paper, indicated as $E_{\|}$. It travels with a <u>different velocity and diverges</u> from the *o*-wave.
  - Angle of refraction of the ***e*-wave** ≠ 0 as required by **Snell's law**.
  - Both waves propagate at <u>different velocities</u> and emerge propagating in the <u>same direction</u> but at orthogonal polarizations.

Retarder Plate



(Negative uniaxial crystal ($n_3(n_e) < n_1(n_o)$))

e-wave with a velocity c/n_e
o-wave with a velocity c/n_o
$n_e < n_o$

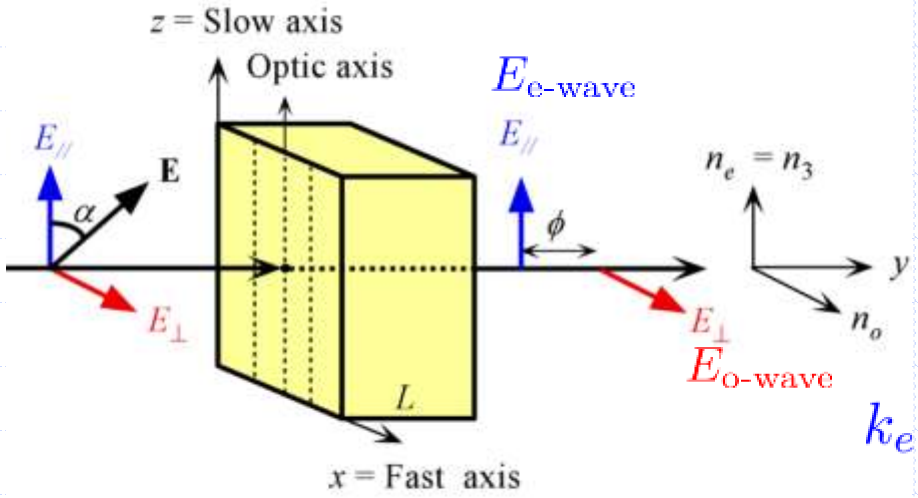
(a) A **birefringent calcite crystal** plate with the optic axis (along *z*) ∥ plate surfaces.

- A ray entering at normal incidence to one of these faces would not diverge into two separate waves.
- The ***o***- and ***e*-waves** would travel in the same direction but with different speeds ⇒ **no double refraction**.

(b) A **birefringent calcite crystal** plate with the optic axis ⊥ plate surfaces.

- Both the ***o*-** and ***e*-waves** would be traveling at the same speed and along the same direction ⇒ **no double refraction.**

Retarder Plate



e-wave with a velocity c/n_e

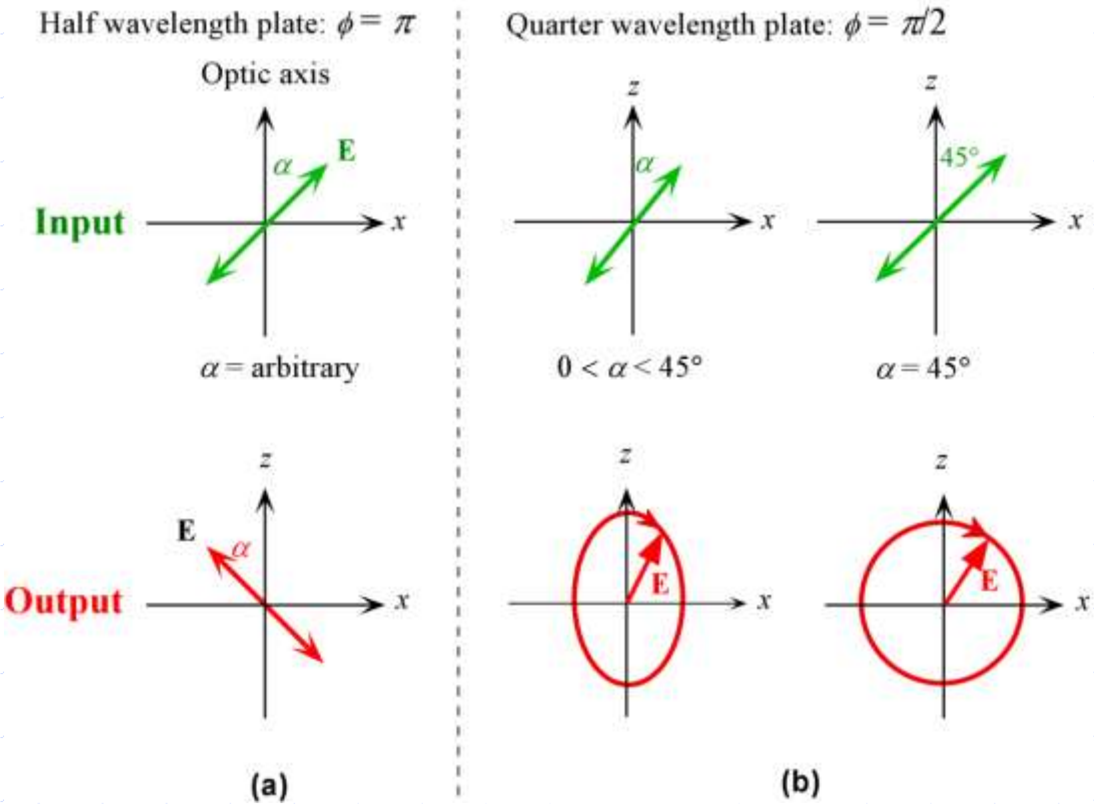
o-wave with a velocity c/n_o

$$n_e > n_o$$

$$k_{e-\text{wave}} = \frac{2\pi}{\lambda} n_e \quad k_{o-\text{wave}} = \frac{2\pi}{\lambda} n_o$$

- Consider a **positive uniaxial crystal** such as quartz ($n_e > n_o$) that has the optic axis parallel to the face plate (plane of incidence) of the light.
  - If **E** is **rotated** w.r.t. the optical axis, then the ***o***- and ***e***-waves propagate through the material at different velocities yielding a phase difference between the perpendicular $\mathbf{E}_\perp$ and the parallel $\mathbf{E}_\parallel$ field.
- The phase difference, ϕ: $\phi = (k_{e-\text{wave}} - k_{o-\text{wave}})L = \frac{2\pi}{\lambda}(n_e - n_o)L$
  - L is the thickness of the plate

Birefringent Retarding Plates

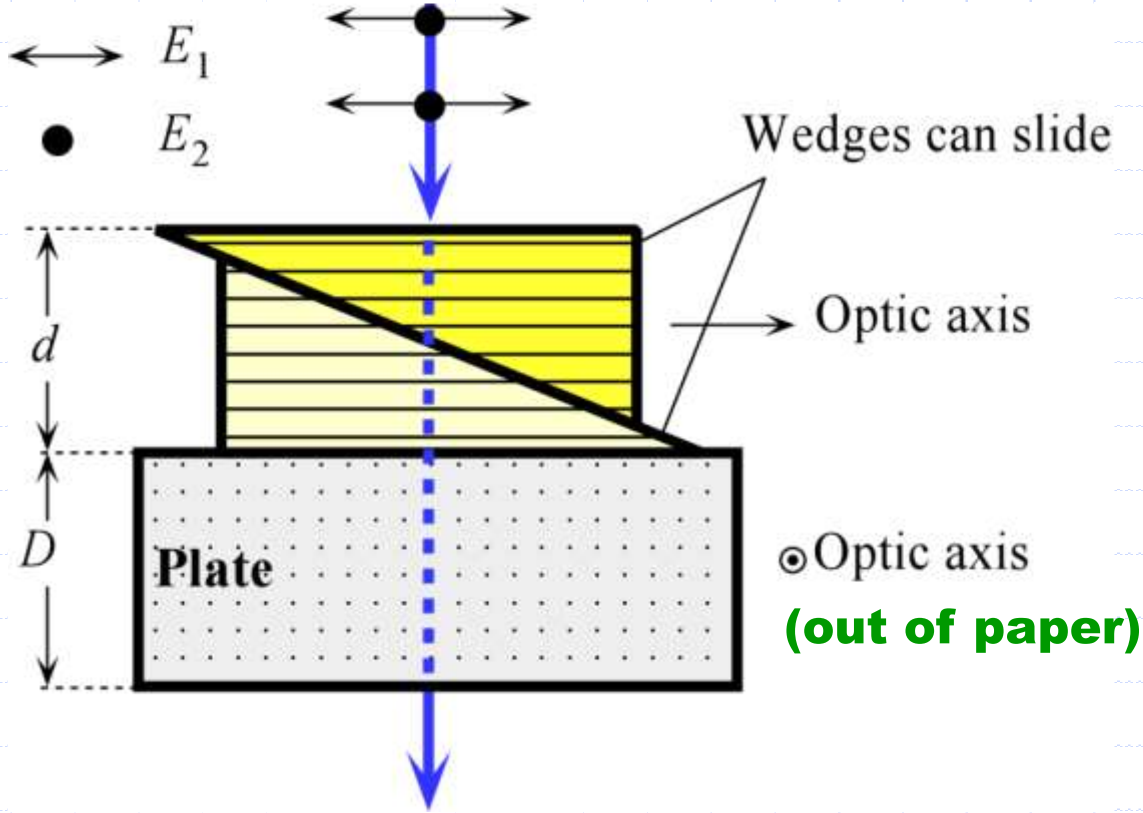


$$\phi = \frac{2\pi}{\lambda}(n_e - n_o)L$$

α =incidence angle of linear polarization

- **Retardation:** phase difference in terms of full wavelengths.
- A $\lambda/2$ plate retarder has a thickness L such that $\phi = 180°$.
  - Resulting wave is linearly polarized and flipped 180° spatially.
- A $\lambda/4$ plate retarder has a thickness L such that $\phi = 90°$.
  - Resulting wave is elliptically polarized for $0 < \alpha < 45°$.
  - Circular polarized for $\alpha = 45°$.

Optical Compensator



$n_e > n_o$

- **Optical compensator:-** is a device that allows one to control retardation between 0 and 2π by precise positioning of one wedge w.r.t. the other.
- Using two birefringent optical wave plates cut such that one when slid across each other, the length of the total retarder is increased or decreased.
- An additional quartz plate is placed under with an orthogonal optical axis.

Optical Compensator

- Suppose that a **linearly polarized** light is incident on this compensator at normal incidence.
  - This light is represented by field oscillations parallel (E_1) and perpendicular (E_2) to the optic axis of the two-wedge block.
- The E_1-polarization travels through the wedges (d) experiencing a refractive index n_e and then travels through the plate (D) experiencing an index n_o (E_1 is perpendicular to the optic axis). Its phase change is

$$\phi_1 = \frac{2\pi}{\lambda}(n_e d + n_o D)$$

- The E_2-polarization wave first experiences n_o through the wedges (d) and then n_e through the plate (D) so that its phase change is

$$\phi_2 = \frac{2\pi}{\lambda}(n_o d + n_e D)$$

- The phase difference $\phi = \phi_2 - \phi_1$ between the two polarizations is

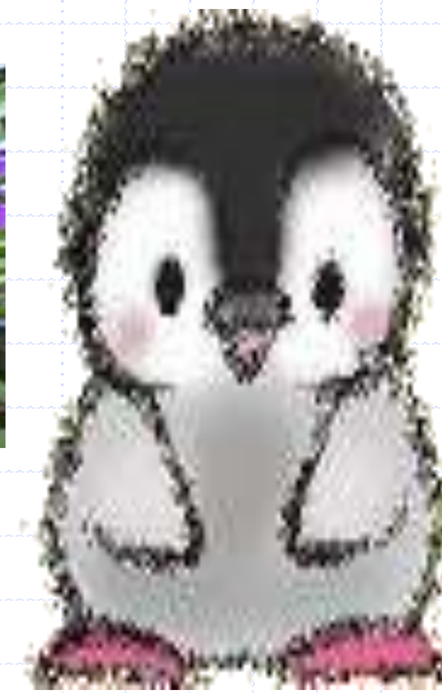
$$\phi = \frac{2\pi}{\lambda}(n_e - n_o)(D - d)$$

Soleil-Babinet Compensator

Courtesy of Thorlabs

https://www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=871

Thank you



Have a nice day.