Lecture 1

The Steady Magnetic Field

Electromagnetic Field Theory

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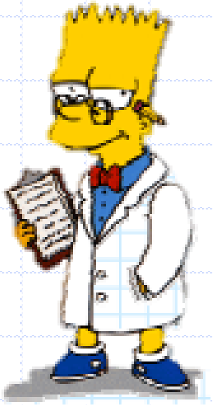
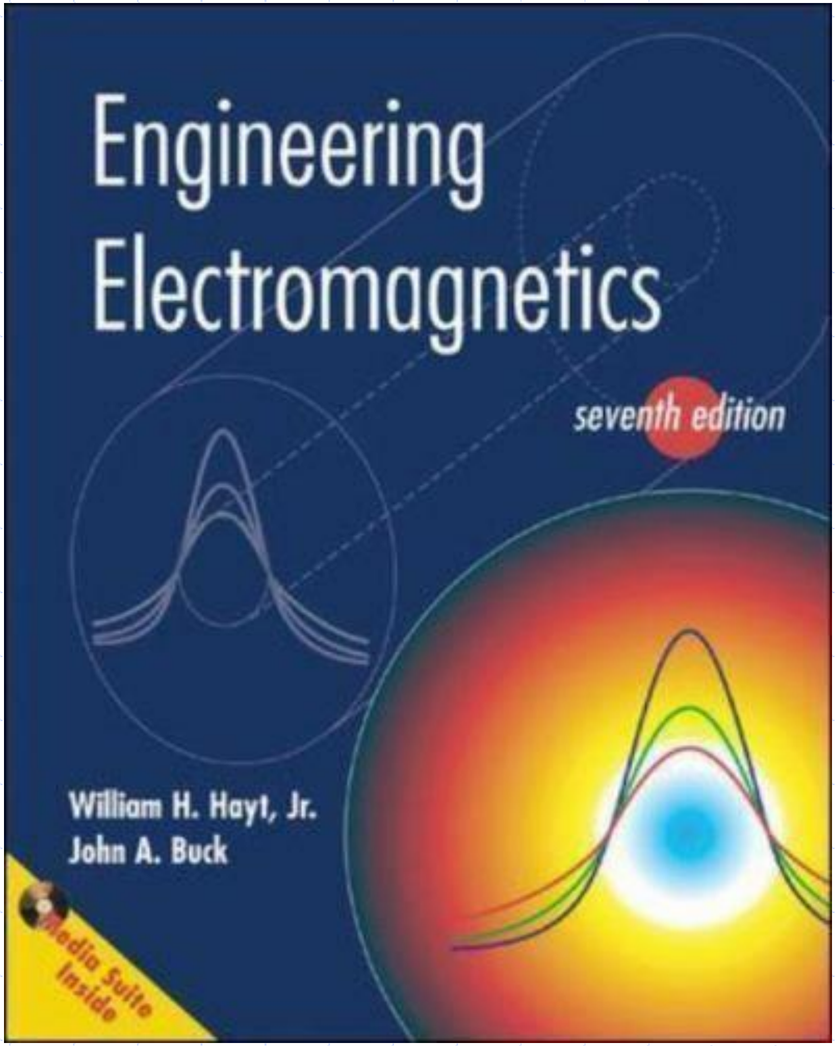
Selected Topics

- The steady magnetic field; Biot Savart and Ampere circuital laws
- Magnetic forces, torque, materials properties.
- Calculations of self and mutual Inductance
- Time varying field and Maxwell s equations
- The uniform plane wave; the transverse Electromagnetic (TEM) Wave
- Poynting theorem, normal and oblique incidence.



Course Textbook



W. Hayt Jr. and J. Buck, Engineering Electromagnetics, McGraw-Hill Education (India) Pvt Limited, 2006.

A very good book on electromagnetics. Easier to read and understand. Recommended ★ ★ ★

Steady Magnetic Field

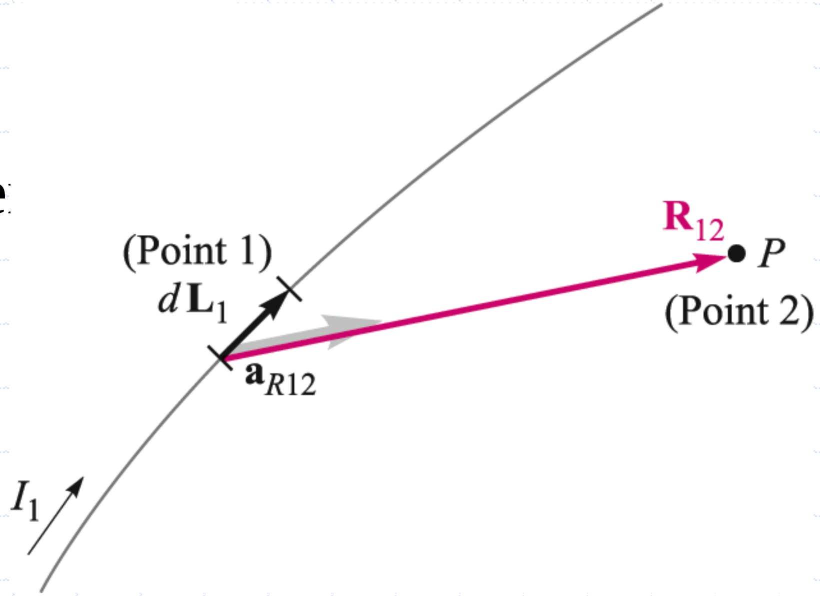
- The source of the steady magnetic field may be:
  - a permanent magnet $\Longrightarrow$ will be ignored
  - an electric field changing linearly with time $\Longrightarrow$ discussed later
  - a direct current.
- Our present study will concern the magnetic field produced by a ***differential dc element*** in free space.
- We may think of this ***differential current element*** as a vanishingly small section of a current-carrying filamentary conductor, where a filamentary conductor is the limiting case of a cylindrical conductor of circular cross section as the **radius approaches zero**.

Biot-Savart Law

The Biot-Savart Law specifies the magnetic field intensity, **H**, arising from a "point source" current eleme of differential length d**L.**

$$d\mathbf{H}_2 = \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2}$$

The units of **H** are [A/m]



Note in particular the inverse-square distance dependence, and the fact that the cross product will yield a field vector that points into the page. This is a formal statement of the right-hand rule

Note the similarity to Coulomb's Law, in which a point charge of magnitude dQ_1 at Point 1 would generate electric field at Point 2 given by:

$$d\mathbf{E}_2 = \frac{dQ_1 \mathbf{a}_{R12}}{4\pi\epsilon_0 R_{12}^2}$$

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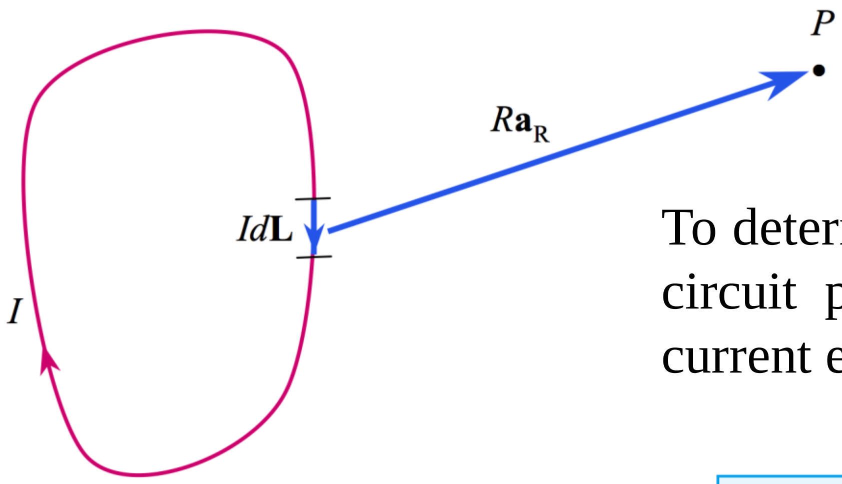
Magnetic Field Compared to Electric Field

- Distance
  - The magnitude of the magnetic field varies as the **inverse** square of the distance from the source.
  - The electric field due to a point charge also varies as the **inverse** square of the distance from the charge.
- Direction
  - The electric field created by a point charge is **radial** in direction.
  - The magnetic field created by a current element is **perpendicular** to both the length element $d\mathbf{L_1}$ and the unit vector a_R.
- Source
  - An electric field is established by an **isolated electric charge**.
  - The current element that produces a magnetic field must be **part of an extended current distribution**.
    - Therefore you must integrate over the entire current distribution.

Magnetic Field Arising From a Circulating Current



At point *P*, the magnetic field associated with the differential current element $Id\mathbf{L}$ is

$$d\mathbf{H} = \frac{I\,d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2}$$

To determine the total field arising from the closed circuit path, we sum the contributions from the current elements that make up the entire loop, or

$$\mathbf{H} = \oint \frac{I\,d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2}$$

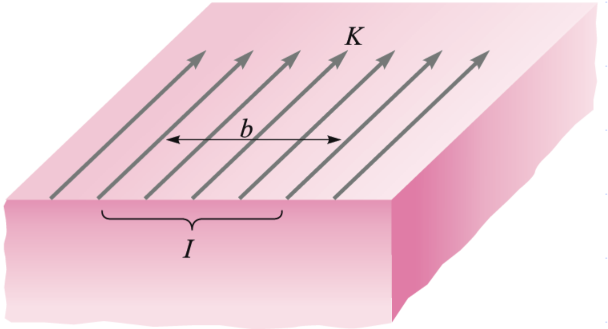
Verified experimentally

The contribution to the field at *P* from any portion of the current will be just the above integral evaluated over just that portion.

2- and 3-Dimensional Currents

On a surface that carries uniform surface current density **K** [A/m], the current within width b is

$$I = Kb$$



.. and so the differential current quantity that appears in the Biot-Savart law becomes:

$$I\,d\mathbf{L} = \mathbf{K}\,dS$$

The magnetic field arising from a current sheet is thus found from the 2-D form of the Biot-Savart law:

$$\mathbf{H} = \int_s \frac{\mathbf{K} \times \mathbf{a}_R\,dS}{4\pi R^2}$$

In a similar way, a **volume current** will be made up of 3-D current elements, and so the Biot-Savart law for this case becomes:

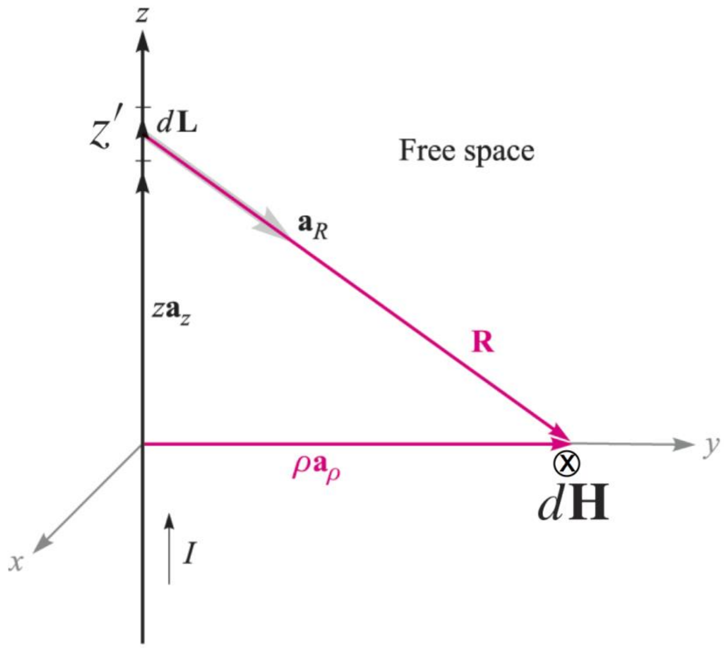
$$\mathbf{H} = \int_{\mathrm{vol}} \frac{\mathbf{J} \times \mathbf{a}_R\,dv}{4\pi R^2}$$

Example of the Biot-Savart Law

In this example, we evaluate the magnetic field intensity on the *y* axis (equivalently in the *xy* plane) arising from a filament current of infinite length in on the *z* axis.

Using the drawing, we identify:

$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = \rho \mathbf{a}_\rho - z' \mathbf{a}_z$$



and so.. $\mathbf{a}_R = \dfrac{\rho \mathbf{a}_\rho - z' \mathbf{a}_z}{\sqrt{\rho^2 + z'^2}}$

so that:

$$d\mathbf{H} = \frac{I d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} = \frac{I\, dz' \mathbf{a}_z \times (\rho \mathbf{a}_\rho - z' \mathbf{a}_z)}{4\pi (\rho^2 + z'^2)^{3/2}}$$

Example: continued

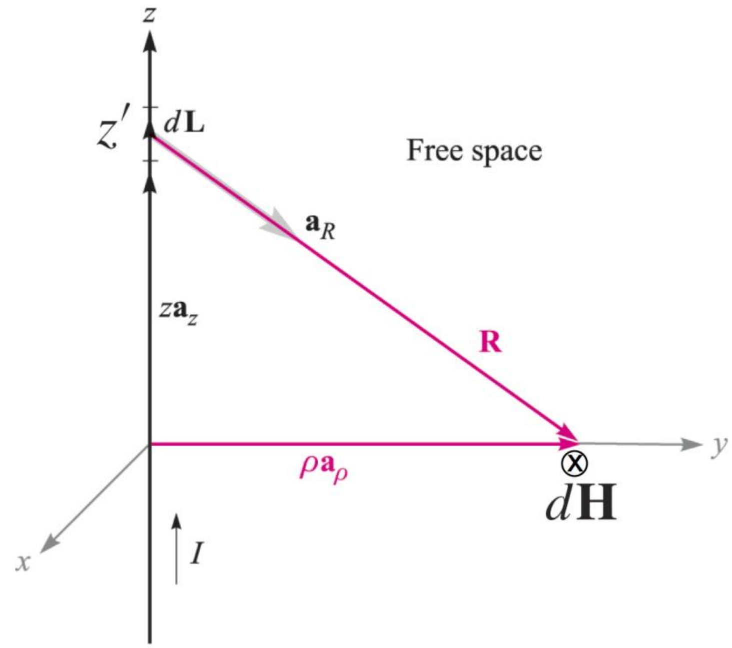
We now have:

$$d\mathbf{H} = \frac{I\,d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} = \frac{I\,dz'\mathbf{a}_z \times (\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi(\rho^2 + z'^2)^{3/2}}$$

Integrate this over the entire wire:

$$\mathbf{H}_2 = \int_{-\infty}^{\infty} \frac{I\,dz'\mathbf{a}_z \times (\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi(\rho^2 + z'^2)^{3/2}}$$

$$\mathbf{H} = \frac{I}{4\pi}\int_{-\infty}^{\infty} \frac{\rho\,dz'\mathbf{a}_\phi}{(\rho^2 + z'^2)^{3/2}}$$



..after carrying out the cross product

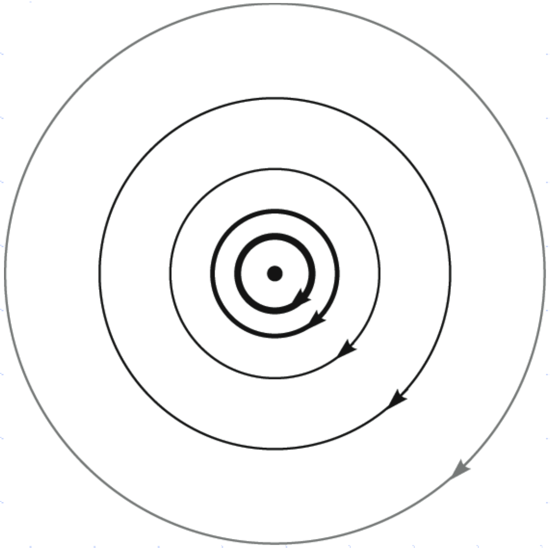
Example: concluded

Evaluating the integral:

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

we have: $$\mathbf{H} = \frac{I}{4\pi}\int_{-\infty}^{\infty} \frac{\rho\, dz'\, \mathbf{a}_\phi}{(\rho^2+z'^2)^{3/2}}$$

$$= \frac{I\rho\mathbf{a}_\phi}{4\pi} \frac{z'}{\rho^2\sqrt{\rho^2+z'^2}}\Bigg|_{-\infty}^{\infty}$$



finally: $$\mathbf{H}_2 = \frac{I}{2\pi\rho}\mathbf{a}_\phi$$

Current is into the page.
Magnetic field streamlines
are concentric circles, whose magnitudes
decrease as the inverse distance from the *z* axis

Field Arising from a Finite Current Segment

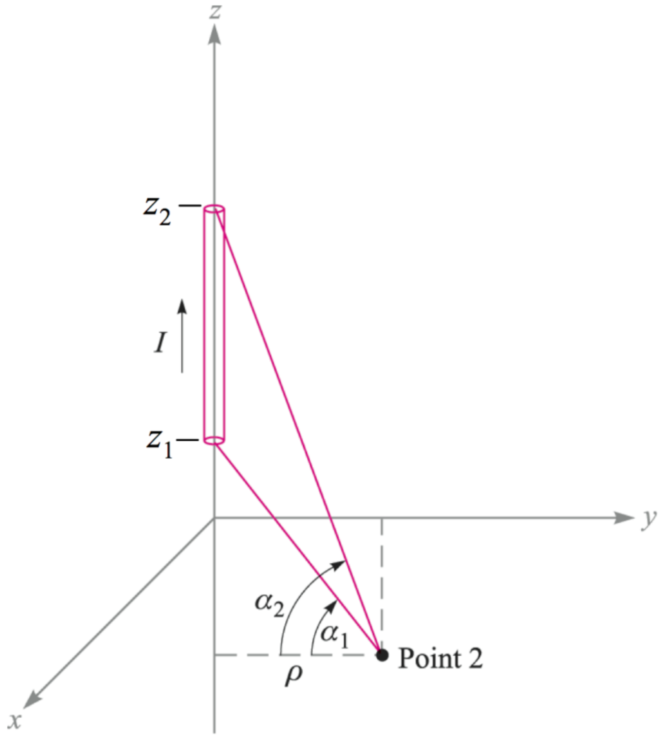
In this case, the field is to be found in the *xy* plane at Point 2. The Biot-Savart integral is taken **over the wire length**:

$$\mathbf{H} = \int_{z_1}^{z_2} \frac{I d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2}$$

$$= \int_{\rho \tan \alpha_1}^{\rho \tan \alpha_2} \frac{I dz \mathbf{a}_z \times (\rho \mathbf{a}_\rho - z \mathbf{a}_z)}{4\pi(\rho^2 + z^2)^{3/2}}$$

..after a few additional steps, we find:

$$\mathbf{H} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\mathbf{a}_\phi$$



$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}}$$

Another Example: Magnetic Field from a Current Loop

Consider a circular current loop of radius *a* in the *x-y* plane, which carries steady current *I*. We wish to find the magnetic field strength anywhere on the *z* axis.

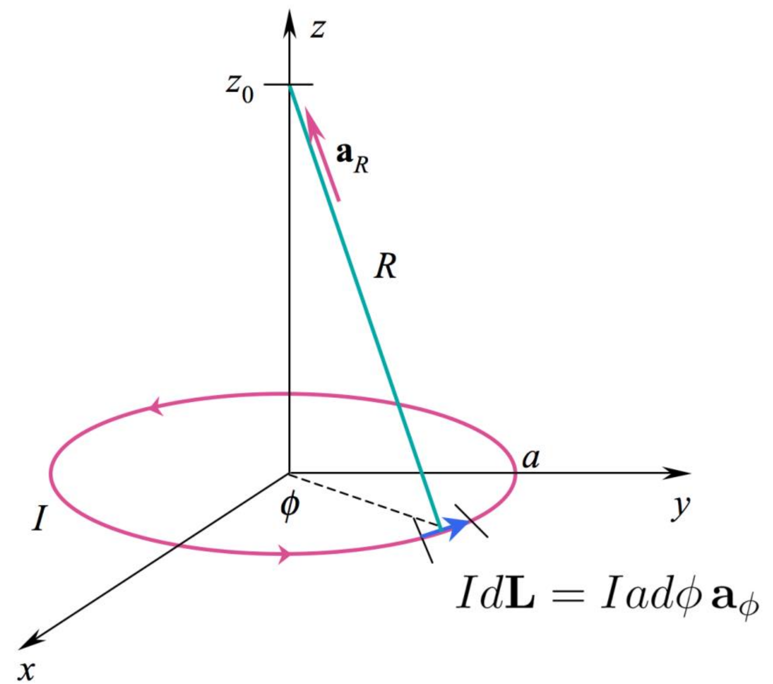
We will use the Biot-Savart Law:

$$\mathbf{H} = \int \frac{Id\mathbf{L} \times \mathbf{a}_R}{4\pi R^2}$$

where: $Id\mathbf{L} = Iad\phi\, \mathbf{a}_\phi$

$$R = \sqrt{a^2 + z_0^2}$$

$$\mathbf{a}_R = \frac{z_0\, \mathbf{a}_z - a\, \mathbf{a}_\rho}{\sqrt{a^2 + z_0^2}}$$



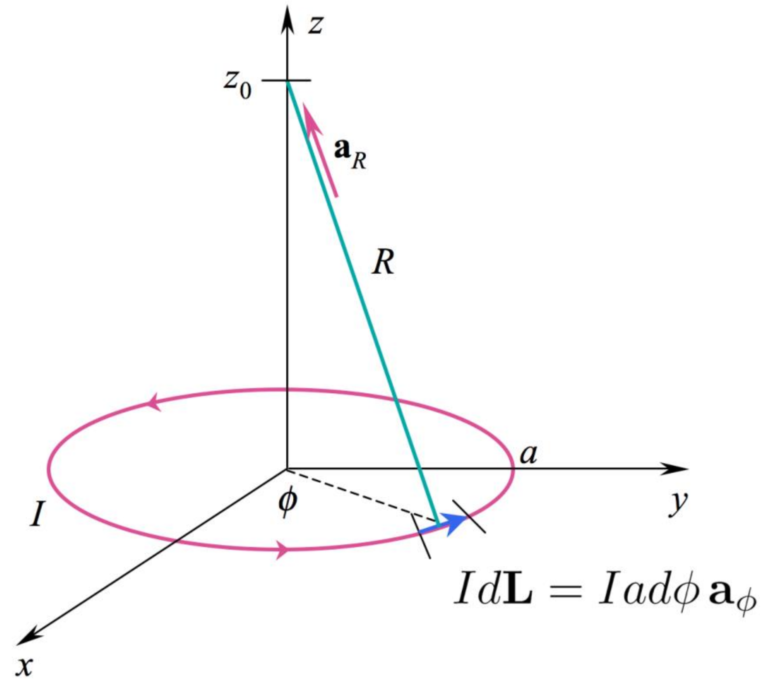
Example: Continued

Substituting the previous expressions, the Biot-Savart Law becomes:

$$\mathbf{H} = \int_0^{2\pi} \frac{Iad\phi\,\mathbf{a}_\phi \times (z_0\,\mathbf{a}_z - a\,\mathbf{a}_\rho)}{4\pi(a^2 + z_0^2)^{3/2}}$$

carry out the cross products to find:

$$\mathbf{H} = \int_0^{2\pi} \frac{Iad\phi\,(z_0\,\mathbf{a}_\rho + a\,\mathbf{a}_z)}{4\pi(a^2 + z_0^2)^{3/2}}$$



but we must include the angle dependence in the radial unit vector:

$$\mathbf{a}_\rho = \cos\phi\,\mathbf{a}_x + \sin\phi\,\mathbf{a}_y$$

with this substitution, the radial component will integrate to zero, meaning that all radial components will **cancel on the *z* axis**.

Example: Continued

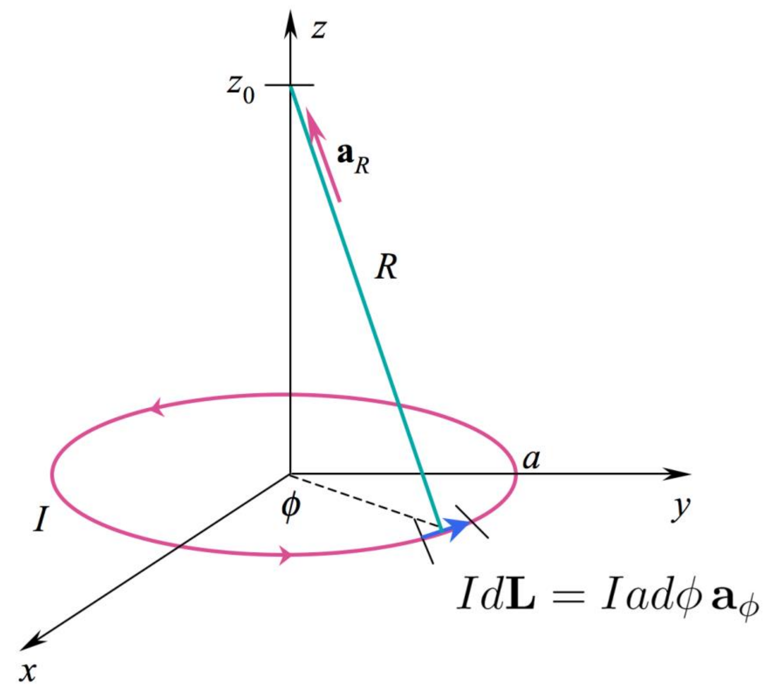
Now, only the *z* component remains, and the integral evaluates easily:

$$\mathbf{H} = \frac{I(\pi a^2)\mathbf{a}_z}{2\pi(a^2 + z_0^2)^{3/2}}$$

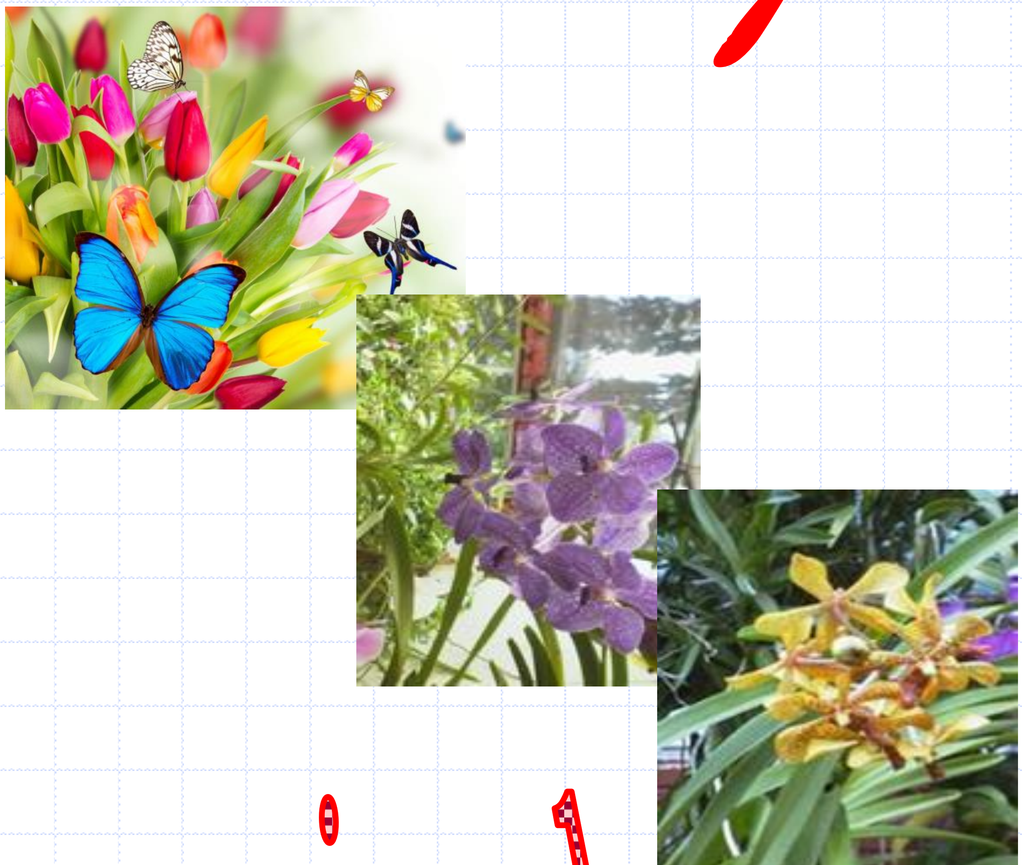
Note the form of the numerator: the product of the current and the loop area.

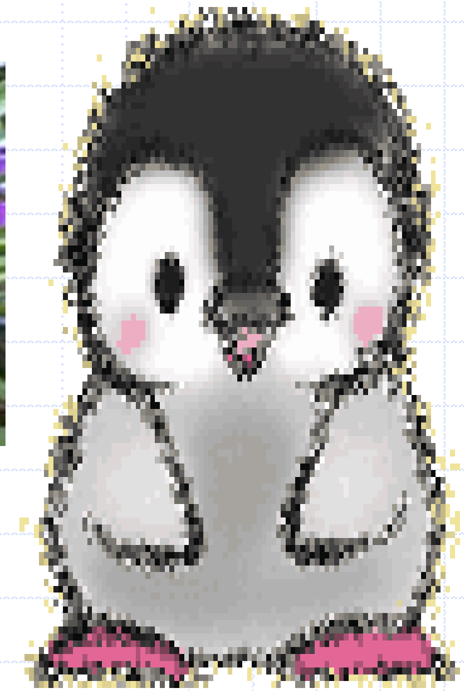
We define this as the ***magnetic moment***:

$$\mathbf{m} = I(\pi a^2)\,\mathbf{a}_z$$



Thank you





Have a nice day.