Lecture 4
MAGNETIC FORCES, MATERIALS AND INDUCTANCE

Electromagnetic Field Theory

Oct. 23, 2019

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MAGNETIC FORCES, MATERIALS AND INDUCTANCE

Force on Moving Charge

Force on a differential current element

Force between two differential current elements

Force and torque in a closed surface

Torque on a differential current loop

Nature of Magnetic Materials

Magnetization and Permeability

Magnetic Boundary Conditions

Magnetic Circuits

Potential Energy and Forces on Magnetic Materials

Inductance and Mutual Inductance

Introduction

- Determining the force and torque excreted by the magnetic field on other charges.
- Electric field excretes a force on stationary or moving charged.
- Magnetic field excretes a force on moving charge **only**.
- Magnetic field produces only form **moving charge** so it excretes only a force on the moving charges.

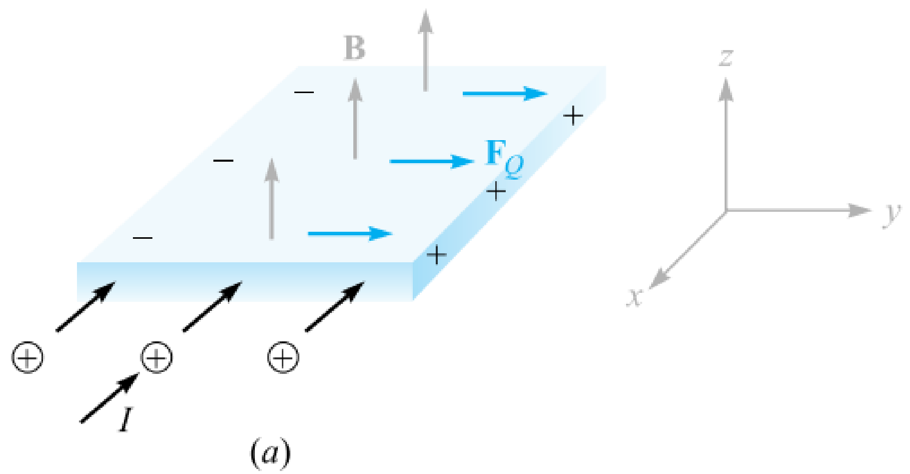
Force on a Moving Charge

Electric Force	Magnetic Force
$F = QE$	$F = Qv \times B$
In the direction of the field	Normal to the direction of field and velocity
Kinetic Energy of the particle can be change	Kinetic Energy of the particle can’t be change
Energy transfer can be occur	Energy transfer can’t be occur

- Solution of Lorentz Force Equation $F = Q(E + v \times B)$ is required in determining
  1. Electron orbits in the Magnetron
  2. Proton Paths in the Cyclotron
  3. Plasma characteristics in Magnetohudrodynamic (MHD)

  Or in general Charged Particle moves in both electric and magnetic field

Force on a Differential current Element



- For electrons move inside conductors as shown.
- The Magnetic field produce a force on the particle cause the particle to change their position.
- But the electric force between +ve and –ve charge contrast this force (**Hall voltage**). The hall voltage normal to both velocity and field and this called **Hall effect.**
- This method can be used to distinguish between n-type and p-type semiconductors.

Force on a Differential current Element

- Using the convection current density equation

$$\mathbf{J} = \rho_v \mathbf{v} \qquad dQ = \rho_v dv \qquad d\mathbf{F} = \rho_v dv\, \mathbf{v} \times \mathbf{B}$$

$$d\mathbf{F} = \mathbf{J} \times \mathbf{B}\, dv \qquad \mathbf{J}\, dv = \mathbf{K}\, dS = I\, d\mathbf{L} \qquad d\mathbf{F} = \mathbf{K} \times \mathbf{B}\, dS \qquad d\mathbf{F} = I\, d\mathbf{L} \times \mathbf{B}$$

$$\mathbf{F} = \oint I\, d\mathbf{L} \times \mathbf{B} = -I \oint \mathbf{B} \times d\mathbf{L} \qquad \mathbf{F} = \int_S \mathbf{K} \times \mathbf{B}\, dS \qquad \mathbf{F} = \int_{\text{vol}} \mathbf{J} \times \mathbf{B}\, dv$$

- The force on straight conductor due to steady uniform field

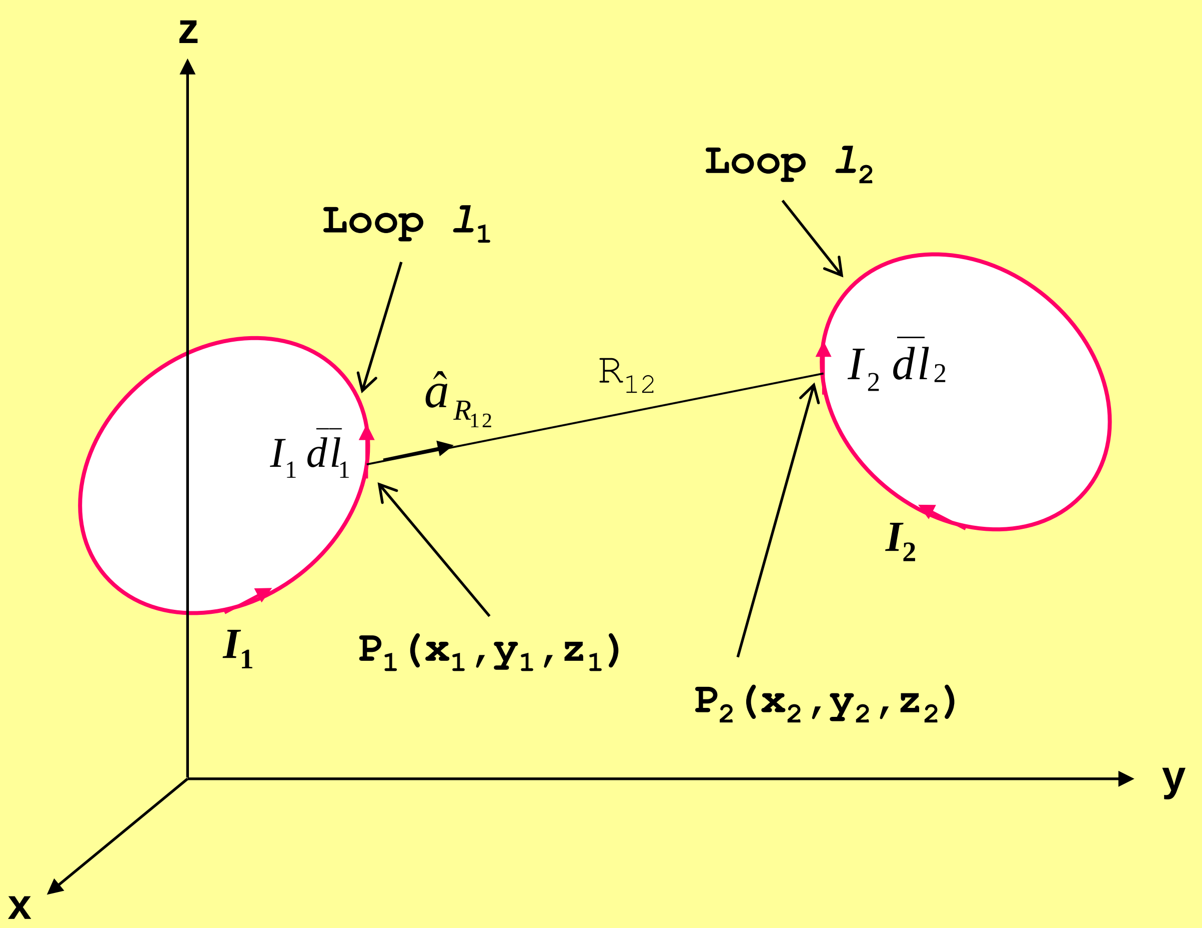
$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$

- The magnitude of the force is

$$F = BIL\sin\theta$$

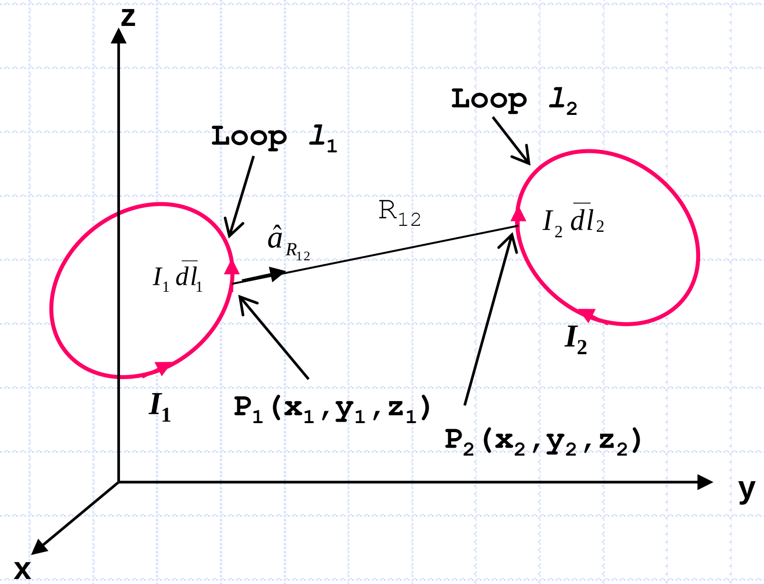
- θ is the angle between the direction of the current flow and the direction of the magnetic field

Force between two differential current element



We have :

$$d\bar{F} = I d\bar{l} \text{ x } \bar{B} \quad \textbf{(N)}$$



The magnetic field at point P_2 due to the filamentary current I_1dl_1 :

$$d\bar{H}_2 = \frac{I_1 d\bar{l}_1 \text{ x } \hat{a}_{R_{12}}}{4\pi {R_{12}}^2} \quad \textbf{(A/m)}$$

$$d\left(d\bar{F}_2\right) = I_2 d\bar{l}_2 \text{ x } \frac{\mu_o I_1 d\bar{l}_1 \text{ x } \hat{a}_{R_{12}}}{4\pi {R_{12}}^2}$$

$$\left(d\bar{F}_2\right) = I_2 d\bar{l}_2 \text{ x } \oint_{l_1} \frac{\mu_o I_1 d\bar{l}_1 \text{ x } \hat{a}_{R_{12}}}{4\pi {R_{12}}^2} = I_2 d\bar{l}_2 \text{ x } \bar{B}_2$$

where $d\bar{F}_2$ is the force due to I_2dl_2 and due to the magnetic field of loop l_1

Integrate:

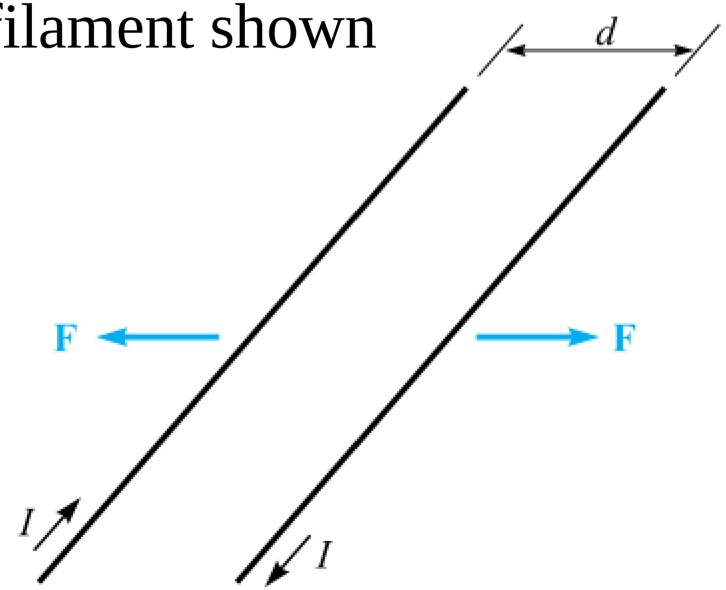
$$\bar{F}_2 = \oint_{l_2} I_2 dl_2 \text{ x } \oint_{l_1} \left[\frac{\mu_o I_1 d\bar{l}_1 \text{ x } \hat{a}_{R_{12}}}{4\pi R_{12}{}^2} \right]$$

$$\bar{F}_2 = \frac{\mu_o I_1 I_2}{4\pi} \oint_{l_2} \left[\oint_{l_1} \frac{\left(\hat{a}_{R_{12}} \text{ x } d\bar{l}_1\right)}{R_{12}{}^2} \right] \text{ x } d\bar{l}_2$$

- The force between two infinite current filament shown

$$\frac{\mu_o I^2}{2\pi d}$$

- The force is repulsive force if the two current in opposite direction and attractive force if the two current are in the same direction



Force and torque in a Closed Surface

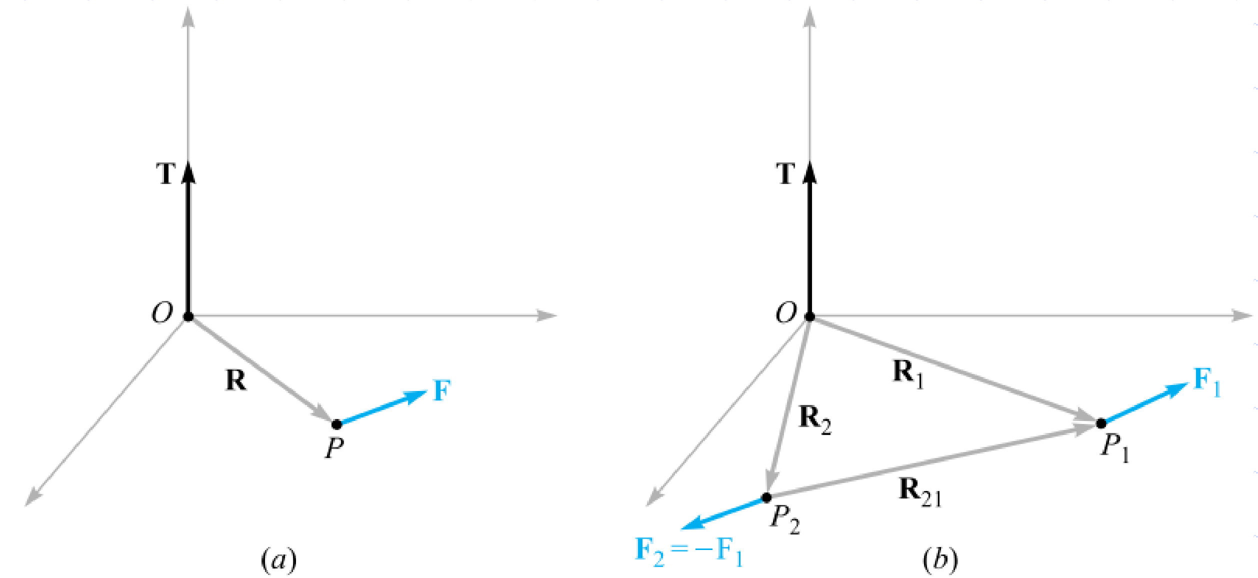
- The force in a filamentary current due to magnetic field is

$$\overline{F} = \oint I\, d\overline{L} \times \overline{B} = -I \oint \overline{B} \times d\overline{L} \quad \text{If the field is uniform} \quad \overline{F} = -I\overline{B} \times \oint d\overline{L}$$

But $\oint d\overline{L} = 0$

- The force in any closed surface carry a direct current (line-surface or volume) due to uniform magnetic field is equal to zero and not equal to zero if the field is non uniform
- Although the force is zero the torque is not equal to zero
- To calculate the torque we must consider
  1. Origin at or about which the torque is calculated
  2. The point at which the field is applied

Force and torque in a Closed Surface



- For part a

$$\mathbf{T} = \mathbf{R} \times \mathbf{F}$$

- For Part b

$$\mathbf{T} = \mathbf{R}_1 \times \mathbf{F}_1 + \mathbf{R}_2 \times \mathbf{F}_2 \qquad \mathbf{F}_1 + \mathbf{F}_2 = 0$$

$$\mathbf{T} = (\mathbf{R}_1 - \mathbf{R}_2) \times \mathbf{F}_1 = \mathbf{R}_{21} \times \mathbf{F}_1$$

- The torque is independent of the choice of the origin, provided that the total force is zero

Torque on Differential current loop

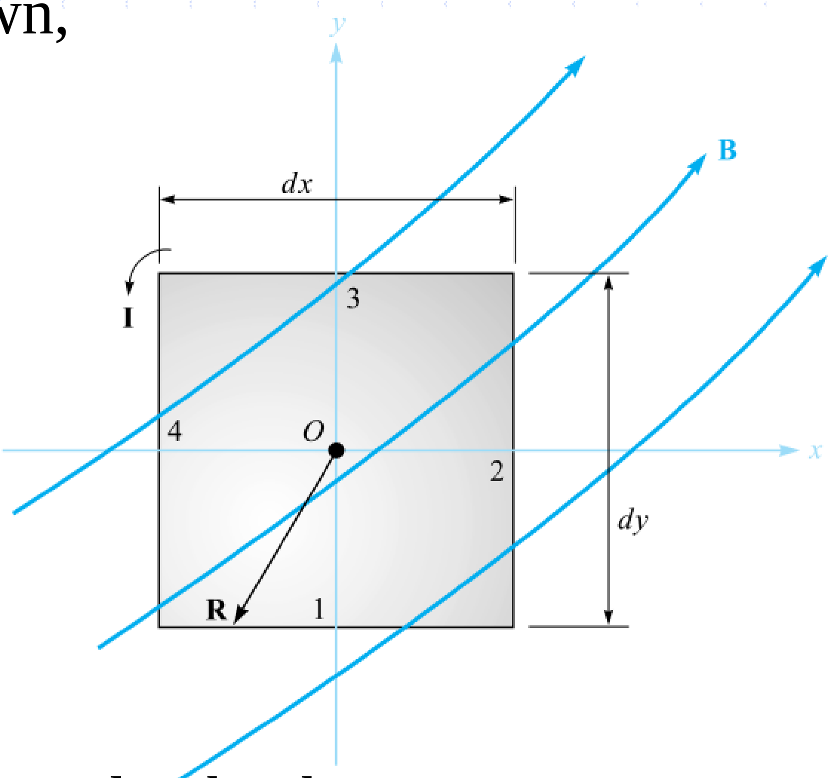
- For the differential current loop shown,

1. The force and torque on side 1

$$d\mathbf{F}_1 = I\,dx\,\mathbf{a}_x \times \mathbf{B}_0$$

or

$$d\mathbf{F}_1 = I\,dx(B_{0y}\mathbf{a}_z - B_{0z}\mathbf{a}_y)$$



- The contribution to the total torque is

$$\begin{aligned} d\mathbf{T}_1 &= \mathbf{R}_1 \times d\mathbf{F}_1 \\ &= -\tfrac{1}{2}dy\,\mathbf{a}_y \times I\,dx(B_{0y}\mathbf{a}_z - B_{0z}\mathbf{a}_y) \\ &= -\tfrac{1}{2}dx\,dy\,I\,B_{0y}\mathbf{a}_x \end{aligned}$$

- The torque contribution on side 3 is found to be the same,

$$\begin{aligned} d\mathbf{T}_3 &= \mathbf{R}_3 \times d\mathbf{F}_3 = \tfrac{1}{2}dy\,\mathbf{a}_y \times (-I\,dx\,\mathbf{a}_x \times \mathbf{B}_0) \\ &= -\tfrac{1}{2}dx\,dy\,IB_{0y}\mathbf{a}_x = d\mathbf{T}_1 \end{aligned}$$

$$d\mathbf{T}_1 + d\mathbf{T}_3 = -dx\,dy\,IB_{0y}\mathbf{a}_x$$

Torque on Differential current loop

- The Torque on side 2 and 4

$$dT_2 + dT_4 = dx\,dy\,IB_{0x}\mathbf{a}_y$$

- The total torque is then

$$d\mathbf{T} = I\,dx\,dy(B_{0x}\mathbf{a}_y - B_{0y}\mathbf{a}_x)$$

- The quantity within the parentheses may be represented by a cross product,

$$d\mathbf{T} = I\,dx\,dy(\mathbf{a}_z \times \mathbf{B}_0)$$

or

$$\boxed{d\mathbf{T} = I\,d\mathbf{S} \times \mathbf{B}}$$

- The Magnetic dipole moment $\boxed{d\mathbf{m} = I\,d\mathbf{S}}$

- The differential Torque due to magnetic and electric field in any loop $\boxed{d\mathbf{T} = d\mathbf{m} \times \mathbf{B}}$ $d\mathbf{T} = d\mathbf{p} \times \mathbf{E}$

- The torque on a planar loop of any size or any shape in **uniform field** B is $\boxed{\mathbf{T} = I\mathbf{S} \times \mathbf{B} = \mathbf{m} \times \mathbf{B}}$

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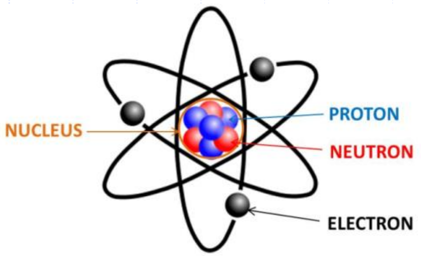
Nature of the Magnetic Material



- Each atom contains **three different type of moments** components and their contribution determine the magnetic characteristics of the material and provide its general magnetic classifications:
  1. **Moment due to electron motion in its orbits:** an electron in an orbits is a analogues to small current loop in which the current is directed opposite to direction of electron travelled and thus **experience a torque** in an external fields
  2. **Moment due to electron spin**, this phenomena can be modeled by considering electron spinning about its own axis
  3. **Moment due to nuclear spin**, provides a negligible effect on the overall magnetic properties but it is the basis of nuclear magnetic resonance imaging (MRI) producer

Types of Magnetic materials

- There are six types of magnetic materials: diamagnetic, paramagnetic, ferromagnetic, antiferromagnetic, ferrimagnetic and supper paramagnetic

Classification	Magnetic Moments	B Values	Comments
Diamagnetic	$\mathbf{m}_{\mathrm{orb}} + \mathbf{m}_{\mathrm{spin}} = 0$	$B_{\mathrm{int}} < B_{\mathrm{appl}}$	$B_{\mathrm{int}} \doteq B_{\mathrm{appl}}$
Paramagnetic	$\mathbf{m}_{\mathrm{orb}} + \mathbf{m}_{\mathrm{spin}} = \mathrm{small}$	$B_{\mathrm{int}} > B_{\mathrm{appl}}$	$B_{\mathrm{int}} \doteq B_{\mathrm{appl}}$
Ferromagnetic	$\lvert\mathbf{m}_{\mathrm{spin}}\rvert \gg \lvert\mathbf{m}_{\mathrm{orb}}\rvert$	$B_{\mathrm{int}} \gg B_{\mathrm{appl}}$	Domains
Antiferromagnetic	$\lvert\mathbf{m}_{\mathrm{spin}}\rvert \gg \lvert\mathbf{m}_{\mathrm{orb}}\rvert$	$B_{\mathrm{int}} \doteq B_{\mathrm{appl}}$	Adjacent moments oppose
Ferrimagnetic	$\lvert\mathbf{m}_{\mathrm{spin}}\rvert \gg \lvert\mathbf{m}_{\mathrm{orb}}\rvert$	$B_{\mathrm{int}} > B_{\mathrm{appl}}$	Unequal adjacent moments oppose; low σ
Superparamagnetic	$\lvert\mathbf{m}_{\mathrm{spin}}\rvert \gg \lvert\mathbf{m}_{\mathrm{orb}}\rvert$	$B_{\mathrm{int}} > B_{\mathrm{appl}}$	Nonmagnetic matrix; recording tapes

Diamagnetic Material

- The magnetic field produces from the electron motion and electron spin are cancel each other (**net field is zero**).
- The permanent magnetic moment of each atom is zero.
- The **external magnetic field** will produce no torque on the atom and no realignment of the dipole field.
- Metallic bismuth shows great diamagnetic than hydrogen , helium and other inherent gases.

Paramagnetic Material

- The field from electron spin and electron motion don't cancel each other **(net field is small).**
- Each atom has a small magnetic field.
- The field due to random variation of atom in a large sample **is zero**.
- **No magnetic effect** in the absence of the external magnetic field.
- With the external field, there is a small torque in each atom (atomic moment), this moment tend to alignment with the external field. This alignment tend to increase B within the material over the external field
- The net result is the **increase the value** of B in the material
- Such as : potassium, oxygen

Ferromagnetic Material

- Each atom has a **relatively large dipole moment** produces from the uncompensated electron motion and electron spin.
- Interatomic forces cause theses moments to **line up in parallel fashion** over regions contains a large number of atoms called domain.
- The domains moment **vary in direction** from one domain to other.
- The **overall effect** cancel each other and the material as a whole has no magnetic moment.
- The **applied external field** increase the size of the domains that have a field in the direction of the applied field and the internal field increase than the external field.
- When the field is removed a completely random domain alignment is not achieved and a **dipole field remain** in the material structure (Hysteresis)
- **Hysteresis**, means that the magnetic status of the material is a function of its magnetic history
- Ferromagnetic material is **not isotropic**
- The only elements that are ferromagnetic are: iron, nickel and cobalt

Antiferromagnetic Material

- Each atom has a **large dipole moment.**
- The force between adjacent atom cause the atomic moment to **line up in antiparallel** fashion and the net magnetic moment is zero.
- The **external field** has a small effect on it.
- Such as, manganese oxide, nickel oxide and cobalt chloride

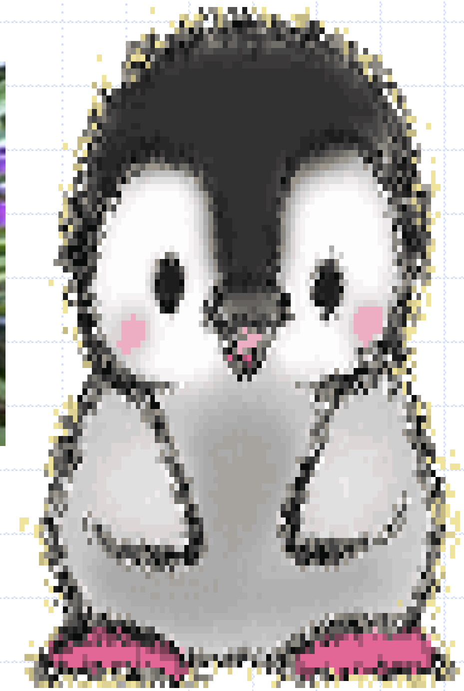
Ferrimagnetic Material

- Each atom has a **large dipole moment.**
- The moment between the adjacent atom is **not zero.**
- The **external field** has large effect but not as ferromagnetic materials
- Such as, ferrite, iron oxide magnetite, nickel-zinc ferrite and nickel ferrite

Superparamagnetic Material

- Each atom has a **large dipole moment.**
- Composed of **assemble of ferromagnetic material** particle in **non ferromagnetic matrix.**
- Such as magnetic tape.

Thank you



Have a nice day.