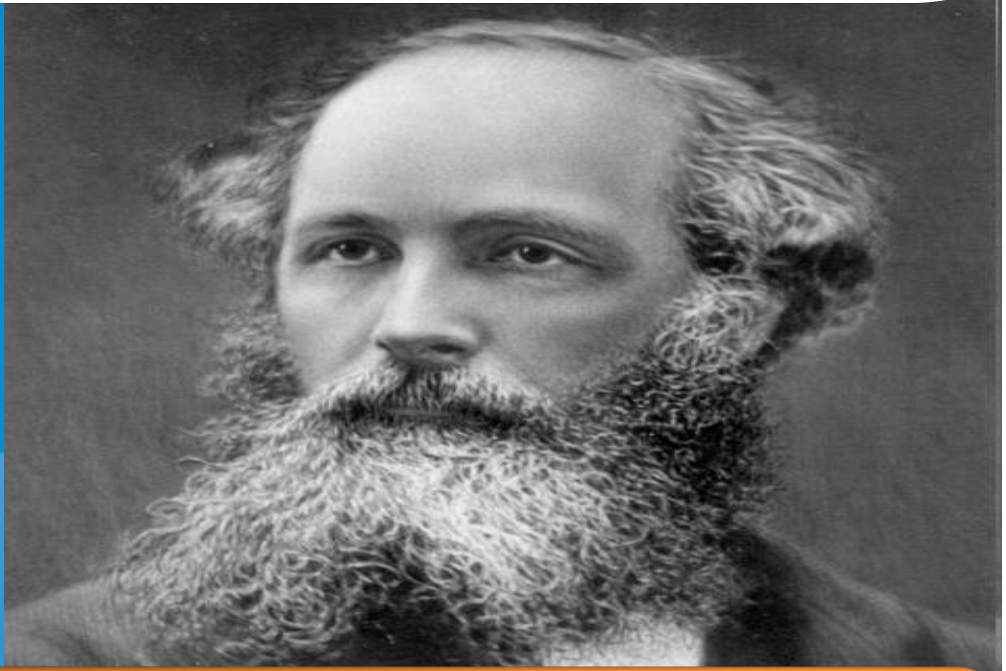
Lecture 6

The Uniform Plane Wave

Electromagnetic Field Theory



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Introduction

- Use Maxwell's Equation to introduce the fundamental theory of **wave motions.**
- The **uniform plane wave** is the simplest form of wave.
- Calculate the **speed** of wave propagation and the **attenuation**.
- Use the **Poynting Vector** to calculate the power.
- Learn how to specify the **polarization** of waves.
- This chapter is the basics for wave reflection, transmission line theory ,waveguide and antenna.

WAVE PROPAGATION IN FREE SPACE

- The medium is **sourceless** ($\rho_v = J = 0$)
- Maxwell's equations

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

- We assume the existence of a **uniform plane wave**,
  - both **E** and **H** lie in the transverse plane—that is, the plane whose normal is the direction of propagation.
  - both **E** and **H** are of constant magnitude in the transverse plane.
    - For this reason, such a wave is sometimes called a **transverse electromagnetic (TEM)** wave.
- The required spatial variation of both fields in the direction normal to their orientations will therefore occur only in the **direction of travel**—or normal to the transverse plane.

WAVE PROPAGATION IN FREE SPACE

- Assume, that
  - $\mathbf{E} = E_x \mathbf{a}_x$, i.e., electric field is polarized in the ***x* direction**.
  - wave travel is in the z direction, i.e., spatial variation of **E** is only with z.
  - Then $\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$ becomes

$$\nabla \times \mathbf{E} = \frac{\partial E_x}{\partial z}\mathbf{a}_y = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} = -\mu_0 \frac{\partial H_y}{\partial t}\mathbf{a}_y \qquad (5)$$

- Using the ***y*-directed magnetic field**, and the fact that it varies only in z, then

$$\nabla \times \mathbf{H} = -\frac{\partial H_y}{\partial z}\mathbf{a}_x = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \epsilon_0 \frac{\partial E_x}{\partial t}\mathbf{a}_x \qquad (6)$$

- The last two equations can be written as

$$\frac{\partial E_x}{\partial z} = -\mu_0 \frac{\partial H_y}{\partial t} \qquad (7)$$

$$\frac{\partial H_y}{\partial z} = -\epsilon_0 \frac{\partial E_x}{\partial t} \qquad (8)$$

WAVE PROPAGATION IN FREE SPACE

- Differentiate $\frac{\partial E_x}{\partial z} = -\mu_0 \frac{\partial H_y}{\partial t}$ **with respect to *z***, obtaining:

$$\frac{\partial^2 E_x}{\partial z^2} = -\mu_0 \frac{\partial^2 H_y}{\partial t \partial z} \qquad (9)$$

- Then, $\frac{\partial H_y}{\partial z} = -\epsilon_0 \frac{\partial E_x}{\partial t}$ is differentiated **with respect to *t***:

$$\frac{\partial^2 H_y}{\partial z \partial t} = -\epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \qquad (10)$$

- Substituting (10) into (9) results in

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \qquad (11)$$

The wave equation for ***x*-polarized TEM** electric field in free space

- From Eq. (11), we further identify the **propagation velocity**:

$$\nu = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s} = c \qquad (12)$$

where *c* denotes the velocity of light in free space.

WAVE PROPAGATION IN FREE SPACE

- A similar procedure, involving differentiating (7) with t and (8) with z, yields the wave equation for the **magnetic field**:

$$\frac{\partial^2 H_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 H_y}{\partial t^2} \qquad (13)$$

- The solution to equations of the form of (11) and (13) will be forward- and backward-propagating waves having the general form

$$E_x(z, t) = f_1(t - z/\nu) + f_2(t + z/\nu)$$

  - where f_1 and f_2 can be any function whose argument is of the form $t \pm z/\nu$.

- The solution to (11) in the form of forward- and backward-propagating cosines.

$$\begin{aligned} E_x(z, t) &= \mathcal{E}_x(z, t) + \mathcal{E}'_x(z, t) \\ &= |E_{x0}| \cos[\omega(t - z/\nu_p) + \phi_1] + |E'_{x0}| \cos[\omega(t + z/\nu_p) + \phi_2] \\ &= \underbrace{|E_{x0}| \cos[\omega t - k_0 z + \phi_1]}_{\text{forward } z \text{ travel}} + \underbrace{|E'_{x0}| \cos[\omega t + k_0 z + \phi_2]}_{\text{backward } z \text{ travel}} \qquad (15) \end{aligned}$$

In free space, phase velocity, $\nu_p = c$

wavenumber in free space in defined as $k_0 \equiv \frac{\omega}{c}$ rad/m

WAVE PROPAGATION IN FREE SPACE

- **ω** is the radian time frequency, measuring phase shift per unit time; it has units of rad/s.
- ***k_0*** the spatial frequency, which measures the phase shift per unit distance along the z direction in rad/m.
- The **wavelength** in free space is the distance over which the spatial phase shifts by **2π radians**, assuming fixed time, or

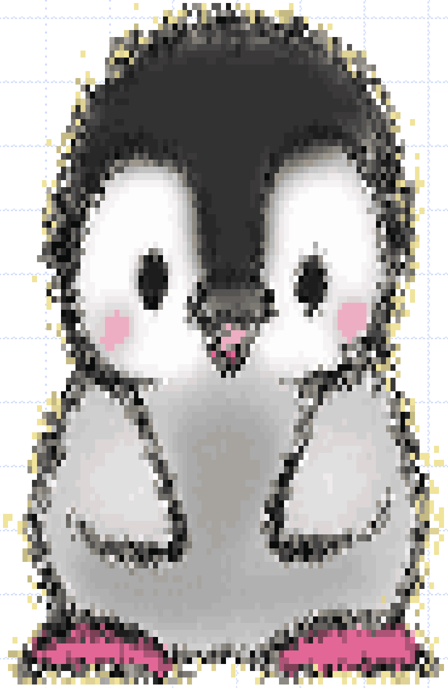
$$k_0 z = k_0 \lambda = 2\pi \quad \rightarrow \quad \boxed{\lambda = \frac{2\pi}{k_0} \quad \text{(free space)}}$$

- The real instantaneous fields of Eq. (15) in terms of their phasor forms.

$$\mathcal{E}_x(z,t) = \frac{1}{2}\underbrace{|E_{x0}|e^{j\phi_1}}_{E_{x0}} e^{-jk_0 z} e^{j\omega t} + c.c. = \frac{1}{2} E_{xs} e^{j\omega t} + c.c. = \text{Re}[E_{xs} e^{j\omega t}] \qquad (19)$$

where ***c.c.*** denotes the **complex conjugate**, and where we identify the *phasor electric field* as $E_{xs} = E_{x0} e^{-jk_0 z}$.

Thank you



Have a nice day.