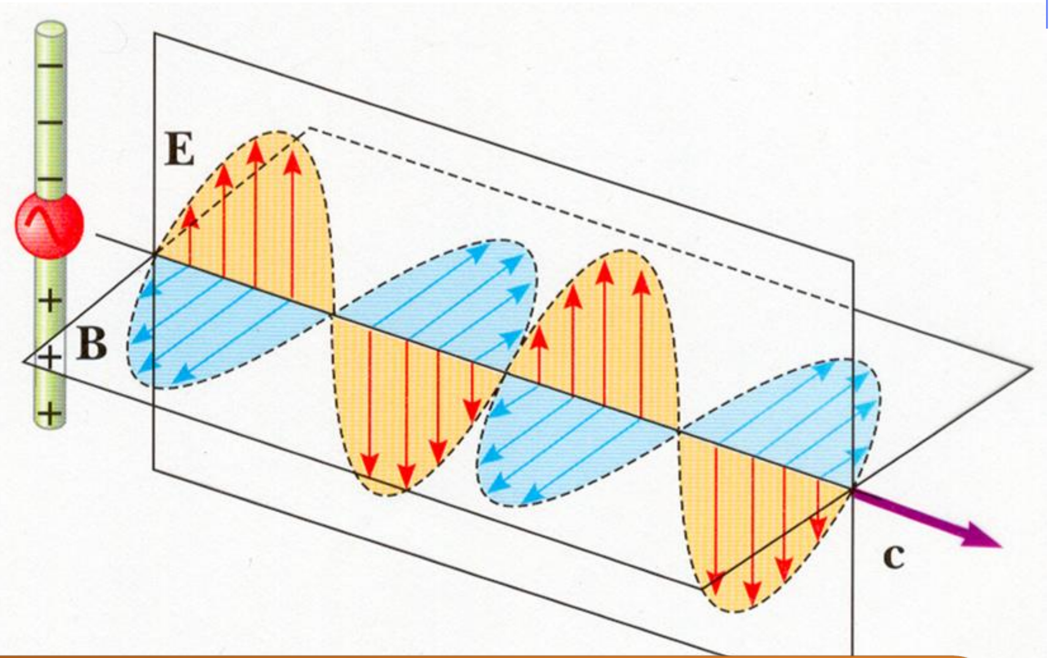
Lecture 9

Wave Polarization

Electromagnetic Field Theory



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WAVE POLARIZATION

- The **wave polarization** is defined as its electric field vector orientation as a function of time at fixed position in space

1. Linearly polarized wave,

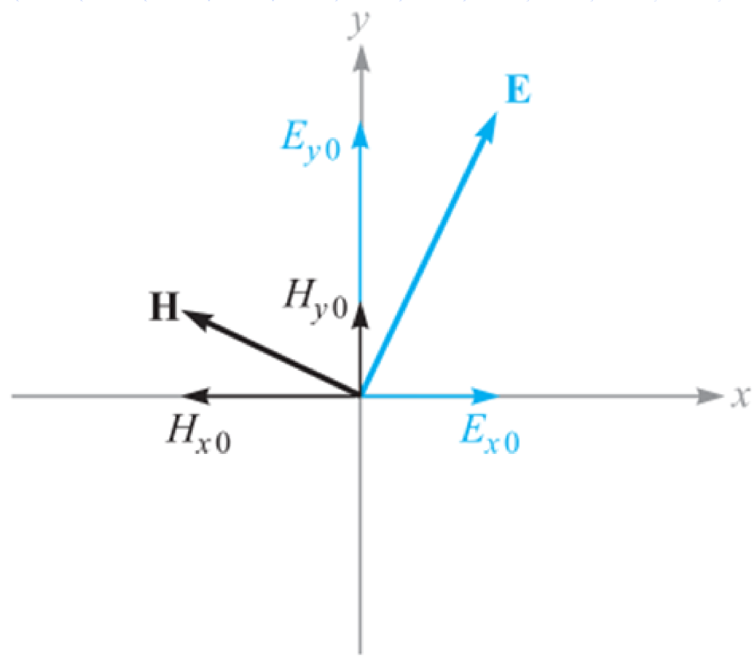
- **E** is in a fixed straight orientation for all times and positions.
- For +ve z propagation, the wave would in general have its **electric field phasor** expressed as

$$\mathbf{E}_s = \left(E_{x0}\mathbf{a}_x + E_{y0}\mathbf{a}_y\right)e^{-\alpha z}\, e^{-j\beta z}$$

- where E_{x0} and E_{y0} are constant amplitudes along x and y.
- The **magnetic field** is readily found by determining its x and y components directly from those of E_s.

$$\mathbf{H}_s = \left[H_{x0}\mathbf{a}_x + H_{y0}\mathbf{a}_y\right]e^{-\alpha z}\, e^{-j\beta z} = \left[-\frac{E_{y0}}{\eta}\mathbf{a}_x + \frac{E_{x0}}{\eta}\mathbf{a}_y\right]e^{-\alpha z}\, e^{-j\beta z}$$

WAVE POLARIZATION



- The figure demonstrates the reason for the minus sign in the term involving E_{y0} .
- The direction of power flow, given by $\mathrm{E} \times \mathrm{H}$, is in the +ve z direction in this case.
  - A component of E in the positive y direction would require a component of H in the negative x direction—thus the minus sign.

WAVE POLARIZATION

- The **power density** in the wave is

$$\langle S_z \rangle = \frac{1}{2}\mathrm{Re}\{\mathbf{E}_s \times \mathbf{H}_s^*\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{E_{x0}H_{y0}^*\left(\mathbf{a}_x \times \mathbf{a}_y\right) + E_{y0}H_{x0}^*\left(\mathbf{a}_x \times \mathbf{a}_y\right)\right\}e^{-2\alpha z}$$

$$= \frac{1}{2}\mathrm{Re}\left\{\frac{E_{x0}E_{x0}^*}{\eta^*} + \frac{E_{y0}E_{y0}^*}{\eta^*}\right\}e^{-2\alpha z}\mathbf{a}_z$$

$$= \frac{1}{2}\mathrm{Re}\left\{\frac{1}{\eta^*}\right\}\left(|E_{x0}|^2 + \left|E_{y0}\right|^2\right)e^{-2\alpha z}\mathbf{a}_z \quad \mathrm{W/m^2}$$

- This result demonstrates the idea that a **linearly polarized plane wave** can be considered as two distinct plane waves having x and y polarizations, whose electric fields are combining in phase to produce the total E.
- The same is true for the magnetic field components.

WAVE POLARIZATION

- Any **polarization state** can be described in terms of mutually perpendicular components of the electric field and their relative phasing.
- We next consider the effect of a phase difference, φ, between E_{x0} and E_{y0}, where $\varphi < \pi/2$.
- Consider propagation in a lossless medium

$$\mathbf{E}_s = \left(E_{x0}\mathbf{a}_x + E_{y0}\mathbf{a}_y\right)e^{-j\beta z}$$

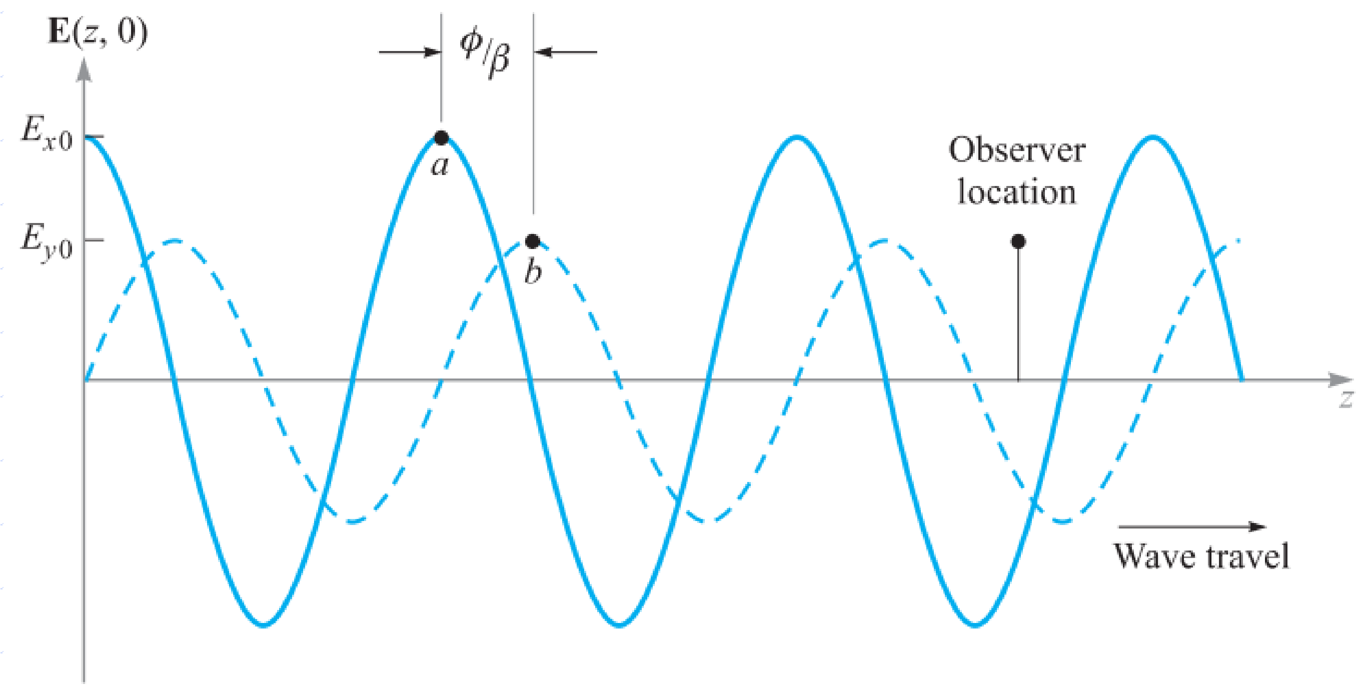
- The real instantaneous form

$$E(z,t) = E_{x0}\cos(\omega t - \beta z)\,\mathbf{a}_x + E_{y0}\cos(\omega t - \beta z + \phi)\,\mathbf{a}_y$$

- E_{x0} and E_{y0} are real.
- Set $t = 0$, and using $\cos(-x) = \cos(x)$

$$E(z,0) = E_{x0}\cos(\beta z)\,\mathbf{a}_x + E_{y0}\cos(\beta z - \phi)\,\mathbf{a}_y$$

WAVE POLARIZATION



- The component magnitudes of $E(z,0)$ are plotted as functions of z.
- Since time is fixed at zero, the wave is frozen in position.
- Consider a crest of E_x, indicated as **point a**.
  - If φ were zero, E_y would have a crest at the same location. Since ϕ is not zero (and positive), the crest of E_y that would otherwise occur at **point a** is now displaced to **point b** farther down z.
- The two points are separated by distance φ/β.
- E_y thus **lags behind** E_x when we consider the spatial dimension.

WAVE POLARIZATION

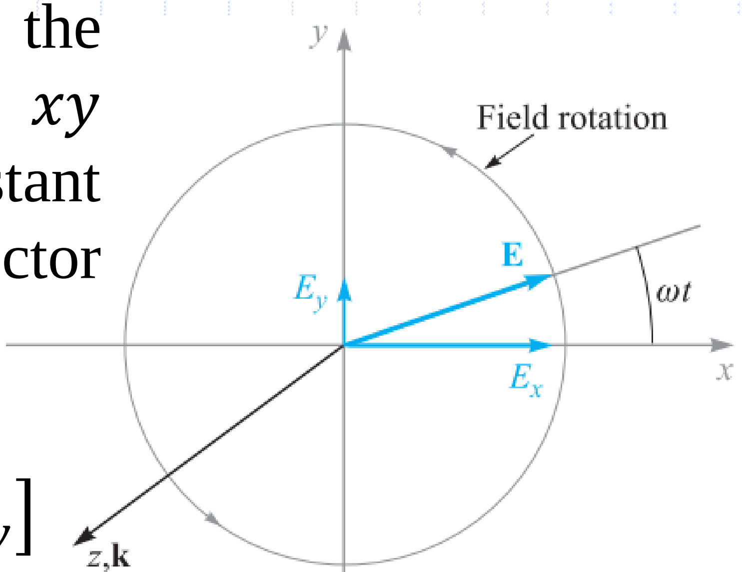
- **Circular polarization:**
- Let $E_{x0} = E_{y0} = E_0$ and $\varphi = \pm\pi/2$.

$$\mathbf{E}(z,t) = E_0\left[\cos(\omega t - \beta z)\,\mathbf{a}_x + \cos(\omega t - \beta z \pm \pi/2)\,\mathbf{a}_y\right]$$
$$= E_0\left[\cos(\omega t - \beta z)\,\mathbf{a}_x \mp \sin(\omega t - \beta z)\,\mathbf{a}_y\right]$$

- If we consider a fixed position along z (such as $z = 0$) and allow time to vary, with $\varphi = -\pi/2$, becomes

$$\mathbf{E}(0,t) = E_0\left[\cos(\omega t)\,\mathbf{a}_x + \sin(\omega t)\,\mathbf{a}_y\right]$$

- The field vector rotates in the **counterclockwise direction** in the xy plane, while maintaining constant amplitude E_0, and so the tip of the vector traces out a **circle**.



- with $\varphi = +\pi/2$

$$\mathbf{E}(0,t) = E_0\left[\cos(\omega t)\,\mathbf{a}_x - \sin(\omega t)\,\mathbf{a}_y\right]$$

- The field vector rotates in the **clockwise direction**.

WAVE POLARIZATION

- The wave exhibits **left circular polarization** (**l.c.p.**) if, when orienting the left hand with the thumb in the direction of propagation, the fingers curl in the rotation direction of the field with time.
- The wave exhibits **right circular polarization** (**r.c.p.**) if, with the right-hand thumb in the propagation direction, the fingers curl in the field rotation direction.

$$\mathbf{E}(0,t) = E_0\left[\cos(\omega t)\,\mathbf{a}_x - \sin(\omega t)\,\mathbf{a}_y\right]$$ left circularly polarized wave

$$\mathbf{E}(0,t) = E_0\left[\cos(\omega t)\,\mathbf{a}_x + \sin(\omega t)\,\mathbf{a}_y\right]$$ right circularly polarized wave

$$\mathbf{E}(z,t) = E_0\left[\cos(\omega t - \beta z)\,\mathbf{a}_x + \cos(\omega t - \beta z \pm \pi/2)\,\mathbf{a}_y\right]$$
$$= E_0\left[\cos(\omega t - \beta z)\,\mathbf{a}_x \mp \sin(\omega t - \beta z)\,\mathbf{a}_y\right]$$

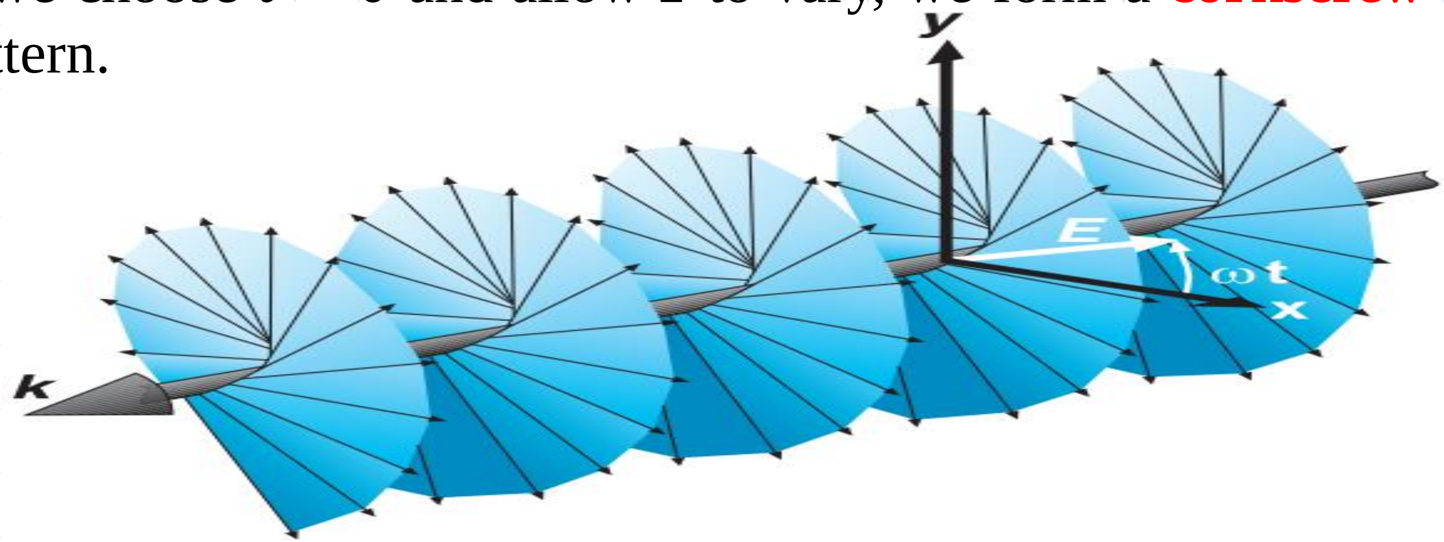
- The instantaneous angle of the field from the x direction

$$\theta(z,t) = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{\mp\sin(\omega t - \beta z)}{\cos(\omega t - \beta z)}\right) = \mp(\omega t - \beta z)$$

WAVE POLARIZATION

$$\theta(z,t) = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{\mp\sin(\omega t - \beta z)}{\cos(\omega t - \beta z)}\right) = \mp(\omega t - \beta z)$$

- where again the **minus sign** (yielding **l.c.p**. for +ve z travel) applies for the choice of $\varphi = +\pi/2$; the **plus sign** (yielding **r.c.p.** for +ve z travel) is used if $\varphi = -\pi/2$.
- If we choose $z = 0$, the angle becomes simply ωt, which reaches 2π (one complete rotation) at time $t = 2\pi/\omega$.
- If we choose $t = 0$ and allow z to vary, we form a **corkscrew-like** field pattern.



Representation of a right circularly polarized wave.

WAVE POLARIZATION

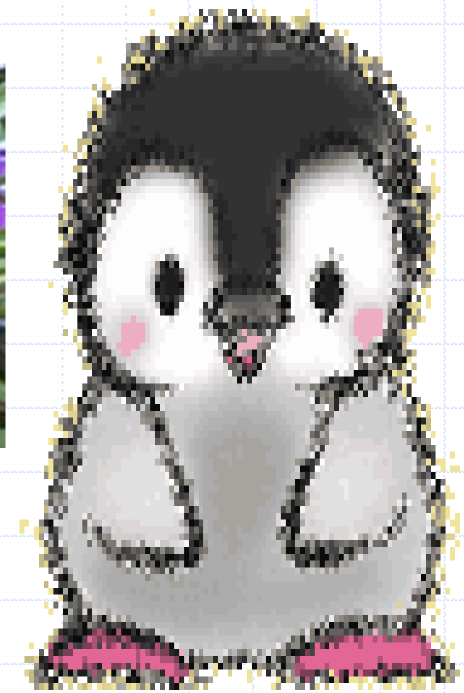
- **Elliptically polarized wave**
- Elliptical Polarization can be attained only if the phase difference between the two components is **odd multiple of $\pi/2$** and their magnitudes **are not equal** or the phase difference is **not multiple of $\pi/2$**.

$$\mathbf{E}_s = \left(E_{x0}\mathbf{a}_x + E_{y0}e^{j\phi}\mathbf{a}_y\right)e^{-j\beta z}$$

$$E(z,t) = E_{x0}\cos(\omega t - \beta z)\,\mathbf{a}_x + E_{y0}\cos(\omega t - \beta z + \phi)\,\mathbf{a}_y$$

$$E(z,0) = E_{x0}\cos(\beta z)\,\mathbf{a}_x + E_{y0}\cos(\beta z - \phi)\,\mathbf{a}_y$$

Thank you



Have a nice day.