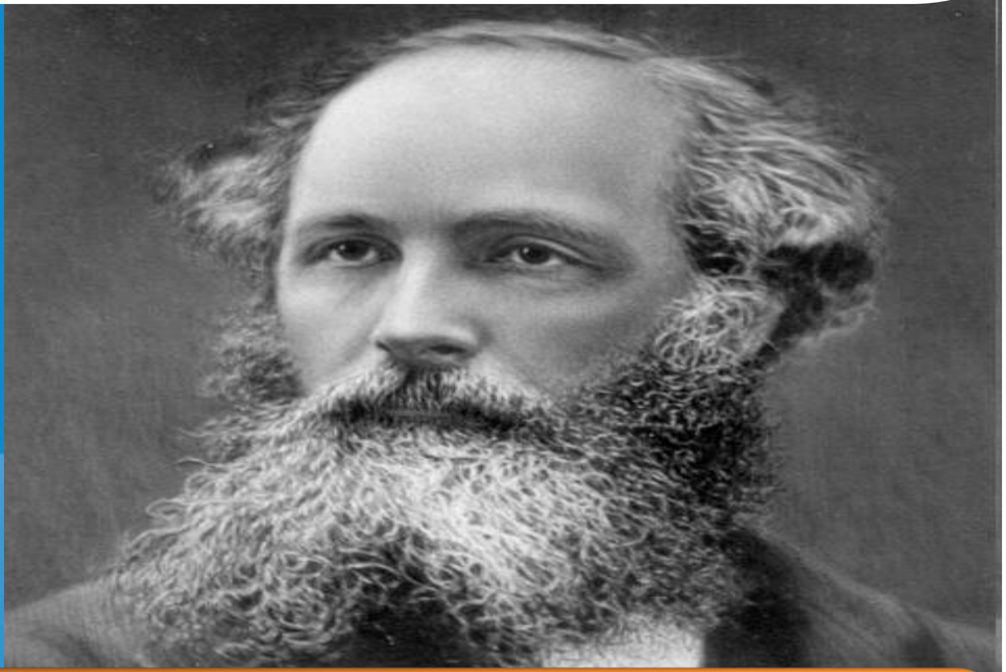
Lecture 5

Time-Varying Fields and Maxwell's Equations

Electromagnetic Field Theory



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Introduction

Time-Varying Fields

Stationary charges → electrostatic fields

Steady currents → magnetostatic fields

Time-varying currents → electromagnetic fields

Only in a **non-time-varying case** can electric and magnetic fields be considered as **independent** of each other. In a **time-varying** (dynamic) case the two fields are **interdependent**.

A changing magnetic field induces an electric field, and vice versa.

Introduction

- In the previous chapter, the steady magnetic field was explained. In this chapter the time varying field will be explained.
- Some of the relationships are not change and most of the relationships are change slightly
- Two concepts will be introduced
  1. Electric field produce by changing magnetic field (Faraday's Law)
  2. Magnetic field produce by changing electric field (Maxwell’s)

Faraday's Law

- **Faraday's goal** was to show that a current could be produced by **magnetism**.
- A time varying magnetic field produces an ***electromotive force* (emf)** which may establish a current in a suitable closed circuit.
- An **emf** is a voltage that arise from conductors moving in a magnetic field or from changing magnetic fields $\text{emf} = -\frac{d\phi}{dt}$ V.
- The above equation required a **closed path** not necessary a **closed conducting path**.
- A non zero value of $\frac{d\Phi}{dt}$ will produces from:
  1. A time varying flux linking a stationary clothed path.
  2. Relative motion between a steady flux and a closed path.
  3. A combination of two.

Faraday's Law

- The **minus sign** is an indication that the emf is in such a direction as to produce a current whose flux, if added to the original flux, would reduce the magnitude of the emf.
  - **Lenz's Law**, states that the induced voltage acts to produces an opposing flux.
- For N turns closed path $\quad \text{emf} = -N\dfrac{d\phi}{dt}$
- The emf in terms of the **electric field** $\quad \text{emf} = \oint \mathbf{E}\cdot d\mathbf{L}$
- For **steady field** the above integral is **zero**.
- Using the equation of $\Phi = \int_S \mathbf{B}\cdot d\mathbf{S} \quad \Rightarrow \text{emf} = \oint \mathbf{E}\cdot d\mathbf{L} = -\dfrac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{S}$
- **Right hand fingers** indicates the direction of path and the **thumb** indicates the direction of $d\mathbf{S}$ which is the same as the direction of **B**.

Faraday's Law

One of Maxwell's equation in Integral form

- For stationary path in time varying fields emf $= \oint \mathbf{E} \cdot d\mathbf{L} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$
- Using the **Stocks theorem**, the equation can be write as

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \quad \Rightarrow (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \quad \Rightarrow \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- For **steady field** the above equations were written as

One of Maxwell's equation in point form

$$\oint \mathrm{E} \cdot d\mathrm{L} = 0 \qquad \Rightarrow \qquad \nabla \times \mathrm{E} = 0$$

- For time constant flux and moving path, we define a **motional electric field** as

$$\mathbf{F} = Q\mathrm{v} \times \mathbf{B} \Rightarrow \frac{\mathbf{F}}{Q} = \mathrm{v} \times \mathbf{B} \Rightarrow \mathrm{E}_m = \mathrm{v} \times \mathrm{B}$$

The ***motional emf*** produced by the moving conductor is then

$$\text{emf} = \oint \mathrm{E} \cdot d\mathrm{L} = \oint \mathrm{E}_m \cdot d\mathrm{L} = \oint (\mathrm{v} \times \mathrm{B}) \cdot d\mathrm{L}$$

- For **time varying** field and **moving path**

$$\text{emf} = \oint \mathbf{E} \cdot d\mathbf{L} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint (\mathrm{v} \times \mathbf{B}) \cdot d\mathbf{L} = -\frac{d\Phi}{dt}$$

Displacement Current

- From **Ampere's circuit law** in point form $\nabla \times \mathbf{H} = \mathbf{J}$
- Taking the **divergence** of both sides $\nabla \cdot \nabla \times \mathbf{H} \equiv 0 = \nabla \cdot \mathbf{J}$
- Using the continuity equation $\mathbf{J} = -\frac{\partial \rho_v}{\partial t}$
- So the point form of Ampere is valid only when $\partial \rho_v / \partial t = 0$
- For time varying field, assume we added an unknown as G

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{G} \quad \Rightarrow \quad \mathbf{0} = \boldsymbol{\nabla} \cdot \boldsymbol{J} + \boldsymbol{\nabla} \cdot \boldsymbol{G} \quad \Rightarrow \quad \boldsymbol{\nabla} \cdot \boldsymbol{G} = \frac{\partial \rho_v}{\partial t}$$

Replacing ρ_v by $\nabla \cdot D$ $\Rightarrow \boldsymbol{\nabla} \cdot \boldsymbol{G} = \frac{\partial}{\partial t}(\nabla \cdot D) = \boldsymbol{\nabla} \cdot \frac{\partial D}{\partial t} \quad \Rightarrow \quad \boldsymbol{G} = \frac{\partial D}{\partial t}$

$$\Rightarrow \quad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial D}{\partial t}$$

Second one of Maxell's equations

The Displacement current density is $J_d = \frac{\partial D}{\partial t} \quad \Rightarrow \nabla \times \mathbf{H} = \mathbf{J} + J_d$

Types of Current

- There are three types for current density
  1. Conduction current density $\mathbf{J} = \sigma \mathbf{E}$
  2. Displacement current density $\mathbf{J}_d = \frac{d\mathbf{D}}{dt}$
  3. Convection current density $\mathbf{J} = \rho_v \mathbf{v}$
- For non conduction medium $\mathbf{J} = \sigma \mathbf{E} \Rightarrow \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$ compare with $\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t}$
- The total displacement current crossing any surfaces is

$$I_d = \int_S \mathbf{J}_d \cdot d\mathbf{S} = \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$$

- The time varying form of Ampere circuit law is found as follow

$$\int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S} = \int_S \mathbf{J} \cdot d\mathbf{S} + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$$

Applying Stocks Theorem

$$\oint \mathbf{H} \cdot d\mathbf{L} = I + I_d = I + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$$

Maxwell's Equation in Point Form

- The changed equations from steady case are

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

- The unchanged equations are

$$\nabla \cdot \mathbf{D} = \rho_v \qquad \nabla \cdot \mathbf{B} = 0 \qquad \mathbf{J} = \sigma \mathbf{E}$$

- The charge density is the source or sink of changed flux lines
- Magnetic charges are not known
- The **auxiliary equations** are

$$\mathbf{D} = \epsilon \mathbf{E} \qquad \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \qquad \mathbf{P} = \chi_e \epsilon_0 \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H} \qquad \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \qquad \mathbf{M} = \chi_m \mathbf{H}$$

- The **Lorentz force per unit volume** is $\mathrm{f} = \rho_v (\mathbf{E} + \mathrm{v} \times \mathbf{B})$

Maxwell's Equation in Integral Form

- The integral form are found from differential forms and applying special theorem

$$\oint \mathbf{H} \cdot d\mathbf{L} = I + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S} \qquad \oint \mathbf{E} \cdot d\mathbf{L} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

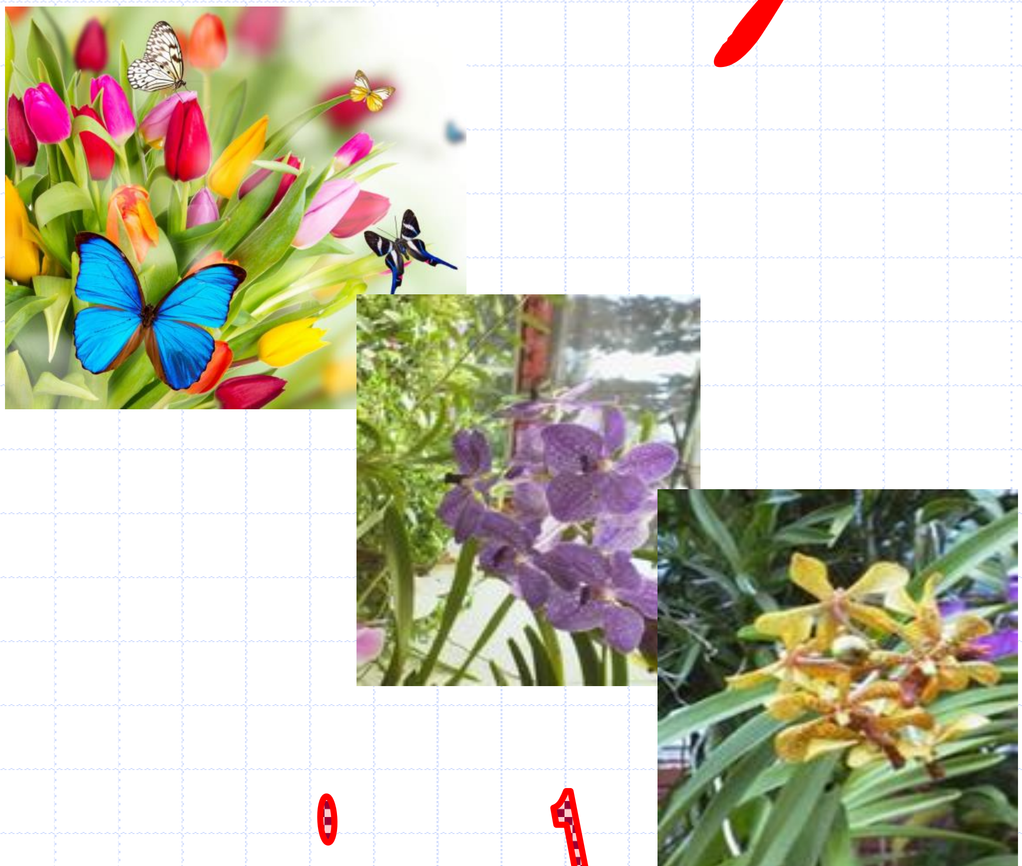
$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 \qquad \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_{\text{vol}} \rho_v dv$$

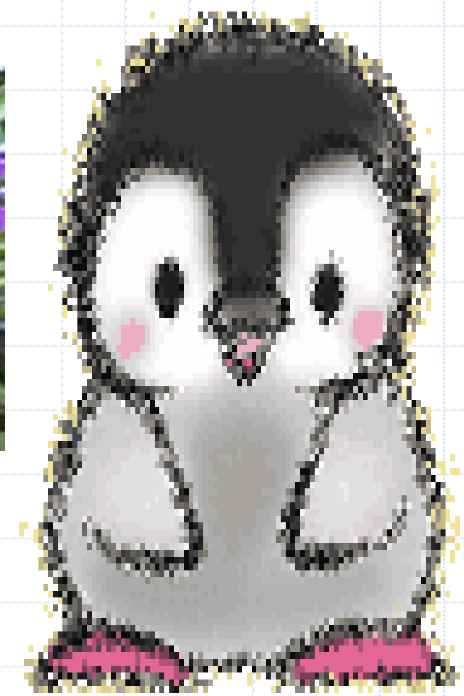
- The boundary conditions are

$$E_{t1} = E_{t2} \qquad H_{t1} = H_{t2}$$

$$D_{N1} - D_{N2} = \rho_S \qquad B_{N1} = B_{N2}$$

Thank you





Have a nice day!